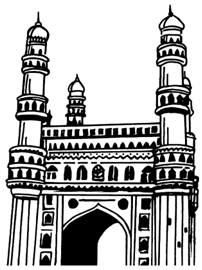
Rahul's ✔
Topper's Voice

AS PER CBCS SYLLABUS



B.Sc.

II Year IV Sem

Latest 2021-22 Edition

WAVES & OPTICS

Physics Paper - IV

- **Study Manual**
- **Solved Problems**
- **Short Question & Answers**
- **Multiple Choice Questions**
- **Fill in the blanks**
- **One Mark Answers**
- **Solved Previous Question Papers**
- **Solved Model Papers**

Price ` 159-00

- by -

WELL EXPERIENCED LECTURER

Rahul Publications™
Hyderabad. Ph : 66550071, 9391018098

B.Sc.

II Year IV Sem

WAVES & OPTICS

Physics Paper - IV

Price `. 159-00

Sole Distributors : ☎ : **66550071, Cell : 9391018098**

VASU BOOK CENTRE

Shop No. 3, Beside Gokul Chat, Koti, Hyderabad.

Maternity Hospital Opp. Lane, Narayan Naik Complex, Koti, Hyderabad.
Near Andhra Bank, Subway, Sultan Bazar, Koti, Hyderabad -195.

CONTENTS

WAVES & OPTICS

Physics Paper - IV

STUDY MANUAL

SOLVED PREVIOUS QUESTION PAPERS

SOLVED MODEL PAPERS

SYLLABUS

UNIT - I

Fundamentals of Waves -Transverse wave propagation along a stretched string, general solution of wave equation and its significance, modes of vibration of stretched string clamped at ends, overtones, energy transport, transverse impedance.
Longitudinal vibrations in bars- wave equation and its general solution. Special cases (i) bar fixed at both ends ii) bar fixed at the mid point iii) bar free at both ends iv) bar fixed at one end.
Transverse vibrations in a bar- wave equation and its general solution. Boundary conditions, clamped free bar, free-free bar, bar supported at both ends, Tuning fork.

UNIT - II

Principle of superposition - coherence - temporal coherence and spatial coherence - conditions for Interference of light.
Interference by division of wave front: Fresnel's biprism - determination of wave length of light. Determination of thickness of a transparent material using Biprism - change of phase on reflection - Lloyd's mirror experiment.
Interference by division of amplitude: Oblique incidence of a plane wave on a thin film due to reflected and transmitted light (Cosine law) - Colours of thin films - Non-reflecting films - interference by a plane parallel film illuminated by a point source - Interference by a film with two non-parallel reflecting surfaces (Wedge shaped film) - Determination of diameter of wire - Newton's rings in reflected light with and without contact between lens and glass plate, Newton's rings in transmitted light (Haidinger Fringes) - Determination of wave length of monochromatic light - Michelson Interferometer-types of fringes - Determination of wavelength of monochromatic light, Difference in wavelength of sodium D_1, D_2 lines and thickness of a thin transparent plate.

UNIT - III

Diffraction

Introduction - Distinction between Fresnel and Fraunhofer diffraction, Fraunhofer diffraction:- Diffraction due to single slit and circular aperture - Limit of resolution - Fraunhofer diffraction due to double slit - Fraunhofer diffraction pattern with N slits (diffraction grating).
Resolving Power of grating - Determination of wave length of light in normal and oblique incidence methods using diffraction grating.
Fresnel diffraction - Fresnel's half period zones - area of the half period zones -zone plate - Comparison of zone plate with convex lens - Phase reversal zone plate - diffraction at a straight edge - difference between interference and diffraction.

UNIT - IV

Polarization

Polarized light : Methods of Polarization, Polarization by reflection, refraction, Double refraction, selective absorption, scattering of light - Brewster's law - Nicol prism polarizer and analyzer - Refraction of plane wave incident on negative and positive crystals (Huygen's explanation) - Quarter wave plate, Half wave plate - Babinet's compensator - Optical activity, analysis of light by Laurent's half shade polarimeter.

Contents

UNIT - I

UNIT - II

Frequently Asked & Important Questions

UNIT - I

1. Derive the differential equation for a Transverse Wave Propagation along a Stretched String.

Ans : **(Imp.)**

Refer Unit-I, Q.No. 2

---

2. Obtain a wave equation for longitudinal vibrations in bars and find its general solution.

Ans : **(Imp.)**

Refer Unit-I, Q.No.10

---

3. Explain longitudinal vibrations in a bar free at both the ends.

Ans : **(Imp.)**

Refer Unit-I, Q.No.13

---

4. Explain longitudinal vibrations when the bar is fixed at one end and free at another end.

Ans : **(Imp.)**

Refer Unit-I, Q.No.14

---

5. Explain transverse vibration in a clamped free bar.

Ans : **(Imp.)**

Refer Unit-I, Q.No.18

---

6. Explain transverse vibrations in a Free - Free Bar.

Ans : **(Imp.)**

Refer Unit-I, Q.No.19

---

7. Write a brief note on tuning fork ?

Ans : **(Imp.)**

Refer Unit-I, Q.No.22

UNIT - II

1. Define the principle of Superposition and explain it.

Ans : **(May-18, Imp.)**

Refer Unit-II, Q.No.1

2. Write about coherent sources and its types. Explain briefly.

Ans : **(Jan.-21, Imp.)**

Refer Unit-II, Q.No.2

3. Explain Fresnel's Biprism method for Determination of Wave Length of Light.

Ans : **(Jan.-21, June-19, June-18, Imp.)**

Refer Unit-II, Q.No.4

4. Determine the thickness of a Transparent material using Biprism.

Ans : **(June-19)**

Refer Unit-II, Q.No.5

5. What is meant by phase change on reflection?

Ans : **(June-18, Imp.)**

Refer Unit-II, Q.No.7

6. Explain Lloyd's Mirror experiments. Determine the wavelength of light using it ? Differentiae Lloyd's and Biprism mirror fringes.

Ans : **(June-18)**

Refer Unit-II, Q.No.8

7. Explain Newton Rings in reflected light with contact between lens and Glass plate ?

Ans : **(Jan.-21, June-18, Imp.)**

Refer Unit-II, Q.No.16

8. Describe Michelson's interferometer with a neat diagram.

Ans : **(June-19, June-18, Imp.)**

Refer Unit-II, Q.No.18

9. Determine the difference in wavelength of sodium D_1, D_2 lines ? and Find the thickness of thin transparent plate.

Ans : **(June-19, June-18, Imp.)**

Refer Unit-II, Q.No.20

UNIT - III

1. What are Fresnel's Diffraction assumptions?

Ans : **(June-18)**

Refer Unit-III, Q.No.2

2. Discuss Fraunhofer diffraction due to a single slit. Explain the distribution of intensity of light in the diffraction pattern.

Ans : **(June-19, June-18)**

Refer Unit-III, Q.No.4

3. Define limit of resolution and obtain an expression for Rayleigh's criterion.

Ans : **(Jan.-21, June-18, May-18)**

Refer Unit-III, Q.No.6

4. Describe the features of a double slit Fraunhofer's diffraction pattern. What is the effect of increasing the slit separation and wavelength.

Ans : **(Jan.-21)**

Refer Unit-III, Q.No.7

5. Explain diffraction of grating by fraunhofer diffraction pattern with N slits.

Ans : **(May-18, Imp.)**

Refer Unit-III, Q.No.9

6. Explain the concept of Fresnel half period zone plates.

Ans : **(Jan.-21, June-19)**

Refer Unit-III, Q.No.13

7. Describe and explain the phenomenon of diffraction due to a straight edge. Explain why the bands are neither equidistant nor equally illuminated.

Ans : **(Jan-21, June-19, June-18)**

Refer Unit-III, Q.No.16

8. Describe and explain the phenomenon of diffraction due to straight edge. Explain why the bands are neither equidistant not equality illuminated.

Ans : **(June-18, Imp.)**

Refer Unit-III, Q.No.17

UNIT - IV

1. Explain various methods to produce the plane polarized light.

Ans : **(June-19)**

Refer Unit-IV, Q.No.2

2. Explain different types of polarization.

Ans : **(June-19, June-18)**

Refer Unit-IV, Q.No.3

3. State and explain Malus Law.

Ans : **(May-18)**

Refer Unit-IV, Q.No.5

4. Explain about nicol prism as a polarizer and analyzer.

Ans : **(May-18, June-18)**

Refer Unit-IV, Q.No.6

5. What is a wave plate? Explain quarter wave plate and half wave plate.

Ans : **(June-19)**

Refer Unit-IV, Q.No.8

6. Explain the construction and working of Babinet's compensator.

Ans : **(Jan.-21)**

Refer Unit-IV, Q.No.10

7. Define polarimeter. Explain Laurent's half-shade polarimeter.

Ans : **(Jan.-21, June-18, May-18)**

Refer Unit-IV, Q.No.12

UNIT - I

Fundamentals of Waves -Transverse wave propagation along a stretched string, general solution of wave equation and its significance, modes of vibration of stretched string clamped at ends, overtones, energy transport, transverse impedance.

Longitudinal vibrations in bars- wave equation and its general solution. Special cases (i) bar fixed at both ends ii) bar fixed at the mid point iii) bar free at both ends iv) bar fixed at one end.

Transverse vibrations in a bar- wave equation and its general solution. Boundary conditions, clamped free bar, free-free bar, bar supported at both ends, Tuning fork.

1.1 Fundamentals of Waves

Q1. Define wave and give its fundamentals.

Ans :

Wave is a physical disturbance that transmits through a medium and transfers energy from one point to another point without causing any permanent displacement of the medium.

Velocity is defined as distance travelled in one second and is given as,

$$\text{Velocity} = \frac{\text{Distance}}{\text{time}} \quad \text{... (1)}$$

Distance travelled in T sec that is in one time period is referred as wavelength, λ and is given by,

$$\text{Velocity} = \frac{\text{Wavelength } (\lambda)}{\text{Timeperiod (T)}} \quad \text{... (2)}$$

Frequency is given by,

$$\text{Frequency (n)} = \frac{1}{\text{Time period (T)}}$$

Therefore velocity can be written as,

$$\text{Velocity } (\nu) = \lambda \times \frac{1}{T}$$

$$= n\lambda$$

$$\boxed{v = n\lambda} \quad \text{... (3)}$$

Above equation gives the relation between wavelength, frequency and velocity of a wave.

1.1.1 Transverse Wave Propagation along a Stretched String

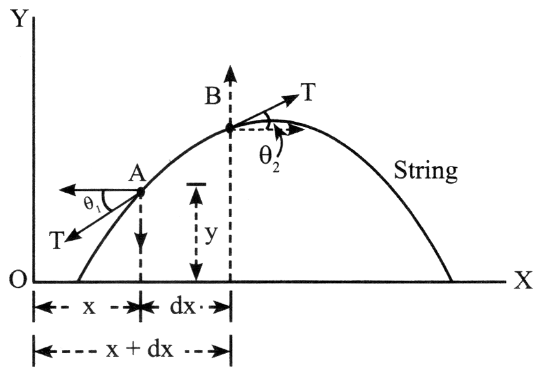
Q2. Derive the differential equation for a Transverse Wave Propagation along a Stretched String.

Ans : **(Imp.)**

If a string stretched between two points is plucked in the direction perpendicular to its length, then transverse vibrations are set up in the string.

Wave Equation

Consider a string element AB of length dx between the coordinates x and x + dx as shown in the figure.



Figure

Let 'y' be the displacement at time 't' and θ_1 and θ_2 be the angles which the tension (T) makes along x-axis.

The components of tension T in respective directions at A and B are as follows,

(i) $T\cos\theta_1$ - Component of tension at A in horizontal direction

(ii) $T\sin\theta_1$ - Component of tension at A in vertical direction

(iii) $T\cos\theta_2$ - Component of tension at B in horizontal direction

(iv) $T\sin\theta_2$ - Component of tension at B in vertical direction.

The horizontal component of tension $T\cos\theta_1$ and $T\cos\theta_2$ are almost equal and balance each other. Therefore, the overall vertical force (F_7) acting in upward direction is given by,

$$F_y = T\sin\theta_2 - T\sin\theta_1 = T[\sin\theta_2 - \sin\theta_1] \quad \text{... (1)}$$

As the displacement AB is small, the angles θ_1 and θ_2 are also small. Therefore,

$$\sin\theta_1 \simeq \tan\theta_1 \simeq \left(\frac{\partial y}{\partial x}\right)_x \quad \text{... (2)}$$

$$\sin\theta_2 \simeq \tan\theta_2 \simeq \left(\frac{\partial y}{\partial x}\right)_{x+dx} \quad \text{... (3)}$$

Substitute equations (2) and (3) in (1),

$$F_y = T\left[\left(\frac{\partial y}{\partial x}\right)_{x+dx} - \left(\frac{\partial y}{\partial x}\right)_x\right] \quad \text{... (4)}$$

Now, expanding the term, $\left(\frac{\partial y}{\partial x}\right)_{x+dx}$ using Taylor's series

$$\left(\frac{\partial y}{\partial x}\right)_{x+dx} = \left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right) dx + \left(\frac{\partial^2 y}{\partial x^3}\right)\frac{(dx)^3}{2!}$$

$$+ \text{...} \left[f(x+\Delta x) = f(x) + f'(x)\Delta x + f'' * (x)\frac{(\Delta x)^2}{2!} + \text{...}\right] \quad \text{... (5)}$$

Neglecting higher powers,

$$\left(\frac{\partial y}{\partial x}\right)_{x+dx} = \left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right) dx \quad \text{... (6)}$$

Substituting equation (6) in equation (4),

$$F_y = T\left[\left\{\left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2}{\partial x^2}\right)dx\right\} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

$$F_y = T\left(\frac{\partial^2 y}{\partial x^2}\right) dx \quad \text{... (7)}$$

Suppose m is the mass per unit length of the wire. Then mass of the element AB will be mdx. Therefore, the force acting on the element AB in upward direction will be,

$$F_y = \text{Mass} \times \text{Acceleration}$$

$$F_y = (mdx) \times \left(\frac{\partial^2 y}{\partial t^2}\right) \qquad \left[\because \text{ Acceleration on element } = \frac{\partial^2 y}{\partial t^2}\right] \quad \text{... (8)}$$

Comparing equations (7) and (8),

$$m\left(\frac{\partial^2 y}{\partial t^2}\right) dx = T\left(\frac{\partial^2 y}{\partial t^2}\right)dx$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{m}\left(\frac{\partial^2 y}{\partial x^2}\right) \quad \text{... (9)}$$

$$\frac{\partial^2 y}{\partial t^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right) \qquad \left[\because \nu = \sqrt{\frac{T}{m}}\right] \quad \text{... (10)}$$

Therefore, equation (10) gives the second order differential wave equation for a transverse wave motion along a stretched string.

From equation (9) and (10) the velocity of transverse wave along the string is given as,

$$v^2 = \frac{T}{m}$$

$$v = \sqrt{\frac{T}{m}}$$

From equation (11) it can be observed that the velocity of transverse vibration of a stretched string is directly proportional to the square root of tension (7) and inversely proportional to the square root of linear mass density (m)

1.1.2 General Solution of Wave Equation

Q3. Derive the general solution of wave equation for a Transverse Wave Propagation along a Stretched String.

Ans : **(Imp.)**

The wave equation for transverse wave motion along a stretched string is given by,

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

Where,

$$v = \sqrt{\frac{T}{m}}$$

Equation (1) has a solution of form,

$$y = f(x, t). \quad \text{... (1)}$$

A wave motion is considered on the string such that every single particle of the string will undergo periodic motion.

In a periodic motion continuous wave pulses should travel one after the other along the length of string at regular interval of time.

Assume that, the left hand of string is at x = 0 and the transverse displacement a sinco/ belongs to it. This transverse displacement produces simple harmonic motion on string with amplitude and angular frequency.

Therefore, at origin of the string, the wave function can be written as,

$$y(0, t) = a \sin\omega t \ \text{...} \quad (2)$$

The particle on right will execute simple harmonic motion but after a time delay. The time delay depends on the distance of this particle from its previous particle. Hence, a phase lag (ϕ) arises between this particle and previous particle. Therefore, the simple harmonic motion of this particle is expressed as,

$$y = A \sin(\omega t - \phi)$$

['–' indicates phase lag] ... (3)

Suppose, this particle is on the left side, it will lead the phase and the motion will be a $\sin(\omega t + \phi)$. The phase angle is proportional to the distance of particle from initially considered particle.

Therefore,

$$\phi \propto x$$

$$\phi \propto kx \quad \text{... (4)}$$

The phase angle is 2π, if the distance travelled (x) by the wave is equal to wavelength (λ).

i.e., $2\pi = k\lambda$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$$

Where,

$$\omega = 2\pi n = \frac{2\pi}{T} \text{ or}$$

$$n = \frac{\omega}{2\pi}$$

Now, $y(x, t) = a\sin(\omega t - kx)$

$$= a \sin\left(\omega t \frac{\omega}{v} x\right) \qquad \left[\because\ k = \frac{\omega}{v}\right]$$

$$y(x, t) = a\sin\omega\left(t - \frac{x}{v}\right) \quad \text{... (5)}$$

Partially differentiating equation (5), with respect to 't'

$$\frac{\partial y}{\partial t} = \left[a\cos\omega\left(t - \frac{x}{y}\right)\right] \times \omega$$

Again partially differentiating above equation,

$$\frac{\partial^2 y}{\partial t^2} = -a \sin\omega \left(t - \frac{x}{y}\right) \times (\omega) \times (\omega)$$

$$= -a\omega^2 \sin\omega \left(t - \frac{x}{y}\right)$$

$$\frac{\partial^2 y}{\partial t^2} = \omega^2 y$$

[$\because$ From equation (5)] ... (6)

Similarly,

$$\frac{\partial^2 y}{\partial x^2} = -a \frac{\omega^2}{v^2} \sin\omega \left(t - \frac{x}{v}\right)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\omega^2}{v^2} y \qquad \text{... (7)}$$

From equation (6) and (7), it can be written as

$$\frac{\partial^2 y}{\partial x^2} = v^2 \left(\frac{\partial^2 y}{\partial x^2}\right) \qquad \text{... (8)}$$

Equation (8) is similar to wave equation. Therefore, general solution of wave equation is,

$$y(x, t) = a \sin\omega\, a \left(t - \frac{x}{v}\right) \qquad \text{... (9)}$$

The wave equation constants can be used to express the general solution or wave function as,

$$y(x, t) = a \sin \left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] \qquad \text{... (10)}$$

$$\therefore \quad y(x, t) = a \sin \frac{2\pi}{\lambda} (vt - x) \qquad \text{... (11)}$$

1.1.2.1 Significance

Q4. Write the significance of wave equation.

Ans :

Consider the general solution of the wave equation to be,

$$y = f\left(t - \frac{x}{v}\right) + g\left(t + \frac{x}{v}\right) \qquad \text{... (1)}$$

The sinusoidal solution of equation (1) is given as,

$$y = a \sin\omega \left(t - \frac{x}{v}\right) + a \sin\omega \left[\left(t + \frac{x}{v}\right) + \phi\right] \qquad \text{... (2)}$$

Where,

α – Amplitude of vibration

ϕ – Phase difference between waves

The first term in equation (1) represents the incident wave and the second term represents the reflected wave. These two waves have same amplitude, frequency and wave length. By super position, these two oppositely moving waves produce stationary waves. Hence this represents the physical significance of the general solution of the wave equation.

1.2 Modes of Vibration of Stretched String Clamped at Ends

Q5. Explain the modes of vibrations of strings clamped at the ends ?

Ans : **(Imp.)**

The general solution of wave equation for uniform string of length "*l*" and mass per unit length and tension produced in the string is "T" given by

$$y = a_1 \sin(\omega t - kx) + a_2 \sin(\omega t + kx) + b_1 \cos(\omega t - kx) + b_2 \cos(\omega t + kx) \qquad \text{... (1)}$$

- where a_1, a_2 & b_1, b_2 are orbitrary constants
- As the string is fixed at both the ends. It has two conditions, which are called as "boundary conditions" given by

(i) y = 0 at x = 0

(ii) y = 0 at x = *l*

Applying 1st boundary condition to equation (1)

$$a_1 \sin\omega t + a_2 \sin\omega t + b_1 \cos\omega t + b_2 \cos\omega t$$

$$(a_1 + a_2)\sin\omega t + (b_1 + b_2)\cos\omega t = 0$$

$$(a_1 + a_2)\sin\omega t = 0 \qquad \& \qquad (b_1 + b_2)\cos\omega t = 0$$

As $\sin\omega t \neq 0;\ a_1 + a_2 = 0 \qquad b_1 + b_2 = 0$

$$a_1 = -a_2 \qquad b_1 = -b_2$$

$$y = a_1 \sin(\omega t - kx) - a_1 \sin(\omega t + kx) + b_1 \cos(\omega t - kx) - b_1 \cos(\omega t + kx)$$

$$y = a_1[\sin(\omega t - kx) - \sin(\omega t + kx) + b_1[\cos(\omega t - kx) - \cos(\omega t + kx)]$$

$$y = a_1 [\sin\omega t \cos kx - \cos\omega t \sin kx - \sin\omega t \cos kx - \cos\omega t \sin kx]$$

$$+b_1[\cos\omega t \cos kx + \sin\omega t \sin kx - \cos\omega t \cos kx + \sin\omega t \sin kx]$$

$$y = -2a_1 \cos\omega t \sin kx + 2b_1 \sin kx \sin\omega t$$

$$y = 2 \sin kx \left[-a_1 \cos\omega t + b_1 \sin\omega t\right] \qquad ...(2)$$

Apply second boundary condition to equation (2)

i.e., $y = 0$ at $x = l$

$\therefore$ Equation (2) becomes

$$2 \sin kl \left[-a_1 \cos\omega t + b_1 \sin\omega t\right] = 0$$

As $\left[-a_1 \cos\omega t + b_1 \sin\omega t\right] \neq 0$ then $2 \sin kl = 0$

$$\sin kl = 0$$

$$\sin kl = \sin(n\pi)$$

$$kl = n\pi$$

$$k = (np/l)$$

But we know that

$$\omega = kv$$

$$k = \omega/v$$

$$\frac{\omega}{v} = \frac{n\pi}{l}$$

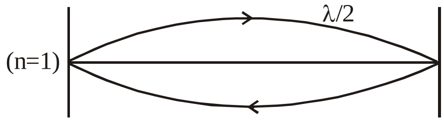
$\omega = 2p\,\nu \Rightarrow$ frequency

$$\frac{2\pi\nu}{\nu} = \frac{n\pi}{l}$$

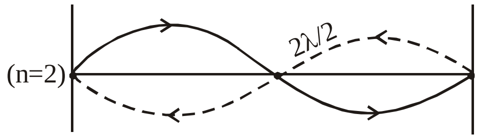
$$\nu = \nu \cdot \frac{n}{2l}$$

$$\boxed{\nu = n\left(\frac{\nu}{2l}\right)}$$

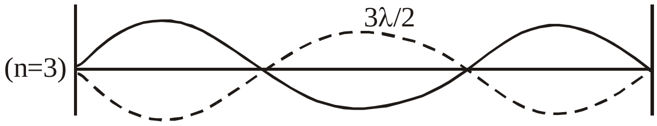
If $n = 1 \Rightarrow \nu_1 = \left(\frac{v}{2l}\right)$



If $n = 2 \Rightarrow \nu_2 = \frac{2v}{2l} = 2v_1$



If $n = 3 \Rightarrow \nu_3 = \frac{3v}{2l} = 3v_1$



1.3 Overtones

Q6. Explain briefly about overtones.

Ans :

Overtones

Overtones are the higher frequencies with which the string vibrates.

The frequency of string fixed at both the ends is given by

$$\nu = n\left(\frac{v}{21}\right) \quad ...(1)$$

Here n = number of modes of vibrations

v = velocity of $\sqrt{T/m}$

l = length of the string

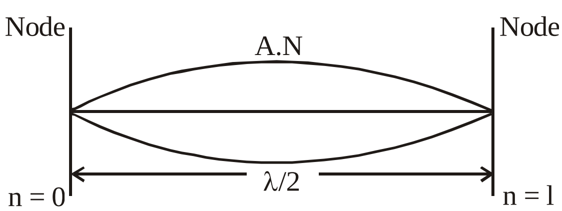
$$\therefore \quad \nu = \frac{n}{2l}\sqrt{T/m} \quad ...(2)$$

Case (i)

When the string is plucked in the middle, then it vibrates with nodes at the ends and antinode at the middle.

If n = l in equation (1)

$$\nu_1 = \frac{\nu}{2l} \Rightarrow \frac{\nu}{2l}\sqrt{T/m}$$



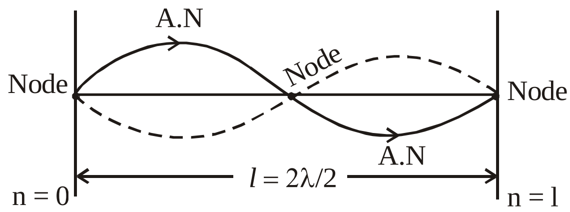
where ν_1 is fundamental frequency (or) 1st harmonic

Case (ii)

When the string plucked at 1/4 of its length then the string vibrates in two segments we get one extranode in the middle.

If n = 2 is usb in equation (1)

$$\nu_2 = \frac{2}{2l}\sqrt{T/m} = 2\nu,$$



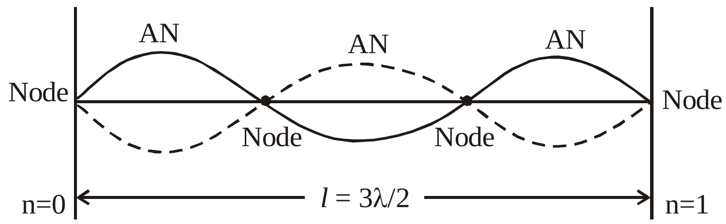
It is known as 1[st] overtone and II - harmonic.

Case (iii)

When the string is plucked at 1/8 of its length then the string vibrates in three segments we get two extra nodes in the middle.

Sub n = 3 in equation (1) we get

$$\nu_3 = \frac{3v}{2l} - 3\nu_1$$



It is known as II-overtone & III - harmonic

If we take the ratios of all overtones we get

$$\nu_1 : \nu_2 : \nu_2 = \nu_1 : 2\nu_1 : 3\nu_1$$

$$\boxed{\nu_1 : \nu_2 : \nu_2 = 1 : 2 : 3}$$

1.4 Energy Transport

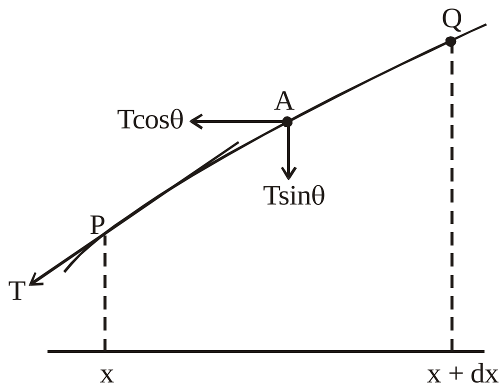
Q7. Derive the expression for energy transport?

Ans :

- Let us consider the energy transport from one point to another point along a vibrating string. Suppose the wave is propagating along the positive x-direction in the string.

- Consider an infinitesimal element PQ of the string having displacement y.
- 'A' is a point on the small element. As the wave is propagating along the positive direction, the tension T sin θ performs some work on the part next to A. The displacement of the string is always transverse and hence horizontal component does not work.
- As the element is small

$$F = -T\sin\theta = -T\tan\theta = -T\left(\frac{\partial y}{\partial x}\right)_{x=0}$$



- Negative sign is used to show that force and displacement are oppositely directed.

Here, $\left(\frac{\partial y}{\partial x}\right)_{x=0} = -A\left(\frac{2\pi}{\lambda}\right)\cos\left(\frac{2\pi\nu t}{\lambda}\right)$

$$F = AT\left(\frac{2\pi}{\lambda}\right)\cos\left(\frac{2\pi\nu t}{\lambda}\right)$$

The input power at a time "t" is given by

$$P(t) = \text{force} \times \text{velocity} = F\left(\frac{\partial y}{\partial t}\right)_{x=0}$$

Here, $\left(\frac{\partial y}{\partial t}\right)_{x=0} = A\left(\frac{2\pi\nu}{\lambda}\right)\cos\left(\frac{2\pi\nu t}{\lambda}\right)$

$$P(t) = A^2T\nu\left(\frac{2\pi}{\lambda}\right)^2\cos^2\left(\frac{2\pi\nu t}{\lambda}\right)$$

$$P(t) = A^2T\nu\left(\frac{2\pi}{\lambda}\right)^2\cos^2\left(\frac{2\pi t}{\tau}\right) \qquad ...(1)$$

($\because \frac{\nu}{\lambda} = \frac{1}{\tau}$, τ = periodic time)

- The mean power is obtained by integrating equation (1) over one periodic time

$$P(t) = A^2 T\nu\left(\frac{2\pi}{\lambda}\right)^2 \frac{\int_0^\tau \cos^2\left(\frac{2\pi t}{\tau} dt\right)}{\int_0^\tau dt}$$

$$= A^2 T\nu\left(\frac{2\pi}{\lambda}\right)^2 \frac{1}{2}\int_0^\tau \frac{1}{2}\left\{1+\cos\left(\frac{4\pi t}{\tau}\right)\right\}dt$$

$$= A^2 T\nu\left(\frac{2\pi}{\lambda}\right)^2 \frac{1}{\tau}\times\frac{\tau}{2}$$

$$= \frac{1}{2}A^2 T\nu\left(\frac{2\pi}{\lambda}\right)^2$$

$$= \frac{1}{2}A^2 T\nu\left(\frac{4\pi^2}{\nu^2}v^2\right) \qquad (\because \nu = v\lambda)$$

$$= \frac{1}{2}A^2 T\nu\left(\frac{4\pi^2 \nu^2 \mu}{T}\right) \qquad (\because \nu = \sqrt{T/\mu})$$

$$= \frac{1}{2}A^2 w^2 \mu\nu$$

$$= 2\pi^2 A^2 \mu\nu^2\nu$$

= Energy per unit length of the string × wave velocity

Hence, time averaged input power is equal to the energy per unit length multiplied by wave velocity. The energy does not stay at the diving agent. We observe the following points :

i) The energy flows along the string and the string acts as a medium for transport of energy from one point to another.

ii) The speed of transport is equal to the wave velocity.

1.5 Transverse Impedance

Q8. Explain the concept of transverse impedance?

Ans :

Transverse Impedance

It is defined to be as the ratio of transverse force to the transverse velocity of particle of the string.

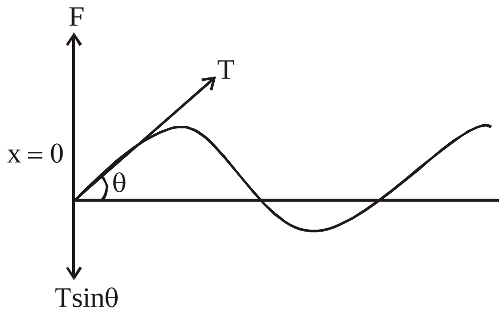
$$\text{Transverse impedance} = \frac{\text{Transverse force}}{\text{Transverse velocity}}$$

- Let us consider a string having linear density μ and tension 'T'.

- An external force "F" is applied at one end of the string so that a transverse wave propagate in it in positive x-direction.
- The mechanical impedance is defined as ratio of external force (F) to the velocity (4)

$$z = F/\mu$$

- The external driving force F_0e^{iwt} and transverse component of tension (Tsinθ) are equal and act in opposite directions as shown diagram.



$$F_y = -T\sin\theta$$

When θ is small

$$\sin\theta = \tan\theta = \frac{\partial y}{\partial x}$$

$$F = -T\frac{\partial y}{\partial x} \text{ and}$$

$$F_0\,e^{-iwt} = -T\frac{\partial y}{\partial t} \qquad \text{... (1)}$$

Using the wave solution given by

$$y(x, t) = Ae^{i(\omega t - kx)}$$

$$\left(\frac{\partial y}{\partial x}\right)_{x=0} = jkAe^{j\omega t} \qquad \text{... (2)}$$

Substituting the equation (2) in (1)

$$F_0\,e^{-jwt} = jkTAe^{j\omega t}$$

But $k = \frac{\omega}{v}$

$$F_0 = \frac{j\omega}{v}AT$$

(or)

$$A = \frac{F_0V}{j\omega T} \qquad \text{... (3)}$$

Substituting equation (3) in $y = A\sin(\omega t - kx)$

$$y(x, t) = \frac{F_0 V}{j\omega T} e^{j(\omega t - kx)} \quad \ldots (4)$$

But velocity $\mu(x, t)$ is defined as $u(x, t) = \frac{\partial y}{\partial t}$

Therefore differentiating equation (4) with respect to "t"

$$u(x, t) = \frac{\partial}{\partial t}\left(\frac{F_0 v}{j\omega T} e^{j(\omega t - kx)}\right)$$

$$= \frac{F_0 v}{j\omega T} j\omega e^{j(\omega t - kx)}$$

$$u(x, t) = \frac{F_0 v}{T} e^{j(\omega t - kx)}$$

Substituting the value of 'F' and u(x, t) in equation

$z = F/u$, the impedance is given by

$$z = \frac{F_0 e^{j\omega t}}{\frac{F_0 v}{T} e^{j\omega t}}$$

$$z = T/v$$

But velocity

$$v = \sqrt{\frac{T}{u}} \text{ (or) } v^2 = \frac{T}{\mu} \text{ or } T = v^2 \mu$$

Substituting "T" value in z

$$z = v\mu$$

∴ Mechanical impedance is the product of velocity & linear density.

1.6 Longitudinal Vibrations in Bars-Wave Equation and its General Solution

Q9. Explain vibrations of bars ?

Ans :

These are three types of vibrations a bar undergoes :

1. Longitudinal
2. Transverse
3. Torsional

1. Longitudinal vibrations

The wave which are produced due to the propagation of wave in parallel direction to the vibration of particles are called as longitudinal vibrations.

2. Transverse vibrations

The wave which are produced due to the propagation of wave in perpendicular direction to the vibration of particles is called as transverse vibrations

3. Torsional vibrations

The vibrations produced in a bar due to the angular displacement is called as torsional vibrations.

- The vibration of a bar depends on external force applied on it and the density of the material.

  Expression for the velocity of longitudinal wave in a bar is given by $\nu = \sqrt{\frac{y}{\rho}}$

where y = young's modulous

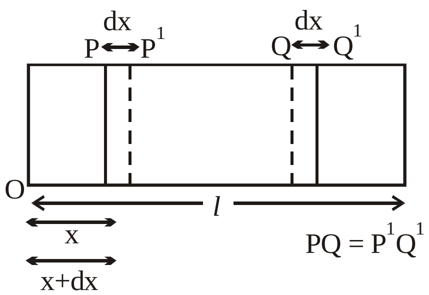
ρ = density.

Q10. Obtain a wave equation for longitudinal vibrations in bars and find its general solution?

Ans : **(Imp.)**

Let us consider a bar of length " l " of uniform cross-sectional area "A" is taken.

- If a longitudinal force (force applied along its length) is applied on the bar then it gets displaced to a distance or length of dx
- The displacement is from "x" to (x+dx)
- If we consider a small length element PQ before displacement at "x", after displacement "p" changes to "p^1" and "Q" changes to "Q^1"
- Due to the application of force the displacement produced at "p" is "dx"



- The displacement at "Q" along z-axis is "dz" total displacement of the element is expanded using Taylor series as

$$z + dz = z + \left(\frac{\partial z}{\partial x}\right)dx$$

$$dz = \frac{\partial z}{dx} \cdot dx \quad ..(1)$$

- Longitudinal strain in the ratio of change in length of the bar by original length of the bar
  Change in length along z-axis $[(z+dz)-z] = dz$
  Change in length along x-axis $[(x+dx)-x] = dx$

$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{dz}{z}, \frac{dx}{x}$$

- Total strain $\frac{dz}{dx} = \frac{\left(\frac{dz}{dx}\right)dx}{dx}$

$$\text{Total strain} = \left(\frac{dz}{dx}\right) \quad ...(2)$$

$$\text{Young's Modulus "Y"} = \frac{\text{longitudinal} \cdot \text{stress}}{\text{longitudinal strain}} = \frac{\text{Force per unit area}}{\left(\frac{\partial z}{\partial x}\right)}$$

$$Y = \frac{-F/A}{\frac{\partial z}{\partial x}}$$

At "p" the force acting is "Fx"

AT "Q" the force acting is $F_x + \left(\frac{\partial F_x}{\partial x}\right)dx$

Resultant force $F = F_x - \left(F_x + \left(\frac{\partial F_x}{\partial x}\right)dx\right)$

$$F = -\left(\frac{\partial F_x}{\partial x}\right)dx$$

From (3)

$$F = -\frac{\partial}{\partial x}\left(-YA\left(\frac{\partial z}{\partial x}\right)dx\right) \Rightarrow F = -\frac{\partial}{\partial x}\left(-YA\frac{\partial z}{\partial x}\right)dx$$

$$F = YA\left(\frac{\partial^2 z}{\partial x^2}\right)dx \quad ...(4)$$

- If "m" is mas per unit length of the bar and "A" is the area of cross section, then volume of the bar is given by

volume = Area × length

= Adx

$$\text{mass} = \text{m/unit length}$$

$$\text{Density "}\rho\text{"} = \frac{\text{mass}}{\text{volume}} = \frac{m}{Adx}$$

$$\rho = \frac{m}{Adx}$$

$$m = \rho Adx \qquad ...(5)$$

From Newton IInd Law

$$F = ma \qquad \text{where } a = \text{acceleration} = \frac{\partial^2 z}{\partial t^2}$$

$$F = (\rho Adx)\frac{\partial^2 z}{\partial t^2} \qquad ...(6)$$

From (4) & (6)

$$YA\left(\frac{\partial^2 z}{\partial x^2}\right)dx = \rho Adx \cdot \frac{\partial^2 z}{\partial t^2}$$

$$\left(\frac{\partial^2 z}{\partial x^2}\right)Y = \frac{\partial^2 z}{\partial t^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{Y}{\rho}\frac{\partial^2 z}{\partial t^2} \qquad ...(7)$$

General

Wave equation is $\Rightarrow$ $\boxed{\frac{\partial^2 z}{\partial x^2} = v^2 \frac{\partial^2 z}{\partial t^2}}$ ...(8)

The above equation is general equation of longitudinal waves in bars by comparing (7) & (8)

$$\nu^2 = \frac{Y}{\rho}$$

$\boxed{\nu = \sqrt{\frac{Y}{\rho}}}$ is the velocity of longitudinal wave.

General Solution of Longitudinal Wave Equation :

The equation of longitudinal wave is given by

$$\frac{\partial^2 z}{\partial t^2} = \left(\frac{Y}{\rho}\right)\frac{\partial^2 z}{\partial x^2}$$

It is differential equation of II-order so it has two solutions

i.e., $z = f_1(ct + x) + f_2(ct - x)$

where "c" is the velocity of electromagnetic wave

T = time period

x = displacement along x-axis.

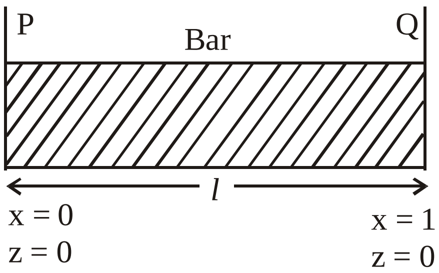
1.6.1 Longitudinal Vibration in Bar Fixed at both Ends

Q11. Explain the longitudinal vibrations when bar fixed at both ends ?

Ans : **(Imp.)**

Consider a bar is fixed at both ends

The general solution of a bar of longitudinal wave in complex from is given by



$Z = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$ ...(1)

$w = 2\pi f$ K = wave vector = w/c

T = time period

We have 2 boundary conditions

i) $z = 0$ at $x = 0$

iii) $z = 0$ at $z = l$

$z = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$

Applying 1st boundary condition two equation (1)

$Ae^{j\omega t} + Be^{j\omega t} = 0$

$e^{j\omega t}(A + B) = 0$

$e^{j\omega t} \neq 0$; $A + B = 0$

$A = -B$ (or) $B = -A$ ... (2)

Equation (1) $\Rightarrow Z = Ae^{j(\omega t - kx)} - Ae^{j(\omega t + kx)}$ ... (3)

II - Boundary Condition

$z = 0$ at $x = l$

$$0 = Ae^{j(\omega t-kl)} - Ae^{j(\omega t+kl)}$$

$$Ae^{j\omega t}(e^{-jkl}) - Ae^{j\omega t}(e^{jkl})$$

$$Ae^{j\omega t}[e^{-jkl} - e^{jkl}] = 0$$

$$-Ae^{j\omega t}[e^{jkl} - e^{-jkl}] = 0 \qquad \text{... (4)}$$

$$e^{ikl} - e^{-jkl} = 2j\sin kl$$

$$2j \sin kl = 0$$

$$\sin kl = 0$$

$$\sin kl = 0$$

$$kl = \sin^{-1}(0)$$

$$kl = n\pi$$

$$k = \frac{n\pi}{l} \qquad \text{...(5)}$$

$$\frac{w}{v} = \frac{n\pi}{l}$$

$$\frac{2\pi f}{v} = \frac{n\pi}{l}$$

$$\boxed{\nu = \left(\frac{n}{2l}\right)v} \qquad \text{... (6)}$$

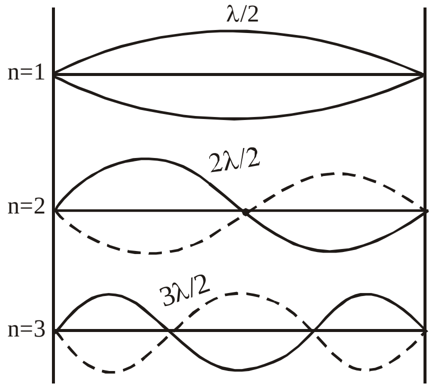
Equation (6) represents the frequency with which the bar vibrates

$$\boxed{\nu = \left(\frac{n}{2l}\right)v}$$

$$\nu = \left(\frac{n}{2l}\right)v$$

If $n=1$; $v_1 = \left(\frac{v}{2l}\right) \Rightarrow$ 1 mode of vibration

and known fundamental frequency or 1st harmonic



If n = 2; $v_2 = \left(\frac{2}{2l}\right)v$. I-overtone, II-harmonic

If n = 3; $v_3 = \left(\frac{3}{2l}\right)v$ II-overtone, III-harmonic.

1.6.2 Longitudinal Vibration in a Bar Fixed at the Mid Point

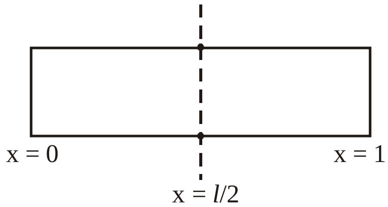
Q12. Explain longitudinal vibration in a bar fixed at the mid point ?

Ans :

Consider a bar is fixed at the midpoint.

The general solution is $z = Ae^{j(\omega t-kx)} + Be^{j(\omega t+kx)}$

It is fixed at point



i) at ends, x = 0 and x = l

If is free $\therefore \frac{\partial z}{\partial x} = 0$

ii) at mid point x = l/2; z = 0

Differentiate equation (1) wr to 'x'

$$\frac{\partial z}{\partial x} = Ae^{j(\omega t-kx)}(-jk) + B\,e^{j(\omega t+kx)}(jk)$$

$$\frac{\partial z}{\partial x} = -Ajk\;e^{i(\omega t-kx)} + Bj\,Ke^{j(\omega t+kx)}$$

Apply 1st boundary conditions is at $x = 0; \frac{\partial z}{\partial x} = 0$

$$-Aj\, ke^{-j\omega t} + Bj\, ke^{j\omega t} = 0$$

$$j\, ke^{i\omega t}\ (-A + B) = 0$$

$$B - A = 0 \Rightarrow B = A$$

Substitute $B = A$ in equation (1)

$$z = Ae^{j(\omega t - kx)} + Ae^{j(\omega t + kx)}$$

$$z = Ae^{j\omega t}(e^{-jkx} + e^{ikx})$$

Applying 2nd boundary conditions

at $x = \frac{1}{2};\ z = 0$

$$0 = Ae^{j\omega t}\left[e^{-jk\,l/2} + e^{jk\,l/2}\right]$$

$$\frac{e^{j\theta - j\theta}}{2} = \cos\theta = e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

If $\theta = \frac{kl}{2}, \left(e^{jk\,l/2} + e^{-jk\,l/2}\right) = 2\cos kl/2$

$$2\cos\frac{kl}{2} = 0$$

$$\cos\frac{kl}{2} = 0$$

$$\frac{kl}{2} = \cos^{-1}(0)$$

$$\frac{kl}{2} = (2n-1)\,\pi/2$$

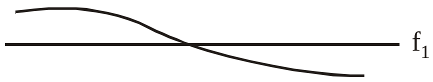
$$kl = (2n-1)\,\pi$$

$$k = \frac{(2n-1)}{l}\cdot\pi$$

$$v = \frac{(2n-1)}{l}\cdot c$$

$$f = \frac{(2n-1)}{l}\cdot c$$

If $n = 1 \Rightarrow f_1 = \frac{c}{l}$



If $n = 2 \Rightarrow f_2 = \frac{2c}{l}$

If $n = 3 \Rightarrow f_3 = \frac{3c}{l}$

1.6.3 Longitudinal Vibrations in a Bar Free at Both the Ends

Q13. Explain longitudinal vibrations in a bar free at both the ends.

Ans :

Consider a bar is free at both the ends

The complete complex harmonic solution of longitudinal wave is

$$z = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)} \quad \text{... (1)}$$

If the bar is free at both the ends then at ends, $\frac{\partial z}{\partial x} = 0$

(1) at $x = 0$, $\frac{\partial z}{\partial x} = 0$

(2) at $x = l$; $\frac{\partial z}{\partial x} = 0$

Applying boundary conditions to equation (1)

$$\frac{\partial z}{\partial x} = Ae^{j(\omega t - kx)}(-jk) + Be^{j(\omega t + kx)}(kj)$$

$$= -jk\,Ae^{j(\omega t - kx)} + B(kj)e^{j(\omega t + kx)}$$

Apply $\frac{\partial z}{\partial x} = 0 = -jk\,Ae^{j(\omega t)} + Bjk\,e^{j\omega t}$

$$-jk\,Ae^{j\omega t} + Bjk\,e^{j\omega t} = 0$$

$$jk\,Ae^{j\omega t}[-A + B] = 0$$

$\therefore \quad A + B = 0$

$B = A$

Substitute $A = B$ in equation (1)

$$Z = Ae^{j(\omega t - kx)} + Ae^{j(\omega t + kx)}$$

$$Z = Ae^{j\omega t} \cdot e^{-jkx} + Ae^{j\omega t} \cdot e^{jkx}$$

$$Z = Ae^{j\omega t}[e^{-jkx} + e^{jkx}]$$

$$Z = Ae^{j\omega t}[2\cos kx]$$

$$e^{j\omega t}[2\cos kx] = z$$

Applying II-boundary conditions

$$\frac{\partial z}{\partial x} = 2e^{j\omega t} \quad [-\sin kx]\, k$$

$$= -2k\, e^{j\omega t} \sin kx$$

$$\text{at } x = l\,;\ \frac{\partial z}{\partial x} = 0$$

$$-2k\, e^{j\omega t} \quad \sin kl = 0$$

$$\sin kl = 0$$

$$kl = \sin^{-1}(0)$$

$$kl = \sin^{-1}(\sin(n\pi))$$

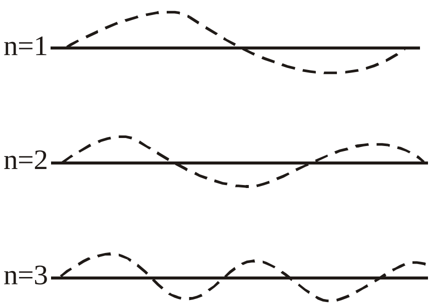
$$kl = n\pi$$

$$k = \frac{n\pi}{l}$$

The general solution is

$$z_n = \sum_{n=0}^{\infty} (A_n \sin wt + B_n \cos wt) \cos k_n x$$

Modes of vibrations



1.6.4 Longitudinal Vibrations when the Bar is Fixed at One End and Free at Another End

Q14. Explain longitudinal vibrations when the bar is fixed at one end and free at another end?

Ans : **(Imp.)**

- Consider a bar is fixed at one end and the another end is free
- This is also known as the case of fixed-free bar

- The boundary condition are

displacement $z = 0$ at $x = 0$

$$\frac{dz}{dx} = 0 \quad \text{at } x = -1$$

- The displacement of superposed wave is given by

$$Z = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)} \qquad \text{... (1)}$$

Applying 1st boundary condition

$$Ae^{i\omega t} + Be^{j\omega t} = 0$$

$$(A + B)e^{j\omega t} = 0$$

Since $e^{j\omega t} \neq 0$

$$(A + B) = 0$$

$$A = -B \text{ or } B = -A$$

Substitute $A = -B$ in (1)

$$Z = Ae^{j\omega t}(e^{-jkt} - e^{jkx}) \qquad \text{... (2)}$$

Applying second boundary condition to equation (2)

$$\frac{\partial z}{\partial x} = -jkAe^{j\omega t}\left[e^{-jkt} + e^{jkt}\right]$$

$$0 = -jkAe^{i\omega t}\left[e^{-jkt} + e^{jkt}\right]$$

$$jkAe^{j\omega t} \neq 0$$

$$(e^{-i\omega t} + e^{j\omega t}) = 0$$

or $\cos kl = 0$

Therefore,

$$kl = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \text{ ...... } (2n-1)\frac{\pi}{2}$$

As k value depends on "n"

$$\text{or} \quad k_n = (2n-1)\frac{\pi}{2l} \qquad \text{... (3)}$$

$$\text{but} \quad k_n = \frac{w_n}{c} \qquad \text{...(4)}$$

From (3) & (4) we have

$$\frac{w_n}{c} = (2n-1)\frac{\pi}{2l} \quad \text{(or)} \quad w_n = (2n-1)\frac{\pi c}{2l}$$

Substituting $w_n = 2\pi f_n$

$$2\pi f_n = \frac{(2n-1)}{l}\cdot\frac{\pi c}{2}$$

or $$f_n = \frac{(2n-1)}{4l}.c$$

for n = 1

$$f_1 = \frac{c}{4l}$$ is called fundamental frequency

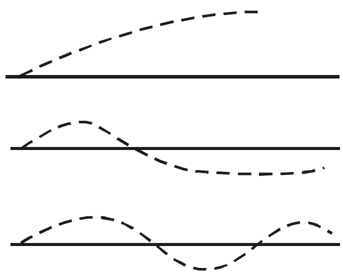
for n = 2

$$f_2 = \frac{3c}{4l}$$ is called first overtime

$$f_2 = 3f_1$$

$$f_3 = \frac{5c}{4l} = 5f_1$$ is called second overtone

Modes of vibrations of Longitudinal waves in a fixed-free bar



Above equation indicates that the frequency of overtones produced in a fixed-free bar are equal to the odd multiples of fundamental frequency. The modes of longitudinal vibrations are as shown in above diagram.

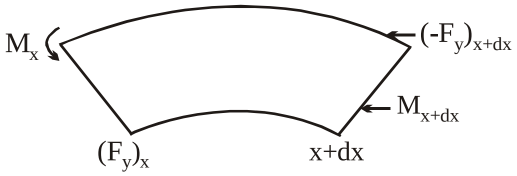
1.7 Transverse Vibrations in a Bar

Q15. Derive the wave equations for transverse vibrations along a stretched string.

Ans :

When a bar is bend as show, beating moments and shearing forces will act on it.

- Bending moment acting on the left side of segment acts in antilockwise direction and hence treated as positive.
- In equilibrium, the turning moment arise due to bending moment and shearing forces on the segment will be zero.



Under the bending condition, net force acting on small segment is zero.

$$dF_y = (Fy)_x - (Fy)_{x+dx}$$

$$dF_y = (Fy)_x - (Fy)_x - \left(\frac{\partial Fy}{\partial x}\right)dx$$

$$dF_y = -\left(\frac{\partial Fy}{\partial x}\right)dx \qquad [\because \ (Fy)_{x+dx} = Fy + \left(\frac{\partial F}{\partial x}\right)dx \ ]$$

Calculating moments from the left side of the segment

$$M_x - M_{x+dx} - (Fy)_{x+dx} + dx = 0 \qquad ...(1)$$

when dx is small

$$M_{x+dx} = M_x + \left(\frac{\partial M}{\partial x}\right)dx \qquad ...(2)$$

and $$(Fy)_{x+dx} = Fy + \left(\frac{\partial F}{\partial x}\right)dx \qquad ...(3)$$

sub (2) & (3) in

$$M_x - M_x - \frac{\partial M}{\partial x}x - Fy\,dx - \frac{\partial F_y}{\partial x}dx \,.\, dx = 0 \qquad ...(4)$$

As dx is small, dx^2 term can be neglected and by simplifying we get

$$Fy = -\frac{\partial M}{\partial x} \qquad ...(5)$$

Substituting $M = YK^2\alpha\left(\frac{\partial^2 y}{\partial x^2}\right)$ in (5) we get

M = bending moment

$$Fy = YK^2\alpha\frac{\partial^2 y}{\partial x^2} \qquad ...(6)$$

$$dF_y = (Fy)_x - F(y)_{x+dx}$$

$$dF_y = -\left(\frac{\partial Fy}{\partial x}\right)dx \qquad ...(7)$$

Sub (6) in (7)

$$(dF_y) = -\frac{\partial}{\partial x}\left(YK^2\alpha\frac{\partial^3 y}{dx^3}\right)dx$$

$$(dF_y) = -\ YK^2\alpha\frac{\partial^4 y}{dx^2}dx$$

The segment vibrates due to force given in the equation (8). If ‘ρ’ is the density and mass is $\rho\alpha dx$ and acceleration $= \frac{\partial^2 y}{dt^2}$

$\therefore$ From Newton Second Law

$$dF_y = (\rho\alpha dx)\frac{\partial^2 y}{\partial t^2} \quad ...(9)$$

Equating (8) & (9)

$$\rho\alpha dx\frac{\partial^2 y}{\partial t^2} = -\ YK^2\alpha\frac{\partial^4 y}{\partial x^4}dx$$

Velocity of Longitudinal wave $= \frac{Y}{\rho}$

$$\frac{\partial^2 y}{\partial t^2} = -\ c^2\, k^2\,\frac{\partial^4 y}{\partial x^4}$$

The above equation is known as wave equation of transverse vibrations in basis.

The general wave equation is

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$$

$\therefore$ velocity of wave = v = $-c^2k^2$ (by comparing both).

1.7.1 Wave Equation and its General Solution

Q16. Obtain a general solution for transverse wave equation ?

Ans :

The general solution of transverse wave is given by

$$Y = \psi(x)e^{j\omega t}$$

Taking derivative w.r. to time ‘t’

$$\frac{\partial^2 y}{\partial t^2} - jw^2\psi\, e^{j\omega t}$$

$$= -w^2\psi\, e^{j\omega t} \quad ... (4)$$

Taking derivative w.r.t to 'x' for equation (1) we get

$$\frac{\partial^4 y}{\partial x^4} = \frac{\partial^4 \psi}{\partial x^4} e^{j\omega t} \qquad ...(3)$$

Substituting (3) & (4) in wave equation we get

$$-\omega^2 \psi e^{j\omega t} = -c^2 k^2 \frac{\partial^4 \psi}{\partial x^4} e^{j\omega t}$$

$$\frac{\partial^4 \psi}{\partial x^2} = \left(\frac{\omega^2}{c^2 k^2}\right)\psi$$

Let velocity of transverse wave $v = \sqrt{wck}$

$$\therefore \quad \frac{\partial^4 \psi}{\partial x^2} = \frac{\omega^2}{v^4}\psi \qquad ...(4)$$

The solution of above equation is $\psi = Ae^{\beta x}$, then

$$\frac{\partial^4 \psi}{\partial x^2} = \beta^4 Ae^{\beta x} \qquad ...(5)$$

Sub (5) in (4) we get

$$\beta^4 Ae^{\beta x} = \frac{\omega^4}{v^4} Ae^{\beta x}$$

$$\beta^4 = \frac{\omega^4}{v^4}$$

or $$\beta^2 = \pm \frac{\omega^2}{v^2}$$

Therefore $\beta^2 = +\frac{\omega^2}{v^2}$ or $\beta^2 = -\frac{\omega^2}{v^2}$

$$\beta = \frac{\omega}{v} \quad \text{and} \quad \beta = -\frac{\omega}{v}$$

$$\beta^2 = -\frac{\omega^2}{v^2}$$

$$\beta^2 = \frac{i^2\omega^2}{v^2} \qquad (\because\ i^2 = 1 \text{ complex number})$$

$$\therefore \quad \beta = +\frac{i\omega}{v} \ \ \& \ \ \beta = -\frac{i\omega}{v} \qquad ...(6)$$

Since β have values gives by equation (6) the general solution of 4 is given by

$$y = e^{i\omega t}\left[Ae^{\omega/vx} + Be^{\left(-\frac{\omega}{v}\right)x} + Ce^{\left(\frac{j\omega}{v}\right)x} + De^{\left(\frac{-j\omega}{v}\right)n}\right]$$

The above equation is the general solution of transverse wave equation in bars.

1.7.2 Boundary Conditions

Q17. Explain the boundary condition in Transverse Vibrations ?

Ans :

The displacement of wave equation is given by

$$y = \cos(wt + \phi)\ [A\cos h(\omega r / u) + B\sin h(\omega x / u)] + [\cos(\omega x/u) + D\sin(\omega x/u)]$$

- The above equation contains four arbitrary constants and hence, four boundary conditions are required for their determination. Actually these conditions are possible as their exist pair of boundary conditions at each of the two ends of the bar.
- Depending on the nature of support, we discuss the following boundary conditions :

i) Free end : There can be neither an external torque nor a shearing force at the free end. i.e., both τ and F_y should be zero, so we have

$$\frac{\partial^2 y}{\partial x^2} = 0 \text{ and } \frac{\partial^3 y}{\partial x^3} = 0 \text{ at all time}$$

ii) Clamped end : When the end of the bar is clamped rigidly, there can be no displacement and slope at all instants of time. Hence, boundary conditions are

$$y = 0 \text{ and } \left(\frac{\partial y}{\partial x}\right) = 0 \text{ at all time}$$

iii) Supported on a knife edge : When the end is supported on a knife edge, the displacement and external torque and zero. Hence, boundary conditions are

$$y = 0 \text{ and } \tau = 0 \text{ (or) } \left(\frac{\partial^2 y}{\partial x^2}\right) = 0 \text{ at all time}$$

1.7.3 Clamped Free Bar

Q18. Explain transverse vibration in a clamped free bar.

Ans : **(Imp.)**

Consider a bar of length '*l*' which is fixed rigidly at x = 0 and free at x = *l*.

Transfer wave equation in a bar is represented as,

$$y = \cos(\omega t + \theta)\left[A\cosh\left(\frac{\omega t}{\nu}\right) + B\sinh\left(\frac{\omega x}{\nu}\right) + C\cos\left(\frac{\omega x}{\nu}\right) + D\sin\left(\frac{\omega x}{n}\right)\right] \quad \text{... (1)}$$

(i) Bar is fixed at x = 0

From boundary conditions,

$$y = 0 \text{ and } \frac{\partial y}{\partial x} = 0$$

Substituting x = 0 and y = 0 in equation (1)

$$0 = A + C$$

$$C = -A \qquad \text{... (2)}$$

Partially differentiating equation (1) with respect to 'x'

$$\frac{\partial y}{\partial x} = \cos(\omega t + \theta)\left[A\frac{\omega}{\nu}\sin\left(\frac{\omega x}{\nu}\right) + B\frac{\omega}{\nu}\cosh\left(\frac{\omega x}{\nu}\right) - C\frac{\omega}{\nu}\sin\left(\frac{\omega x}{\nu}\right) + D\frac{\omega}{\nu}\cos\left(\frac{\omega x}{\nu}\right)\right] \qquad \text{... (3)}$$

Substituting x = 0 and $\frac{\partial y}{\partial x} = 0$ in equation (3)

$$0 = \frac{\omega}{\nu}(B + D)$$

$$\Rightarrow \quad B = -D \qquad \text{... (4)}$$

Substituting equations (2) and (4) in equation (1)

$$y = \cos(\omega t + \theta)\left[A\left\{\cosh\left(\frac{\omega x}{\nu}\right) - \cos\left(\frac{\omega x}{\nu}\right)\right\} + B\left\{\sinh\left(\frac{\omega x}{\nu}\right) - \sin\left(\frac{\omega x}{\nu}\right)\right\}\right] \qquad \text{... (5)}$$

(ii) Bar Free at x = *l*

From boundary conditions,

$$\frac{\partial^2 y}{\partial x^2} = 0 \text{ and } \frac{\partial^3 y}{\partial x^3} = 0$$

Partially differentiating equations (5) twice with respect to 'x'.

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{\omega}{y}\right)^2 \cos(\omega t + \theta)\left[A\left(\cosh\left(\frac{\omega x}{\nu}\right) + \cos\left(\frac{\omega x}{\nu}\right)\right) + B\left(\sinh\frac{\omega x}{\nu} + \sin\frac{\omega x}{\nu}\right)\right] \qquad \text{... (6)}$$

Substituting x = *l* and $\frac{\partial^2 y}{\partial x^2} = 0$ in equation (6)

$$0 = A\left[\cosh\left(\frac{\omega l}{\nu}\right) + \cos\left(\frac{\omega l}{\nu}\right)\right] + B\left[\sinh\frac{\omega l}{\nu} + \sin\frac{\omega l}{\nu}\right]$$

$$\Rightarrow \quad A\left[\left(\cosh\frac{\omega l}{\nu}\right) + \cos\left(\frac{\omega l}{\nu}\right)\right] = -B\left(\sinh\frac{\omega l}{\nu} + \sin\frac{\omega l}{\nu}\right) \qquad \text{... (7)}$$

Partially differentiating equation (5) thrice with respect to 'x'

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{\omega}{y}\right)^2 \cos(\omega t + \theta) \left[A\left(\sinh\frac{\omega x}{\nu} - \sin\frac{\omega x}{\nu}\right) + B\left(\cosh\frac{\omega x}{\nu} + \cos\frac{\omega x}{\nu}\right)\right] \quad \text{... (8)}$$

Substituting $x = l$ and $\frac{\partial^3 y}{\partial x^3} = 0$ in equation (8)

$$0 = A\left[\sinh\frac{\omega l}{\nu} - \sin\frac{\omega l}{\nu}\right] + B\left[\cosh\frac{\omega l}{\nu} + \cos\frac{\omega l}{\nu}\right]$$

$$\Rightarrow \quad A\left(\sinh\frac{\omega l}{\nu} - \sin\frac{\omega l}{\nu}\right) = B\left(\cosh\frac{\omega l}{\nu} + \cos\frac{\omega l}{\nu}\right) \quad \text{... (9)}$$

Dividing equation (7) with equation (9),

$$\left(\cosh\frac{\omega l}{\nu} + \cos\frac{\omega l}{\nu}\right)^2 = \sin^2 h\frac{\omega l}{\nu} - \sin^2\frac{\omega l}{\nu}$$

Expanding above equation and substituting $\cos^2 h\frac{\omega l}{\nu} - \sin^2 h\frac{\omega l}{\nu} = 1$ and $\cos^2\frac{\omega l}{\nu} + \sin^2\frac{\omega l}{\nu} = 1$

$$\Rightarrow \quad \cosh^2\frac{\omega l}{\nu} + \cos^2\frac{\omega l}{\nu} + 2\cosh\frac{\omega l}{\nu}\cos\frac{\omega l}{\nu} = \sinh^2\frac{\omega l}{\nu} - \sin^2\frac{\omega l}{\nu}$$

$$2 + 2\cosh\frac{\omega l}{\nu}\cos\left(\frac{\omega l}{\nu}\right) = 0$$

$$\Rightarrow \quad \cosh\left(\frac{\omega l}{\nu}\right)\cos\left(\frac{\omega l}{\nu}\right) = -1$$

$$\Rightarrow \quad \left(\cosh^2 + \frac{\omega l}{2\nu} + \sinh^2 + \frac{\omega l}{2\nu}\right)\left(\cos^2\frac{\omega l}{2\nu} - \sin^2\frac{\omega l}{2\nu}\right) = -1 \qquad \left|\ \because\ \cosh^2\theta = \cosh^2\frac{\theta}{2} + \sinh^2\frac{\theta}{2},\quad \cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\ \right|$$

$$\Rightarrow \quad \frac{\left(\cosh^2\frac{\omega l}{2\nu} + \sinh^2\frac{\omega l}{2\nu}\right)\left(\cos^2\frac{\omega l}{2\nu} - \sin^2\frac{\omega l}{2\nu}\right)}{\cosh^2\left(\frac{\omega l}{2\nu}\right)\cos^2\left(\frac{\omega l}{2\nu}\right)} = \frac{-1}{\cosh^2\left(\frac{\omega l}{2\nu}\right)\cos^2\left(\frac{\omega l}{2\nu}\right)}$$

$$\Rightarrow \quad \left[\sec^2\left(\frac{\omega l}{2\nu}\right) + \tanh^2\left(\frac{\omega l}{2\nu}\right)\sec^2\left(\frac{\omega l}{2\nu}\right)\right]\left[\cos^2\frac{\omega l}{2\nu} - \sin^2\frac{\omega l}{2\nu}\right] = -\text{sech}^2\left(\frac{\omega l}{2\nu}\right)\sec^2\left(\frac{\omega l}{2\nu}\right)$$

$$\Rightarrow \quad 1 - \tan^2 \frac{\omega l}{2v} + \tanh^2\left(\frac{\omega l}{2v}\right) - \tanh^2\left(\frac{\omega l}{2v}\right) \tan^2 \frac{\omega l}{2v} = -\left[1 - \tanh^2 \frac{\omega l}{2v}\right]\left[1 + \tan^2 \frac{\omega l}{2v}\right]$$

$$\left| \because \begin{array}{l} \text{sech}^2\theta + \tanh^2\theta = 1 \\ \sec^2\theta - \tan^2\theta = 1 \end{array} \right|$$

$$\Rightarrow \quad 1 + \tan^2 \frac{\omega l}{2v} - \tan^2 \frac{\omega l}{2v} - \tanh^2\left(\frac{\omega l}{2v}\right) \tan^2 \frac{\omega l}{2v} = -1 + \tanh^2 \frac{\omega l}{2v} - \tan^2 \frac{\omega l}{2v} + \tanh^2 \frac{\omega l}{2v} \tan^2 \frac{\omega l}{2v}$$

$$\Rightarrow \quad 2 = 2 \tanh^2\left(\frac{\omega l}{2v}\right) \tan^2\left(\frac{\omega l}{2v}\right)$$

$$\Rightarrow \quad \tanh^2\left(\frac{\omega l}{2v}\right) \tan^2\left(\frac{\omega l}{2v}\right) = 1$$

$$\Rightarrow \quad \tanh^2\left(\frac{\omega l}{2v}\right) = \cot^2\left(\frac{\omega l}{2v}\right)$$

$$\Rightarrow \quad \cot \frac{\omega l}{2v} = \pm \tanh \frac{\omega l}{2v}$$

The variation of cot $\frac{\omega l}{2v}$ and $\pm$ tanh $\frac{\omega l}{2v}$ is plotted against $\frac{\omega l}{2v}$ as shown in the figure (1)

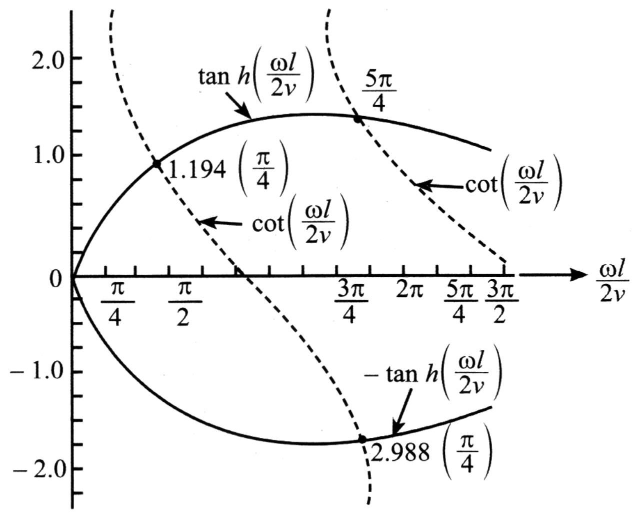


Figure : (1)

The intersecting point of cot $\left(\frac{\omega l}{2v}\right)$ graph and tanh $\left(\frac{\omega l}{2v}\right)$ graph gives the values of $\frac{\omega l}{2v}$ and from the value bar vibrating frequencies can be determined. From the above figure the intersecting values are obtained as,

$$1.194\pi, \; 2.988 \; \frac{\pi}{4}, \; \frac{5\pi}{4}, \; ...$$

$\Rightarrow \quad \frac{\omega l}{2\nu} = \frac{\pi}{4}$ (1.194, 2.988, 5, 7 ...)

$\Rightarrow \quad \frac{\omega l}{2\sqrt{\omega ck}} = \frac{\pi}{4}$ (1.194, 2.988, 5, 7 ...)

Squaring on both sides

$\Rightarrow \quad \omega = \frac{\pi^2}{4l^2}$ ck (1.194^2, 2.988^2, 5^2, ...)

$\Rightarrow \quad 2\pi f = \frac{\pi^2}{4l^2}$ ck (1.1944, 2.988^2, 5^2, ...)

$f = \frac{\pi}{8l^2}$ ck (1.194^2, 2.988^2, 5^2, ...)

$\therefore$ Fundamental frequency $(f_1) = \frac{\pi ck}{8l^2}(1.194)^2$

First overtone $(f_2) = \frac{\pi ck}{8l^2}(2.988)^2$

$\Rightarrow \quad f_2 = 6.267 \times f_1$

Second overtone $(f_3) = \frac{\pi ck}{8l^2}(5^2)$

$= 17.557\, f_1$

The modes at transverse vibration of a clamped free bar is an shown in the figure (2)

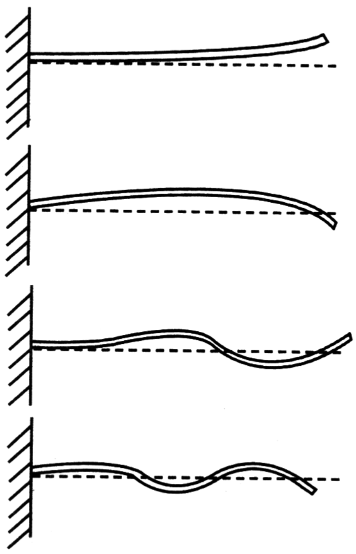


Fig. (2) : Modes of Vibration of a Bar Fixed at One End and Free at the Other End

Therefore it can be concluded that in a bar clamped at one end, the overtones are not harmonic with fundamental frequency.

1.7.4 Free - Free Bar

Q19. Explain transverse vibrations in a Free - Free Bar.

Ans : **(Imp.)**

Consider a bar is free at both the ends

If the two ends of bar are free, the bending moment and shearing forces are zero. The following boundary conditions are hold good.

i) At $x = 0; \frac{\partial^2 y}{kdx^2} = 0$

ii) At $x = 0; \frac{\partial^3 y}{dx^3} = 0$

Applying boundary condition to general solution fromc condition (1)

$x = 0; \frac{\partial^2 y}{dx^2} = 0$ we get

$A - C = 0$

$A = C$

From condition (2)

$B - D = 0$ or $B = D$

Secondary conditions are

iii) $x = l; \frac{\partial^2 y}{dx^2} = 0$ iv) $x = l; \frac{\partial^4 y}{dx^4} = 0$

Applying condition (iii) for general solution equation and simplifying

$$A\left(\cos h\left(\frac{\omega l}{v}\right) + \cos\left(\frac{\omega l}{v}\right)\right) - B\left(\sin h\left(\frac{\omega l}{v}\right) - \sin\left(\frac{\omega l}{v}\right)\right) = 0$$

$$A\left(\cos h\left(\frac{\omega l}{v}\right) + \cos\left(\frac{\omega l}{v}\right)\right) = B\left(\sin h\left(\frac{\omega l}{v}\right) - \sin\left(\frac{\omega l}{v}\right)\right) \quad ...(1)$$

By applying condition (iv) we get

$$A\left(\sin h\left(\frac{\omega l}{v}\right) + \sin\left(\frac{\omega l}{v}\right)\right) = -B\left(\cos h\left(\frac{\omega l}{v}\right) - \cos\left(\frac{\omega l}{v}\right)\right)$$

Dividing equations (1) & (2) and cross multiplying

$$\left[\left(\cos h\left(\frac{\omega l}{v}\right) - \cos\left(\frac{\omega l}{v}\right)\right)\right]^2 = \sin^2 h\left(\frac{\omega l}{v}\right) - \sin^2\left(\frac{\omega l}{v}\right)$$

We know that

$$\cos^2 h\left(\frac{\omega l}{v}\right) - \sin^2 h\left(\frac{\omega l}{v}\right) = 1$$

and $\cos^2\left(\frac{\omega l}{v}\right) + \sin^2\left(\frac{\omega l}{v}\right) = 1$

By using above values we get

$$\cos h\left(\frac{\omega l}{v}\right) \; \cos\left(\frac{\omega l}{v}\right) = 1$$

It is a transcendental equation

$$\tan\left(\frac{\omega l}{v}\right) = \pm \tan h\left(\frac{\omega l}{v}\right)$$

A plot of tan $\left(\frac{\omega l}{v}\right)$ and tan h $\left(\frac{\omega l}{v}\right)$ against $\left(\frac{\omega l}{v}\right)$ is drawn

The intersecting points gives allowed frequencies

$$\frac{\omega l}{v} = \frac{\pi}{4}\,(3.012,\,5,\,7,\,9...) \qquad ...(3)$$

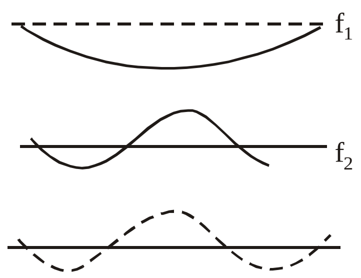
Substituting above values

$$v = \sqrt{\omega ck} \quad c = \sqrt{\frac{y}{\rho}}$$

Substitute (v) & (c) values in (3)

$$f = \frac{\pi ck}{8l^2}\,\left((13.0112)^2,\,5^2,\,7^9,\,9^2...\right)$$

If f_1 is fundamental frequency



Modes in
Free-Free bar

$$f_1 = \frac{\pi ck}{8l^2}(3.0112)^2$$

First overtone $f_2 = \frac{\pi ck}{8l^2}$

Second overtone $f_3 = \frac{\pi ck}{8l^2}\, 7^2$

$$\left(\frac{f_2}{f_1}\right) = \left(\frac{5}{3.0111}\right)^2 = 2.756$$

$f_2 = 2.756\, f_1$

$f_3 = 5.404\, f_1$

In a free-free bar the vibrational frequencies of overtones are not harmonics of fundamental frequency.

The mode of vibrations are shown in diagrams.

1.7.5 Bar Supported at Both Ends

Q20. Explain transverse vibrations in a bar supported at both ends.

Ans :

For bar supported at both ends, the following boundary conditions are valid.

i) at $x = 0$; $y = 0$ and $\frac{\partial y}{\partial x} = 0$

ii) at $x = l$; $y = 0$ and $\frac{\partial y}{\partial x} = 0$

Applying 1st boundary condition to transverse wave equation.

$A + C = 0$; $A = -C$

$C = -A$

and

Since $\frac{\partial y}{\partial x} = 0$ at $x = 0$

$B + D = 0$

$D = -B$ Substituting c & D values in general solution we get

$$Y = \cos(\omega t + \theta)\left(A\left(\cos h\frac{\omega x}{v} - \cos\frac{\omega x}{v}\right) + B\left(\sin h\frac{\omega x}{v} - \sin\frac{\omega x}{v}\right)\right)$$

Here y = 0

$$\therefore \quad \cos(\omega t+\theta)\left(A\left(\cos h\frac{\omega x}{v}-\cos\frac{\omega x}{v}\right)\right)+B\left(\sin h\left(\frac{\omega x}{v}\right)-\sin\frac{\omega x}{v}\right)=0$$

As $\cos(\omega t+\theta)\neq 0$

$$A\left(\cos h\left(\frac{\omega x}{v}\right)-\cos\left(\frac{\omega x}{v}\right)+B\left(\sin h\left(\frac{\omega x}{v}\right)-\sin\left(\frac{\omega x}{v}\right)\right)\right)=0$$

$$A\left(\cos h\left(\frac{\omega x}{v}\right)-\cos\left(\frac{\omega x}{v}\right)\right)=-B\left(\sin h\left(\frac{\omega x}{v}\right)-\sin\left(\frac{\omega x}{v}\right)\right) \qquad \text{... (1)}$$

Applying second boundary conditions we get

at $x = l$; $y = 0$ and $\frac{\partial y}{\partial x}=0$

$$\text{cost}(\omega t+\theta)\left(\frac{\omega}{v}\right)A\left(\sin h\left(\frac{\omega l}{v}\right)-\sin\left(\frac{\omega l}{v}\right)\right)+B\left(\cos\left(\frac{\omega l}{v}\right)\right)=0$$

Since, $\cos(\omega t+\theta)\left(\frac{\omega}{v}\right)\neq 0$

$$A\left(\sin h\left(\frac{\omega l}{v}\right)+\sin\left(\frac{\omega l}{v}\right)\right)+B\left(\cos h\left(\frac{\omega l}{v}\right)-\cos\left(\frac{\omega l}{v}\right)\right)=0$$

or

$$A\left(\sin h\left(\frac{\omega l}{v}\right)+\sin\left(\frac{\omega l}{v}\right)\right)=-B\left(\cos h\left(\frac{\omega l}{v}\right)-\cos\left(\frac{\omega l}{v}\right)\right) \qquad \text{... (2)}$$

Dividing equation (1) & (2) and cross multiplying

$$\cos h\left(\frac{\omega l}{v}\right)\cos\left(\frac{\omega l}{v}\right)=1$$

(or)

$$\tan\left(\frac{\omega l}{2v}\right)=\pm\tan h\left(\frac{\omega l}{2v}\right)$$

Therefore allowed frequency in case of bar fixed at both ends are identical to the variations of free - free bar.

Q21. Write the differences between longitudinal and Transverse vibrations ?

Ans :

S.No.	Longitudinal vibrations	Transverse vibrations
1.	Particle vibrate in parallel direction to wave direction	Particles vibrate in perpendicular direction to wave direction
2.	Equation of longitudinal wave $\Rightarrow \frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$	Equation of transverse wave $\Rightarrow \frac{\partial^2 y}{\partial t^2} = v^2 k^2 \frac{\partial^2 y}{\partial x^2}$
3.	Velocity of longitudinal wave $\Rightarrow \nu = \sqrt{\frac{y}{\rho}}$	Velocity of transverse wave $v = \sqrt{\frac{T}{m}}$
4.	Velocity is independent of frequency Ex : sound wave	Velocity depends of frequency Ex : Light (or) Electromagnetic wave

1.8 Tuning Fork

Q22. Write a brief note on tuning fork ?

Ans : **(Imp.)**

A tuning fork is in the form of a υ - shaped bar of elastic metal (usually steel). If resonants at a specific constant pitch when set vibrating by striking it against a surface or with an object. It emits a pure musical tone after waiting a moment. The pitch that a particular tuning fork generates depends on the length of the two prongs.

The reason for using the fork shape is that, when vibrating, there is a node in the vibration pattern at the bend of the "u" where the handle is attached, so the handle doesn't vibrate. This allows it to be held there without damping the vibration.

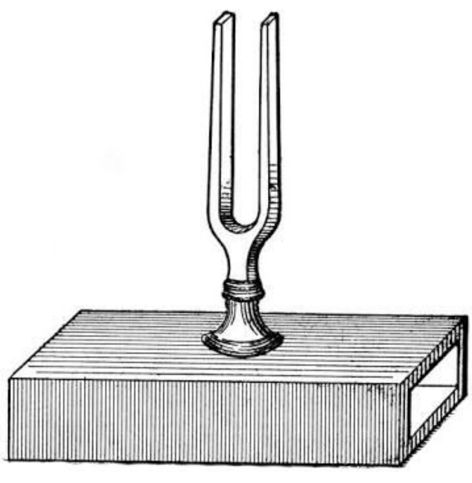


Fig. : Tuning fork on a resonance box

Frequency : The frequency of tuning fork depends on its dimensions and the material from which is made. The frequency is given by

$$f \alpha \frac{1}{l^2}\sqrt{\frac{AE}{\rho}}$$

and where the tins are cylindrical means

$$f = \frac{r}{\pi l^2}\sqrt{\frac{E}{\rho}}$$

Where

f is the frequency

A $\Rightarrow$ cross-sectional area of tuning fork

l $\Rightarrow$ length of the fork times

E $\Rightarrow$ young's modulus of the material of fork

p $\Rightarrow$ density of material

r $\Rightarrow$ radius of times.

Q23. Write the uses of tuning fork ?

Ans :

(i) In Musical Instruments

A number of keyboard musical instruments using construction similar to tuning fork have been made, the most popular of them bring the Rhodes piano, which has hammers hitting constructions working on the same principle as tuning forks.

(ii) In Electromechanical Watches

Electromechanical watches developed by Max Hetzel for Bulova used a 360 Hertz tuning fork with a battery to make a mechanical watch keep time with great accuracy. The production of the Bulova Accutron, as it was called ceased in 1977.

(iii) Medical Uses

Tuning forks, usually e-512, are used by medical practitioners to access a patient's hearing. Lower-pitched ones (usually c-128) are also used to check vibration sense as part of the examination of the peripheral nervous system.

Problems

1. A travelling wave propagates according to the expression $y = 0.03 \sin(3x - 2t)$ where "y" is the displacement at position x at time "t".

Taking the units to be in SI, determine

(a) Amplitude

(b) Wavelength

(c) Frequency

(d) Period of the wave

Sol :

We know that $y = a \sin(kx - \omega t)$

Comparing this equation with the given equation, we get

(a) Amplitude $a = 0.03$ metre

(b) Wavelength $\lambda = \left(\frac{2\pi}{k}\right) = \frac{2 \times 3.14}{3} = 2.09$ metre

(c) Frequency $\nu = \frac{w}{2\pi} = \frac{2}{2\pi}$ Hz $= 0.31$ Hz

(d) Period $\tau = \frac{1}{\nu} = \frac{2\pi}{2} = 3.14$ sec

---

2. A string vibrates according to the equation $y = 5\sin\left(\frac{\pi x}{3}\right)\cos(40\pi t)$ where x, y are in cm and 't' in sec. Find the distance between two successive nodes and the speed of particle of the string at a position $x = 1.5$ cm, when $t = 9/8$ sec.

Sol :

At nodes $y = 0$, thus $\sin\left(\frac{\pi x}{3}\right) = 0$

$\therefore \quad \frac{\pi x}{3} = n\pi$ where $n = 0, 1, 2, 3.....$

or $x = 3n = 0, 3, 6, 9.....$

So, the distance between two successive nods = 3 cm

Velocity of particle $= \frac{dy}{dt} = -5 \sin\left(\frac{\pi x}{3}\right) \sin(40\pi t)(40)$

When

$x = 1.5$ cm and $t = 9/8$ sec, then

$$\frac{dy}{dt} = -5 \sin\left(\frac{\pi \times 1.5}{3}\right) 40\pi \times \sin(45\pi)$$

$$= -5 \sin(\pi/2) \times 40\pi \times \sin(45\pi)$$

$$= -5 \times 40\pi \times 0 = 0$$

Hence, the particle is at rest at that instant.

3. The fundamental frequency of vibration of a stretched string of length 1m is 256 Hz. Find the frequency of the same string of half the original length under identical conditions.

Sol :

Here given $\nu_1 = 256$; $l_1 = 1$; $l_2 = \frac{1}{2}$

For same tension and for the same linear density

$$\nu_1 l_1 = \nu_2 l_2$$

$$256 \times 1 = \nu_2 \times \frac{1}{2}$$

$$\nu_2 = 512 \text{ Hz}$$

4. A steel wire 50 cm long has mass of 5 gms. It is stretched with a tension of 400N. Find the frequency of wire in fundamental mode of vibration.

Sol :

$$\nu = \frac{1}{2l}\sqrt{T/m}$$

Here, $l = 50$ cm $= 0.5$ m

$$m = 5 \times 10^{-3} \text{ kg}$$

$$= 0.5 \times 10^{-2} \text{ kg}$$

$$\therefore \quad \nu = \frac{1}{2 \times 0.5} \times \sqrt{\frac{400}{10^{-2}}}$$

$$= \frac{1}{1.0}\sqrt{4000 \times 10^2}$$

$$= 200 \text{ Hz}$$

5. A flexible string of length 1m and mass 1gm is stretched by a tension T. The string is found to vibrate in three segments at a frequency of 512 Hz. Calculate the tension.

Sol :

$$V_3 = \frac{3}{2l}\sqrt{T/m}$$

Here, l = lm, m = 10^{-3} kg/1m $\Rightarrow$ 10^{-3} kg/m

$$V_3 = 512$$

$$512 = \frac{3}{2\times 1}\sqrt{\frac{T}{10^{-3}}}$$

$$\left(\frac{512\times 2}{3}\right)^2 = \frac{T}{10^{-3}}$$

$$T = \left(\frac{512\times 2}{3}\right)^2 \times 10^{-3}$$

$$= 116.49 \text{ newton}$$

6. Two identical guitar string are tuned to the same frequency of 300 Hz. The tension of one of the string is increased by 2%. How many beats per sec. will be heard when the two strings are sounded together.

Sol :

Let v_2 be the frequency of the second string when the tension is increased by 2%. Here we have

$$\nu_1 = \frac{1}{2l}\sqrt{T/m} \quad ...(1)$$

and

$$\nu_2 = \frac{1}{2l}\sqrt{\frac{(102T)}{(100m)}} \quad ...(2)$$

Dividing equation (2) by (1), we get

$$\frac{\nu_2}{\nu_1} = \sqrt{\left(\frac{102}{100}\right)}$$

$$\therefore \quad \nu_2 = 300\sqrt{\frac{102}{100}} = 300\sqrt{1.02} = 302.9 \text{ Hz}$$

$\therefore$ Number of beats = 302.9 – 300 = 2.9 Hz = 3 beats per sec.

7. The speed of a transverse wave on a stretched string is 500 m/s, when it is stretched under a tension of 19.6N. If the tension is altered to a value of 78.4N, what will be the speed of the wave ?

Sol :

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\nu_2 = \nu_1\sqrt{\frac{T_2}{T_1}}$$

$$\nu_2 = 500 \times \sqrt{\frac{78.4}{19.6}}$$

$$= 500 \times \sqrt{4} = 500 \times 2 = 1000 \text{ m/s}$$

8. The fundamental frequency a sonometer wire increases by 5 Hz if its tension is increased by 21%. How will the frequency be affected if its length is increased by 10% ?

Sol :

The fundamental frequency is given by

$$\nu = \frac{1}{2l}\sqrt{T/m} \quad \text{...(1)}$$

when the tension is increased by 21%, the new tension will be 1.21 T

$$\nu+5 = \frac{1}{2l}\sqrt{\left(\frac{1.21T}{m}\right)} \quad \text{...(2)}$$

Dividing equation (2) by (1) we get

$$\frac{\nu+5}{\nu} = \sqrt{(1.21)} = 1.1$$

Solving we get $\nu = 50$ Hz

when the length is increased by 10%, the new frequency ν^1 is given by

$$\nu^1 = \frac{1}{2(1.1l)}\sqrt{T/m} = \frac{\nu}{1.1}$$

$$\nu^1 = 45.45 \text{ Hz}$$

9. A flexible string of length 1m and mass 1gm is stretched to a tension T. The string is found to vibrate in three segments at a frequency of 612 Hz. Calculate the tension in the string.

Sol :

We know that

$$\nu_3 = \frac{3}{2l}\sqrt{\frac{T}{m}}$$

Here ν_3 = 612 Hz, t = 1 metre and m = 10^{-3} kg

= 10^{-3} kgm^{-1}

$$\therefore \quad 612 = \frac{3}{2\times 1}\sqrt{\frac{T}{10^{-3}}}$$

$$\frac{612\times 2}{3} = \sqrt{\frac{T}{10^{-3}}}$$

$$T = \left(\frac{612\times 2}{3}\right)^2 \times 10^{-3}$$

T = 166.46 Newton

10. A string of length 0.5m and linear density 0.0001 kgm^{-1} kept under a tension $\perp$ N. Find the first three overtones of the string when it is plucked at its mid-point.

Sol :

The fundamental frequency of the string

$$\nu = \frac{l}{2l}\sqrt{\frac{T}{m}}$$

Here T = 1N, length l = 0.5m

m = 0.0001 kgm^{+}

$$v = \frac{1}{2\times 0.5}\sqrt{\frac{1}{0.0001}} = 100 \text{ Hz}$$

When the string is plucked at its mid point even overtones are absent. So the string vibrates with 3v, 5v, 7v.

Frequency of first overtone = 3v = 3 × 100

= 300 Hz

Frequency of second overtone = 5v = 5 × 100

= 500 Hz

Frequency of third overtone = 7v = 7 × 100

1 = 700 Hz

11. The density of aluminium is 2.8×10^3 kg/m³ and its young's modulus is 7 × 10^{10} pascals. If the frequency of the aluminum rod si 500 Hz, calculate the velocity of sound and wavelength through the rod.

Sol.:

The velocity of longitudinal wave is given by

$$\nu = \sqrt{\frac{Y}{\rho}}$$

Given that y = 7 × 10^{10} pascal

$= 7 \times 10^{10}$ N/m and $\rho = 2.8 \times 10^3$ kg/m³

$$\nu = \sqrt{\left(\frac{7 \times 10^{10}}{2.8 \times 10^3}\right)} = 5 \times 10^3 \text{ M/S}$$

12. A brass rod of length 1m is damped at its midpoint . If it is made to vibrate longitudinally, find the first overtone frequency. (Y for brass = 10×10^{10} N/m², density of brass is 8.3×10^3 kg/m³)

Sol.:

The frequency of n^{th} mode is given by

$$\nu_n = \frac{(2n-1)}{2l}\sqrt{\frac{Y}{\rho}} \qquad n = 1,2,3....$$

For first overtone (n=2)

$$\therefore \qquad \nu_2 = \frac{2 \times 2 - 1}{2 \times 1}\sqrt{\left(\frac{10 \times 10^{10}}{8.3 \times 10^3}\right)} = \frac{3}{2}\sqrt{\frac{10^8}{8.3}}$$

$$= \frac{3}{2} \times \frac{10^4}{\sqrt{8.3}}$$

$= 0.536 \times 10^4$ Hz

13. A brass rod 2m length is clamped at it s mid point. On end is excited to produce longitudinal vibrations. The frequency of the emitted note is found to be 1000 Hz. Calculate the velocity of longitudinal waves and young's modulus of the brass assuming the density to be 8500 kg/m³.

Sol.:

We know that in case of rod clamped at mid point, the frequency of n^{th} mode is given by

$$\nu_n = \frac{(2n-1)}{4l}\sqrt{\frac{Y}{\rho}}$$

In case of fundamental frequency, n = 1

$$\therefore \qquad \nu_1 = \frac{1}{4l}\sqrt{\frac{Y}{\rho}} \quad \text{or} \quad 1000 \times \frac{1}{4\times 2}\sqrt{\left(\frac{Y}{8500}\right)}$$

Solving, we get

$Y = 5440 \times 10^8$

$Y = 5.4 \times 10^{11}\ N/m^2$

$$\text{velocity} \qquad \nu = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{5.4\times 10^{11}}{8500}}$$

$$= \sqrt{6353} \times 10^2$$

$$= 79.71 \times 10^2 \text{ M/s}$$

14. A copper bar of length 1m is free at both ends. The diameter of bar is 0.01 cm. Calculate fundamental frequency of longitudinal and transverse vibrations. Sound velocity is copper is 3500 m/sec.

Sol.:

Bar length 1 = 1m

Bar diameter = 0.01 m

Therefore radius R = 0.005m

Geometric radius = K = R/2 = 0.025m

Wave velocity v = 3560 m/sec

Fundamental frequency in longitudinal mode of free-free bar f_c is given by

$$f_W = \frac{v}{2L} \qquad \therefore \quad f_L = \frac{3560}{2\times 1} = 1780 \text{ Hz}$$

Fundamental frequency in transverse mode of free - free bar

$$f_T = \frac{\pi ck}{8l^2}(3.0112)^2$$

$$f_T = \frac{\pi\times 3560\times 0.025}{8\times 1^2}(3.0112)^2$$

$$= 31.68 \text{ Hz}$$

15. In steel sound velocity in 5050 m/sec. If steel density is 7700 kg/m³ then determine young modulus of a steel.

Sol.:

Sound velocity in steel C = 5050 m/sec

Steel density ρ = 7700 kg/m³

Wave velocity $\therefore$ C = $\sqrt{\frac{Y}{\rho}}$

Young modulous Y = $c^2\rho$

= $(50\ 50)^2 \times 7700\ N/m^2$

= $19.64 \times 10^{11}\ N/m^2$

16. In a circular copper bar of length 1cm and radius 0.004 m which is free at both the ends. Calculate fundamental and first overcome of transverse vibration in bar (Sound velocity in bar is 3560 m/sec)

Sol.:

Bar length l = m Bar radius R = 0.004 m

Geometric radius of circular bar K = R/2 = 0.002m

Wave velocity in bar C = 3560 m/sec

Fundamental frequency of transverse mode of vibration

$$f_1 = \frac{\pi ck}{8l^2}(3.0112)^2$$

$$f_1 = \frac{\pi \times 3500 \times 0.002}{8 \times l^2}(3.0112)^2$$

$$f_1 = 25.35\ Hz$$

First overtone $f_2 = \frac{\pi ck}{8l^2}(5)^2$

$$\frac{f_2}{f_1} = \left(\frac{5}{3.0112}\right)^2$$

or $$f_2 = \left(\frac{5}{3.0112}\right)^2 \times f_1$$

$$= \left(\frac{5}{3.0112}\right)^2 \times 25.35$$

$$= 68.86\ Hz$$

17. A square shape steel bar of length 20cm is fixed at both the ends. Its cross section is a square with side length "a" m. The fundamental frequency of transverse vibration is 250Hz. Then find the value "a" (In steel sound velocity 4990 m/sec)

Sol.:

Steel bar length l = 20 cm = 0.2 m

Geometric radius of square shape bar with side length "a" is

$$k = \frac{a}{\sqrt{12}}$$

Wave velocity C = 4990 m/sec

Transverse wave fundamental frequency f_1 = 250 Hz

$$f_1 = \frac{\pi ck}{8l^2}(3.0112)^2$$

$$\therefore \quad a = \frac{f_1 \times 8 \times l^2}{\pi c} \times \frac{\sqrt{2}}{(3.0112)^2}$$

$$= \frac{250 \times 8 \times (0.2)^2 \times \sqrt{12}}{\pi \times 4990 \times (3.0112)^2}$$

$$= 0.0195 \text{ m or } 195 \text{ cm}$$

18. Calculate at what frequency transverse and longitudinal velocities are equal in a nickel root of 0.01m radius.

Nickel density ρ = 8800 kg/m³

Young's Modulus Y = 21×10¹⁰ N/m²

Sol.:

In a rod longitudinal wave velocity $C = \sqrt{\frac{Y}{P}}$ m/sec

$$\therefore \quad C = \sqrt{\frac{21 \times 10^{10}}{8800}} = 4885 \text{ m/sec}$$

Rod radius R = 0.01 m

Isometric radius $K = R/2 = \frac{0.01}{2} = 0.005$ m

Let the angular frequency is w

Transverse wave velocity $v = \sqrt{wck}$

According to given problem v = c

$$c = \sqrt{wck}$$

$$c^2 = wck \text{ or } c = wk \text{ or } w = \frac{c}{k}$$

Frequency $f = \frac{w}{2\pi} = \frac{c}{2\pi k}$

$$= \frac{4885}{2 \times 3.12 \times 0.005}$$

$= 155494.4$ Hz

$= 0.155 \times 10^6$ Hz

$f = 0.155$ MHz

At frequency 0.155 MHz, in Nickel rod both longitudinal and transverse wave velocity are equal.

Short Question & Answers

1. Define the laws of vibrations in strings fixed at both ends ?

Ans :

Laws of transverse vibrations in strings can be stated as follows :

1st Law

The frequency of vibrations is inversely proportional to the length of string, when its tension and linear density are constant.

$$v \alpha \frac{1}{l} \text{ (T, } \mu \text{ constant)}$$

2nd Law

The frequency of vibration is directly proportional to the square root of tension, when the length and linear density of string are constant

$$v \alpha \sqrt{T} \quad \text{(1, } \mu \text{ are constants)}$$

3rd law

The frequency of vibration is inversely proportional to the square root of linear density when length and tension are constant.

$$v \alpha \frac{1}{\sqrt{\mu}} \text{ (} l \text{, T are constants)}$$

---

2. Define the terms Transverse Waves, Overtones, Transverse impedances?

Ans :

Transverse Waves

The waves which oscillates about its mean position in the direction perpendicular to the direction of clave propagation are known as transverse waves.

Overtones

Overtones are the higher frequencies with which the string vibrates.

Transverse Impedance

It is defined as to be as the ratio of transverse force to the transverse velocity of particle of the string.

$$\text{Transverse impedance} = \frac{\text{Transverse force}}{\text{Transverse velocity}}.$$

3. Explain briefly about overtones.

Ans :

Overtones

Overtones are the higher frequencies with which the string vibrates.

The frequency of string fixed at both the ends is given by

$$\nu = n\left(\frac{v}{21}\right) \qquad \text{... (1)}$$

Here n = number of modes of vibrations

v = velocity of $\sqrt{T/m}$

l = length of the string

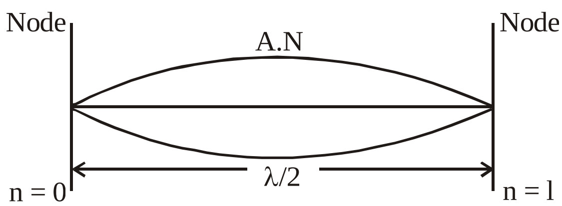
$$\therefore \qquad \nu = \frac{n}{2l}\sqrt{T/m} \qquad \text{... (2)}$$

Case (i)

When the string is plucked in the middle, then it vibrates with nodes at the ends and antinode at the middle.

If n = l in equation (1)

$$\nu_1 = \frac{\nu}{2l} \Rightarrow \frac{\nu}{2l}\sqrt{T/m}$$



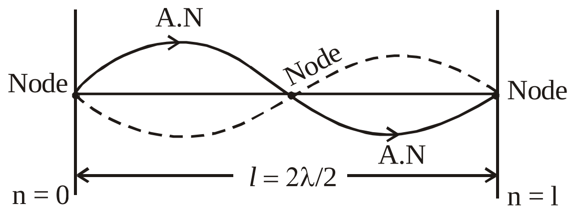
Where ν_1 is fundamental frequency (or) 1st harmonic

Case (ii)

When the string plucked at 1/4 of its length then the string vibrates in two segments we get one extranode in the middle.

If n = 2 is usb in equation (1)

$$\nu_2 = \frac{2}{2l}\sqrt{T/m} = 2\nu,$$



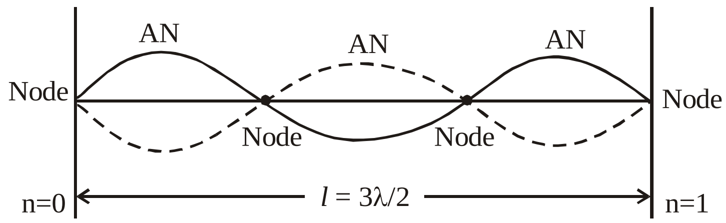
It is known as 1st overtone and II - harmonic.

Case (iii)

When the string is plucked at 1/8 of its length then the string vibrates in three segments we get two extra nodes in the middle.

Sub n = 3 in equation (1) we get

$$\nu_3 = \frac{3v}{2l} - 3\nu_1$$



It is known as II-overtone & III - harmonic

If we take the ratios of all overtones we get

$$\nu_1 : \nu_2 : \nu_2 = \nu_1 : 2\nu_1 : 3\nu_1$$

$$\boxed{\nu_1 : \nu_2 : \nu_2 = 1 : 2 : 3}$$

4. Explain transverse wave propagation along a stretched string ?

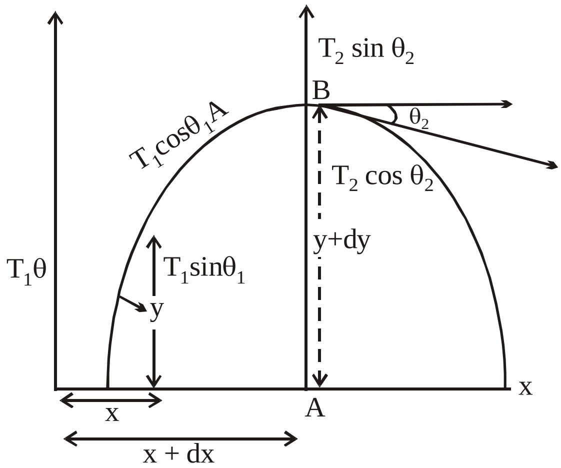
Ans :

- Consider a string stretched between two points A & B
- At "A" the displacement along x-axis is "x" & along y-axis is "Y"
- At $-_B$ "the displacement are x + dx, y + dy
- If tension at both ends is equal then $T_2 = T_1 = T$
- As horizontal components are equal to the force along "x" distance is zero.

$$Fx_1 = 0,\ T\cos\theta_1 = 0 = T\cos\theta_2 \quad \text{... (1)}$$

- Force along vertical direction

$$F_y = T(\sin\theta_2 - \sin\theta_1) \quad \text{... (2)}$$



As w.r.to when θ_1 & θ_2 <<< $\sin\theta_1 \simeq \text{Tan}\,\theta \simeq \left(\frac{\partial y}{\partial x}\right)x$

$$\sin\theta_2 \simeq \text{Tan}\,\theta \simeq \left(\frac{\partial y}{\partial x}\right)_{x+dx}$$

$$F_y = T\left[\left(\frac{\partial y}{\partial x}\right)_{x+dx} - \left(\frac{\partial y}{\partial x}\right)x\right] \quad \text{... (3)}$$

Expanding $\left(\frac{\partial y}{\partial x}\right)_{x+dx}$ using Taylor series

$$\left(\frac{\partial y}{\partial x}\right)_{x+dx} = \left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right)\frac{(dx)^1}{1!} + \left(\frac{\partial^3 y}{\partial x^3}\right)\frac{(dx)^2}{2!} + ...$$

By neglecting higher order terms

$$\left(\frac{\partial y}{\partial x}\right)_{x+dx} = \left[\left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right)dx\right] \quad \text{... (4)}$$

Substituting this values in (3) we get,

$$F_y = T\left[\left(\frac{\partial y}{\partial x}\right)_{x+dx} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

$$F_y = T\left[\left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right)dx - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

$$F_y = T\left(\frac{\partial^2 y}{\partial x^2}\right)dx \quad \text{...(5)}$$

From Newton 2nd Law F = ma

Where,

m = mass per unit length

a = acceleration = $\frac{\partial^2 y}{\partial t^2}$

$\therefore$ F = mdx $\left(\frac{\partial^2 y}{\partial t^2}\right)$ ... (6)

(5) = (6) $m\frac{\partial^2 y}{dt^2}dx = T\left(\frac{\partial^2 y}{dx^2}\right)dx$

$$m\frac{\partial^2 y}{dt^2} = T\frac{\partial^2 y}{dx^2}$$

$$\frac{\partial^2 y}{dt^2} = T/m\frac{\partial^2 y}{dx^2}$$

We now that frequency $\nu = \sqrt{T/m} \Rightarrow \nu^2 = T/m$

$$\boxed{\therefore \frac{\partial^2 y}{\partial t^2} = \nu^2\frac{\partial^2 y}{\partial x^2}}$$

$\nu = \sqrt{T/m}$ is known as the velocity of transverse vibration long a stretched string.

5. Obtain a general solution for transverse wave equation.

Ans :

The equation of transverse wave

$$\frac{\partial^2 y}{\partial t^2} = \nu^2\frac{\partial^2 y}{\partial x^2}$$

Its solution is given by

y = A sin(ωt – kx) A sin(ωt + kx)

Here "ω" angular frequency given by

ω = 2πx or 2πf

$\omega = \frac{2\pi}{T}$ ($\because$ T = 1/f)

- We know that any arbitrary functions either ($\omega t + x$) or ($\omega t - x$) will be solution of the wave equation. Hence, the most general solution would be a linear combination of them.

$$\boxed{y = A_1 \sin(\omega t - x) + A_2 \sin(\omega t + x)}$$

Physical Significance

- So, general solution of wave equation is given by

$$y = F(t - x/v) + g(t + x/v) \qquad ...(*)$$

where f(t – x/v) is "y" value initially then after sometime having phase difference of ϕ we get f(t+x/v)

$$y = A\sin\omega(t - x/v) + A\sin\omega[(t - x/v) + \phi]$$

- Equation (*) represents the incident wave

[1st term & 2nd term] represents the reflected wave.

- The two waves have same amplitude frequency and the wavelength but, in opposite direction, combine to produce stationary wave :

---

6. Explain vibrations of bars ?

Ans :

These are three types of vibrations a bar undergoes :

1. Longitudinal vibrations

The wave which are produced due to the propagation of wae in parallel direction to the vibration of particles are called as longitudinal vibrations.

2. Transverse vibrations

The wave which are produced due to the propagation of wave in perpendicular direction to the vibration of particles is called as transverse vibrations

3. Torsional vibrations

The vibrations produced in a bar due to the angular displacement is called as torsional vibrations.

- The vibration of a bar depends on external force applied on it and the density of the material.

Expression for the velocity of longitudinal wave in a bar is given by $\nu = \sqrt{\frac{y}{\rho}}$

where y = young's modulous

ρ = density.

7. Write the differences between longitudinal and Transverse vibrations ?

Ans :

S.No.	Longitudinal vibrations	Transverse vibrations
1.	Particle vibrate in parallel direction to wave direction	Particles vibrate in perpendicular direction to wave direction
2.	Equation of longitudinal wave $\Rightarrow \frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$	Equation of transverse wave $\Rightarrow \frac{\partial^2 y}{\partial t^2} = v^2 k^2 \frac{\partial^2 y}{\partial x^2}$
3.	Velocity of longitudinal wave $\Rightarrow \nu = \sqrt{\frac{y}{\rho}}$	Velocity of transverse wave $v = \sqrt{\frac{T}{m}}$
4.	Velocity is independent of frequency Ex : sound wave	Velocity depends of frequency Ex : Light (or) Electromagnetic wave

8. Write a brief note on tuning fork ?

Ans :

A tuning fork is in the form of a υ - shaped bar of elastic metal (usually steel). If resonants at a specific constant pitch when set vibrating by striking it against a surface or with an object. It emits a pure musical tone after waiting a moment. The pitch that a particular tuning fork generates depends on the length of the two prongs.

The reason for using the fork shape is that, when vibrating, there is a node in the vibration pattern at the bend of the "u" where the handle is attached, so the handle doesn't vibrate. This allows it to be held there without damping the vibration.

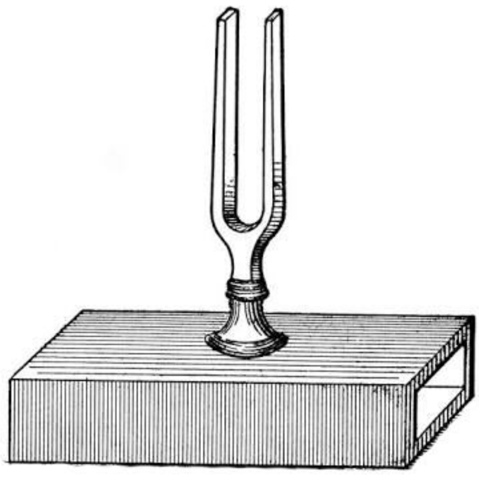


Fig. : Tuning fork on a resonance box

Frequency : The frequency of tuning fork depends on its dimensions and the material from which is made. The frequency is given by

$$f \,\alpha\, \frac{1}{l^2}\sqrt{\frac{AE}{\rho}}$$

and where the tins are cylindrical means

$$f = \frac{r}{\pi l^2}\sqrt{\frac{E}{\rho}}$$

Where

f is the frequency

A $\Rightarrow$ cross-sectional area of tuning fork

l $\Rightarrow$ length of the fork times

E $\Rightarrow$ young's modulus of the material of fork

p $\Rightarrow$ density of material

r $\Rightarrow$ radius of times.

9. Write the uses of tuning fork ?

Ans :

In Musical Instruments :

A number of keyboard musical instruments using construction similar to tuning fork have been made, the most popular of them bring the Rhodes piano, which has hammers hitting constructions working on the same principle as tuning forks.

In Electromechanical Watches :

Electromechanical watches developed by Max Hetzel for Bulova used a 360 Hertz tuning fork with a battery to make a mechanical watch keep time with great accuracy. The production of the Bulova Accutron, as it was called ceased in 1977.

Medical Uses :

Tuning forks, usually e-512, are used by medical practitioners to access a patient's hearing. Lower-pitched ones (usually c-128) are also used to check vibration sense as part of the examination of the peripheral nervous system.

10. Explain the boundary condition in Transverse Vibrations ?

Ans :

The displacement of wave equation is given by

$$y = \cos(wt + \phi)\,[A \cos h(\omega r / u) + B \sin h(\omega x / u)] \; + \; [\cos(\omega x/u) + D \sin(\omega x/u)]$$

- The above equation contains four arbitrary constants and hence, four boundary conditions are required for their determination. Actually these conditions are possible as their exist pair of boundary conditions at each of the two ends of the bar.
- Depending on the nature of support, we discuss the following boundary conditions :

i) Free end : There can be neither an external torque nor a shearing force at the free end. i.e., both τ and F_y should be zero, so we have

$$\frac{\partial^2 y}{\partial x^2} = 0 \text{ and } \frac{\partial^3 y}{\partial x^3} = 0 \text{ at all time}$$

ii) **Clamped end :** When the end of the bar is clamped rigidly, there can be no displacement and slope at all instants of time. Hence, boundary conditions are

$$y = 0 \text{ and } \left(\frac{\partial y}{\partial x}\right) = 0 \text{ at all time}$$

iii) **Supported on a knife edge :** When the end is supported on a knife edge, the displacement and external torque and zero. Hence, boundary conditions are

$$y = 0 \text{ and } \tau = 0 \text{ (or) } \left(\frac{\partial^2 y}{\partial x^2}\right) = 0 \text{ at all time}$$

11. Obtain a general solution for longitudinal wave equations.

Ans :

The longitudinal wave equation is similar to transverse wave equation in case of string. So, the general solution of the wave equation for the transverse vibrations of strings may also be applied in case of longitudinal waves. hence

$$y = f_1(\nu t - x) + f_2(\nu t + x)$$

- Here we assume that "y" varies as a harmonic function of time : the simple harmonic solution may be expressed as

$$y = a_1 \sin(\omega t - kx) + a_2 \sin(\omega t + kx) + b_1 \cos(\omega t - kx) + b_2 \cos(\omega t + kx)$$

Where

a_1, a_2, b_1 and b_2 are amplitude constants

We know that $k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{\nu}$

- Here, k is the propagation constant, "ω" the angular frequency ($2\pi\nu$) and v, the velocity, the velocity of longitudinal waves.

Boundary Conditions

The following boundary conditions are applied :

i) At a point where the bar is fixed, the displacement is zero at all time i.e.,

$$y = 0 \text{ (at all time)}$$

ii) At the free end, there can be no internal elastic force, hence $\dfrac{dy}{dx} = 0$ at all time $\dfrac{dy}{dx} = 0$

By supporting a bar on soft supports placed at some distance from the ends, we can realize the free end condition.

Choose the Correct Answer

1. Transverse wave velocity in a string is directly proportional to [c]
   (a) tension of wire (b) linear density
   (c) square root of tension (d) square root of linear density

2. Transverse wave velocity in a string is inversely proportional to [d]
   (a) tension of wire (b) linear density
   (c) square root of tension (d) square root of linear density

3. If a transverse wave is represented by the equation [a]
   $y = 5 \sin(100t - 4\pi x)$. The velocity of wave is equal to
   (a) 25 (b) 50
   (c) 75 (d) 100

4. A wire of 1m length is vibrating with a frequency of 100Hz. If the length is doubled what is resonance frequency. [d]
   (a) 200 (b) 100
   (c) 75 (d) 50

5. A wire of 1m length is vibrating with a frequency of 200Hz. If the linear density is increased by 4 times, how the vibrational frequency changes [c]
   (a) 200 Hz (b) 400 Hz
   (c) 100 Hz (d) 300 Hz

6. If the tension of wire is increased by 4 times what happens to the vibrational frequency [b]
   (a) increases by three times (b) doubled
   (c) reduced to half (d) remains the same

7. If tension of wire is 100 N, velocity of transverse wave is 200 ms^{-1}, find mechanical impedance [a]
   (a) 0.5 (b) 0.75
   (c) 1 (d) zero

8. If a mass of 10 kg weighed at the end of a wire having 1m length, find the velocity, given the linear density 0.01 kgm^{-1}, assume $g = 10\ ms^{-2}$ [a]
   (a) 100 ms^{-1} (b) 1000 ms^{-1}
   (c) 10 ms^{-1} (d) none

9. The fundamental frequency of vibrating string is 300 Hz. Find its length, given the velocity is 300 ms^{-1}. [b]
   (a) 1m (b) 1/2 m
   (c) 2m (d) none

10. A wave equation is represented by $y = 50e^{0.5j(t-x/100)}$. The velocity and angular frequency are respectively [d]
    (a) 50, 05 (b) 50, 100
    (c) 100, 50 (d) 100, 05

11. The allowed frequencies of longitudinal vibrations in a bar [b]

(a) $\frac{2l}{c}$ (b) $\frac{c}{2l}$

(c) $\frac{l}{2}$ (d) c/2

12. The allowed frequencies of longitudinal waves in free-free bar are integral multiplies of [b]

(a) 21/2 (b) c/21

(c) l / 2 (d) c/2

13. The allowed frequencies of longitudinal waves in fixed-free bar are proportional to [d]

(a) c/2l (b) 2l/c

(c) c/l (d) (2n–1) c/2l

14. The allowed frequencies of longitudinal waves in a bar fixed in mid-point are proportional to which of the following [d]

(a) c/2l (b) 2l/c

(c) c/l (d) (2n–1) c/2l

15. The transverse vibrational frequency of first overtone is a clamped free bar is ______ times the fundamental frequency. [b]

(a) 2.988 (b) 6.267

(c) 4 (d) 6.0

16. The transverse vibrational frequency of 2nd overtone in a free-free bar is ______ times the fundamental frequency [d]

(a) 5.0 (b) 5.5

(c) 5.25 (d) 5.404

17. The ratio of angular velocity/frequency (w) and propagation constant is equal to [d]

(a) wavelength (b) phase

(c) amplitude (d) velocity

18. If f_0 is the frequency of bar, the frequency of tuning fork made up with this bar is ______ times to [c]

(a) 3/2 (b) 1/2

(c) 2/3 (d) 1/4

19. At the stem point of tuning fork the ______occurs [b]

(a) node (b) Antinode

(c) both (d) none

20. If young's modulus of material is 10×10^{-12} Nm^{-2} and its density is $0.1 \times$ gm^{-2}, the velocity of longitudinal wave in the bar is [a]

(a) 10×10^{-6} ms^{-1} (b) 10×10^{-5} ms^{-1}

(c) 10×10^{-7} ms^{-1} (d) none

Fill in the blanks

1. Maximum value of power transported by a transverse wave in a string is ____________
2. The vibrational frequency of a stretched string is equal to ____________
3. The ratio of overtones in a stretched string is ____________
4. The frequency of a string corresponding to n^{th} mode of vibrations is ____________
5. The natural frequency of vibrating string is called as is its ____________ frequency.
6. Fundamental frequency (or) first harmonic is ____________
7. If the number of loops in string is two, the frequency is known as ____________
8. The average power in a string is directly proportional to ____________ of frequency.
9. Velocity of energy transport is equal to ____________ velocity
10. Characteristic impedance of a string depends on ____________ and ____________
11. The frequency of tuning fork is related to its length as ____________
12. Expression for frequency of transverse waves in a bar is ____________
13. The equation for transverse vibrations in a bar is ____________
14. Allowed frequencies of transverse waves in a clamped free bar ____________
15. Fundamental frequency of transverse wave in a clamped bar ____________
16. The frequency of transverse waves in a tree-free bar is given by ____________
17. The fundamental frequency of transverse wave in a tree-free bar is ____________
18. Velocity of longitudinal wave in a bar is directly proportional to ____________
19. The ratio of angular velocity and wave velocity is ____________
20. Expression for velocity of a transverse wave in a bar is ____________

ANSWERS

1. $\frac{4\pi^2 \nu^2 A^2}{v} \cdot T$
2. $\frac{\lambda}{2l}$
3. 1 : 2 : 3 : 4
4. $\frac{n}{2l}\sqrt{T/\mu}$
5. Fundamental

6. $\frac{1}{2l}\sqrt{T/\mu}$

7. 1st overtone (or) second harmonic

8. Square

9. Wave

10. Linear density, velocity

11. Inversely

12. $f_n = \frac{\pi ck}{8l^2}$ $((3.0112)^2, 3^2, 7^2, 9^2)$

13. $\frac{\partial^2 y}{\partial t^2} = -c^2k^2\frac{\partial^4 y}{\partial x^2}$

14. $\frac{\pi ck}{8l^2}$ $(1.194^2, 2.988^2, 5^2, 7^2)$

15. $\frac{\pi ck}{8l^2}$ $(1.194)^2$

16. $f = \frac{\pi ck}{8l^2}(3.0112^2, 5^2, 7^2, 9^2,...)$

17. $f = \frac{\pi ck}{8l^2}$ $(3.0112)^2$

18. Square root of young's modulus of bar

19. Propagation constant

20. $v = \sqrt{wck}$

UNIT - II

Principle of superposition - coherence - temporal coherence and spatial coherence - conditions for Interference of light.

Interference by division of wave front: Fresnel's biprism - determination of wave length of light.

Determination of thickness of a transparent material using Biprism - change of phase on reflection - Lloyd's mirror experiment.

Interference by division of amplitude: Oblique incidence of a plane wave on a thin film due to reflected and transmitted light (Cosine law) - Colours of thin films - Non-reflecting films - interference by a plane parallel film illuminated by a point source - Interference by a film with two non-parallel reflecting surfaces (Wedge shaped film) - Determination of diameter of wire - Newton's rings in reflected light with and without contact between lens and glass plate, Newton's rings in transmitted light (Haidinger Fringes) - Determination of wave length of monochromatic light - Michelson Interferometer-types of fringes - Determination of wavelength of monochromatic light, Difference in wavelength of sodium D_1, D_2 lines and thickness of a thin transparent plate.

2.1 Principle of Superposition

Q1. Define the principle of Superposition and explain it.

(OR)

Write short a note on Principle of Superposition.

Ans : **(May-18, Imp.)**

If two or more light waves of the same frequency overlap at a point, the resultant effect depends on the phases of the waves as well as their amplitudes. The resultant wave at any point, at any instant of time is governed by the principle of superposition.

- In simple way, we can say that the principle of superposition of waves states that the resultant displacement at a point is equal to the vector sum of the displacements of different waves states that the resultant displacement at a point is equal to the vector sum of the displacements of different waves at that point.
- The principle of superposition applies to electromagnetic waves also and is the most important principle in wave optics
- Interference is an important consequence of superposition of coherent waves

Superposition of two sinusodial Waves

consider two sinusodial waves travelling in z – direction. The two waves are represented by the wave equations given below

First wave : $\frac{d^2x_1}{dz^2} - \frac{1}{V_1^2}\frac{d^2x_1}{dt^2}$ & Second Wave : $\frac{d^2x_2}{dz^2} - \frac{1}{V_2^2}\frac{d^2x_2}{dt^2}$

The solutions for the above equations represent the displacement produced by each of the waves

Let the solutions are given by

$x_1(t) = a_1 \cos(\omega t + \theta_1)$ and

$x_2(t) = a_2 \cos(\omega t + \theta_2)$

We are assuming that the displacements are in the same direction, however they may have different amplitudes and different initial phases. Now, according to the superposition principle the resultant displacement x(t) would be given by

$x(t) = x_1(t) + x_2(t)$

$x(t) = a_1 \cos(\omega t + \theta_1) + a_2\cos(\omega t + \theta_2)$

$x(t) = a_1 (\cos\omega t \cos\theta_1 - \sin\omega t \sin\theta_1) + a_2 (\cos\omega t\cos\theta_2 - \sin\omega t\sin\theta_2)$

$x(t) = \cos\omega t (a_1\cos\theta_1 + a_2\cos\theta_2) - \sin\omega t (a_1\sin\theta_1 + a_2\sin\theta_2)$ —— (I)

Let $a\cos\theta = a_1\cos\theta_1 + a_2\cos\theta_2$ — (1)

and $a\sin\theta = a_1\sin\theta_1 + a_2\sin\theta_2$ — (2)

sub (1) and (2) in (I)

$\therefore$ $x (t) = a\cos\omega t\cos\theta - a\sin\omega t\sin\theta$

$= a(\cos qt\cos\theta - \sin\omega t \sin\theta)$

$= a \cos (\omega t + \theta)$

by squaring and adding equations (1) and (2) we get

$a = (a_1^2 + a_2^2 + 2a_1a_2 \cos (\theta_1 - \theta_2))1/2$

on dividing (2) by (1) we get

$$\tan q = \frac{(a_1 \sin\theta_1 + a_2 \sin\theta_2)}{(a_1 \cos\theta_1 + a_2 \cos\theta_2)}$$

or $$\boxed{\theta = \frac{\tan^{-1}(a^1 \sin\theta_1 + a_2 \sin\theta_2)}{(a_1 \cos\theta_1 + a_2 \cos\theta_2)}}$$ ... II

Here from ((II) we can easily get ‘θ’ value

we find that if

$\theta_1 \sim \theta_2 = 0, 2\pi, 4\pi ..... = 2\pi n, n = 0,1,2,3$

Then $a = a_1 + a_2$

2.2 Coherence

2.2.1 Temporal Coherence and Spatial Coherence

Q2 Write about coherent sources and its types. Explain briefly.

(OR)

Explain the terms temporal and spatial coherence.

Ans : **(Jan.-21, Imp.)**

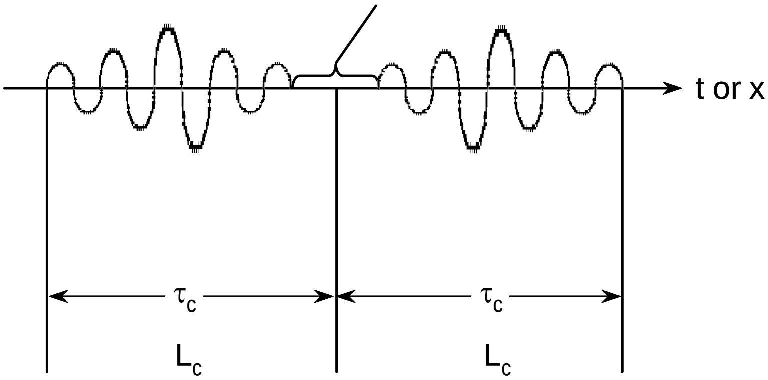
In order to produce a stable interference pattern the individual waves must maintain a constant phase relationship with one another i.e., the two sources must emit waves having a constant phase difference between them such sources are called coherent sources.

Coherence is two types

i) Temporal coherence

ii) Spatial coherence

- The phenomenon is known as coherence
- Light waves are said to be coherent. If they maintain constant phase difference over a period of time. A constant phase difference is maintained easily if the waves are harmonic waves. But real light waves are not perfectly harmonic waves for all values of time.
- The phase relationship fluctuates irregularly, from one wave train to another wave train as shown in diagram.



(i) Temporal Coherence

Definition

If two waves maintain a definite relationship between their phases at a given time and at certain time later, then the waves are said to be temporally coherent, this phenomenon is known as temporal coherence.

- Consider a point source of quasi monochromatic lights, which emit light in all directions as shown in diagram
- Consider the light travelling along the path SP_1P_2.
- The phase relationship between the points P_1 and P_2 depends on the distance P_1P_2 and the coherence length of the light beam. The coherence length L, of the light is defined by the equation.

$$L = C\,T_C$$

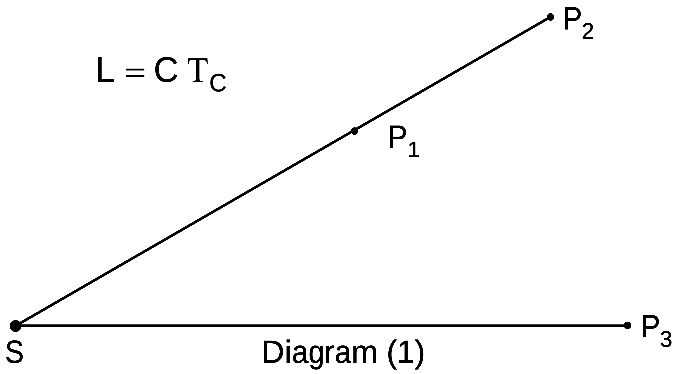


Diagram (1)

Where τ_c is called the coherence time and c is the speed of light.

- Temporal coherence is characterized by two parameters namely coherence length (L) and coherence time (τ_c)

Coherence Time (tc) and Coherence Length (L_c)

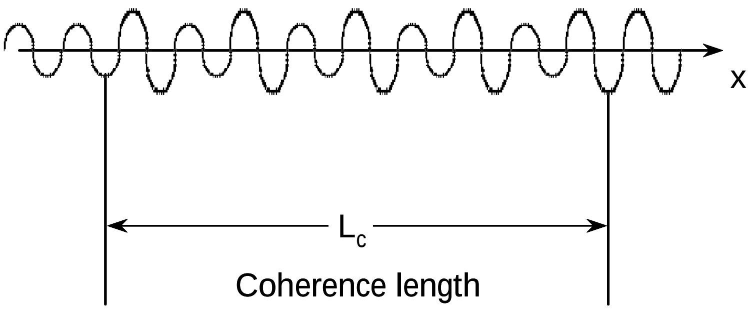
(a) Coherence Time (τ_c)

In conventional light sources, light is emitted in the form of short wave trains and the phase of one wave train would remain constant with respect to the phase of another wave train only for about 10^{-10} sec.

- The average time during which the ideal sinusodial charmonic wave emission exists is called coherence time (τ_c).

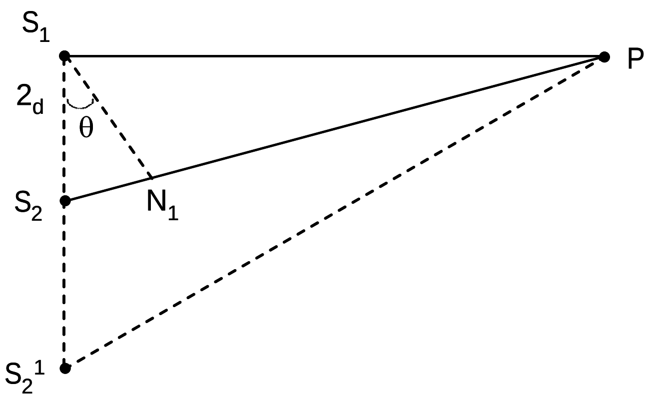
(b) Coherence Length (L_c)

It is the length of the wave packet over which it may be assumed to be sinusodial and has predictable phase. It is denoted by L_C.



(ii) Spatial Coherence

Spatial coherence refers to the continuity and uniformity of a wave in a direction perpendicular to the direction of propagation. If the phase difference for any two fixed points in a plane normal to the wave propagation does not vary with time, then the wave is said to exhibit spatial coherence



- From diagram (1) $SP_1 = SP_3$ so, the fields at points P_1 & P_2 have same phase. Thus an ideal point source exhibits spatial coherence if the waves produced by it are likely to have the same phase at points in space which are equidistant from the source.

2.2.2 Conditions for Interference of Light

Q3. Explain the conditions for sustainable Interference of Light.

Sol : **(June-19)**

The conditions for obtaining a distinct well-defined interference pattern are,

(i) Conditions for Sustained Interference

- The frequency of light waves from two sources must be same
- The two light waves must be coherent to each other
- The coherence length (l_{coh}) must be greater than the path difference (Δ) between the overlapping waves. i.e., $l_{coh} > \Delta$.
- The polarization planes of two sets of waves must be same.

(ii) Conditions for the Formation of Distinct Fringe Pattern

- The clearly observe fringes, the distance between two coherent sources must be small.
- The screen must be placed at a large distance from the two sources.
- In dark regions, the vector sum of overlapping electric field vectors must be zero.

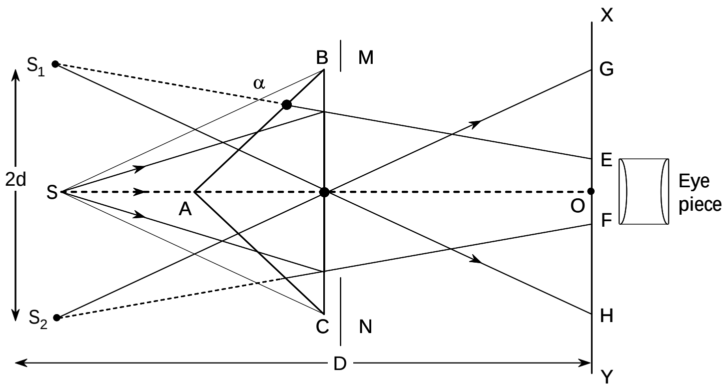
2.3 Interference by Division of Wave Front

2.3.1 Fresnel's Biprism

Q4. Explain Fresnel's Biprism method for Determination of Wave Length of Light.

Ans : **(Jan.-21, June-19, June-18, Imp.)**

Frensel divised a simple arranged for the production of interference pattern. He used a biprism, which consists of two prisms of very small refracting angles joined base to base. In practice, a thin glass plate is taken and one of its faces is ground and polished till a prism is formed with an obtuse angle of about 179° and two side angles of the order of 30'.



- As shown in diagram the biprism ABC consists of two acute angled prisms placed base to base. (Actually, it is constructed as a single prism of obtuse angle of about 179°). The acute angle 'α' on both sides is about 30' (or $\frac{1}{2}°$). The prism is placed with its refracting edge parallel to the line of source (S) slit, such that SA is normal to the face BC of the prism

- When light from S falls on the lower portion of the prism it is bent upwards and appears to come from the virtual source S_2. Similarly light from S falling on the upper portion of the prism is bent downwards & appears to come from the virtual source S_1.
- Therefore S_1 & S_2 act as two coherent sources, if a screen XY is placed at O, interferences fringes of equal width are observed between E & F out beyond E & F fringes of large width are observed. These are due to diffraction. MN is a stop to limit the rays. The fringes can be viewed by replacing screen with an eye piece.
- The theory of the interference and fringe promotion in case of Fresnel biprism is the same as that for the double – slit. As the point 0 is equidistant from S_1 and S_2, the central bright fringe of maximum intensity occurs there. On both sides of "0", alternate bright & dark fringes are formed. We know that the width of the dark or bright fringe is given by equation.

$$\beta = \frac{\lambda D}{d}$$

- Where D = (a + b) is the distance of the source from the eye piece. Here a is the distance between source and biprism and b is the distance between biprism and eyepiece, λ is the wavelength of the light used and d is the distance between two coherent sources S_1 & S_2.

Determination of fringe width (β)

- When the fringes are observed in the field view of the eyepiece, the vertical cross-wire is made to coincide with the centre of one of the bright fringes. The position of the eyepiece is read on the scale, say x_0. The micrometer screw of the eye piece is moved slowly and the number of the bright fringes "N" that pass across the cross – wire is counted. The position of the cross – wire is again read say X_N.

  The fringe width is given by $\beta = \frac{(X_N - X_0)}{N}$

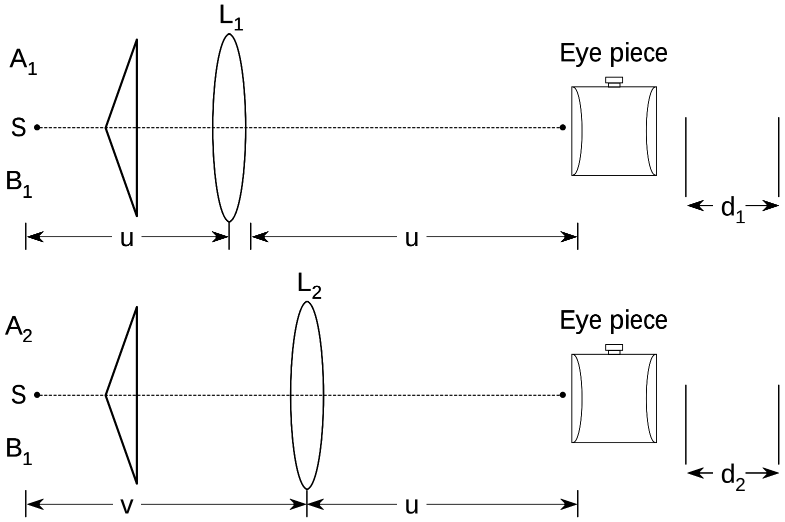
Determination of d (lens displacement method)

- A convex lens (L_1) of short focal length is placed between the slit and the eyepiece without distributing their positions. The lens is moved back and forth near the biprism till a pair of sharp and real images of slits are obtained in the field of view of the eye piece. The distance between these images is measured with the help of the micrometer eyepiece. Let it be denoted by d_1. If u is the distance of slit and v that of the eyepiece from the lens then the magnification is

$$\frac{v}{u} = \frac{d_1}{d} \quad .... (1)$$

- The lens is then moved to a position nearer to the eyepiece, where pair of images of slits are obtained in the field of view of eyepiece. The distance between these images is measured with the help of micrometer eyepiece. Let it be denoted by d_2. Again the magnification is given by

$$\frac{V}{u} = d_2/d$$



- Note that magnification in one position is the reciprocal of the magnification in the other position

  on multiplying (1) and (2) we obtain

$$\frac{d_1 d_2}{d^2} = 1$$

$$\therefore \; d = \sqrt{d_1 d_2}$$

- The procedure of determining d_1 and d_2 is repeated at least 3 times by moving eyepiece into different points and the average value of d is found

- The value of D, which is the distance between the source and eye piece can be measured directly from the scale attached to the optical bench. Now the values of β, d and D, The wavelength λ can be computed.

2.3.2 Determination of Wave Length of Light Determination of thickness of a Transparent Material using Biprism

Q5. Determine the thickness of a Transparent material using Biprism.

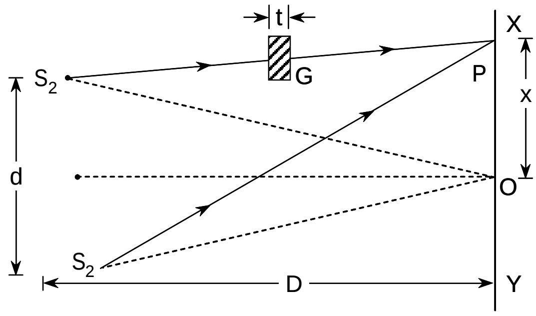
Ans : **(June-19)**

- The thickness of a given thin sheet of transparent material such as glass (or) mica can be determined using biprism experiment

- If a thin transparent sheet is introduced in the path of one of the two interfering beams, the interference pattern gets displaced towards the beam in whose path the sheet is introduced. By measuring the amount of displacement, the thickness of the sheet can be determined.

Theory

Suppose S_1 & S_2 are the virtual coherent monochromatic sources. The point O is euqidistant from S_1 & S_2, where we obtain the central bright fringe.



- Let a transparent material G of thickness t and refractive index μ be introduced in the path of one of the beams.
- The optical path lengths S_1O and S_2O are now not equal and the central bright fringe shifts to P from O. The light waves from S_1 to P travel partly in air and partly in the material G; the distance travelled in air is $(S_1P - t)$ and that in the sheet is 't'.

The optical path $\Delta_{S_1P} = (S_1P - t) + \mu t = S_1P = (\mu - 1)\, t$

The optical path $\Delta_{S_2P} = S_2P$

- The optical path difference at P is $\Delta S_1P - \Delta S_2P = 0$, since in the presence of the thin material, the optical path lengths S_1P & S_2P are equal and central zero fringe is obtained at P.

$$\therefore\ \Delta_{S_2P} = S_2P$$

$$(S_1P + (\mu - 1)t) = S_2P$$

$$S_2P - S_1P = (\mu - 1)\, t$$

- But according to the relation for optical path difference between the waves at P. We have $S_2P - S_1P = X\, d/D$, where x is the lateral shift of the central fringe due to the introduction of the thin material.

$$(\mu - 1)t) = \frac{Xd}{D}$$

Hence, the thickness of the thin material

$$t = \frac{Xd}{D(\mu - 1)}$$

$$x = \frac{(\mu - 1)\, Dt}{d}$$

If displacement $x = n\beta$, β is fringe width then

$$x = n\beta$$

$$\therefore\ n\beta = \frac{D}{d}(\mu - 1)\, t$$

$$\text{(or) } t = \frac{n\beta d}{D(\mu-1)} = \frac{n\lambda}{(\mu-1)} \qquad \left(\because \beta = \frac{\lambda D}{d}\right)$$

So, by knowing λ, μ and n also we can calculate the thickness of the plate.

Q6. Explain the features of Interference pattern produced with a Biprism.

Ans : **(Jan.-21)**

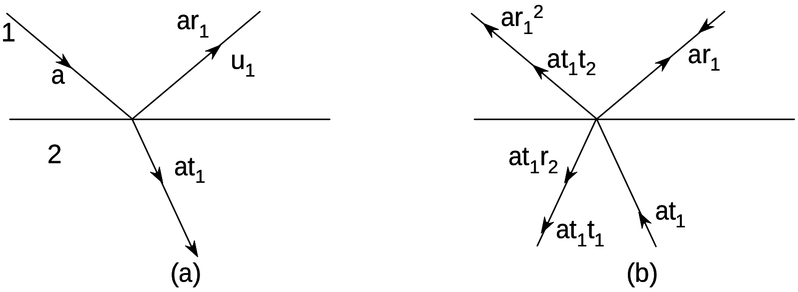
1. In a biprims experiment, the interference pattern consists of coloured fringes on both sides of white fringe when a white light is illuminated from the slit.
2. In this, monochromatic light produces only bright fringes of same colour.
3. The thickness of a thin sheet of transparent material can be determined through biprism experiment, when it is introduced in the path of two interfering beams.

2.3.3 Change of Phase on Reflection

Q7. What is meant by phase change on reflection?

Ans : **(June-18, Imp.)**

Consider a light ray incident on an interface of two media of refractive indices μ_1 and μ_2. Let medium 2 be optically denser than medium 1 ($\mu_2 > \mu_1$)



- The ray is partly reflected and partly refracted. Let r_1 is the reflection coefficient (i.e., the fraction of the amplitude reflected) and 't_1' is the transmission coefficient. Thus, if the amplitude of the incident ray is a, then the amplitudes of the reflected and refracted beam would be ar_1 and at_1 respectively.
- If the paths of the reflected ray and the refracted ray are reversed, then by the principle of optical reversibility they should give rise to the wave of original amplitude a, provided there is no dissipation of energy by absorption.
- We know reverse the rays and consider a ray of amplitude at_1 incident on medium 1 and a ray of amplitude ar_1 incident on medium 2 as shown in diagram. The ray of amplitude at_1 will rive rise to a reflected ray of amplitude at_1r_2 and a transmitted ray of amplitude at_1t_2, where r_2 and t_2 are the reflection and transmission coefficients when a ray is incident from medium 2 to medium 1. Similarly, the ray of amplitude ar_1 will give rise to a ray of amplitude ar_1^2 and a refracted ray of amplitude ar_1t_1.

  According to the principle of optical reversibility, we have

$$ar_1^2 + at_1t_2 = a$$

or $\quad t_1t_2 = 1 - r_1^2$

Further the two rays of amplitudes at_1r_2 and ar_1t_1 must cancel each other;

i.e., $at_1 r_2 + ar_1 t_1 = 0$

(or) $r_2 = -r_1$

From the equation $t_1t_2 = 1 - r_1^2$, we have

$r_1^2 = 1 - t_1t_2$. Also we have $r_1^2 + t_1^2 = 1$

From these two equations we get $t_1 = t_2$

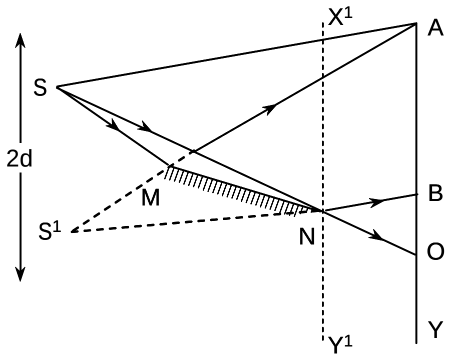
➢ Whereas in the equation $r_2 = -r_1$, the negative sign represents that an abrupt phase change of Π occurs when light gets reflected by a denser medium and that no such abrupt phase change occurs when light gets reflected by a rarer medium.

2.3.4 Lloyd's Mirror Experiment

Q8. Explain Lloyd's Mirror experiments. Determine the wavelength of light using it ? Differentiae Lloyd's and Biprism mirror fringes.

Ans : **(June-18)**

➢ Lloyd designed a simple method to obtain interference fringes with the help of a single plane mirror. Here also, the method employed is the technique of division of wave front.



➢ Lloyd designed a simple method to obtain interference fringes with the help of a single plane mirror. Here also, the method employed is the technique of division of wavefront.

➢ In this arrangement mono chromatic light from a slit S is allowed to fall on a plane mirror M at grazing incidence.

➢ The mirror M is either a flat polished metal or a piece of optically plane glass plate blackened at the back surface so that no reflection takes place from the back surface of the mirror. The reflected light appears to be diverging from S', i.e., the virtual image of source S. S and S' serve as two coherent sources and interference fringes are observed on a screen xy with in the region AB.

Determination of Wavelength

➢ For this purpose Lloyd's mirror is mounted vertically on an upright two of an optical bench with its surface parallel to the length of the bench. On first upright, a narrow vertical slit is mounted and is illuminated with mono chromatic light whose wave length is to be determined. Micrometer eye piece is mounted on upright four. Now the mirror is rotated about an axis parallel to the length of the bench until distinct fringes are observed throughout eye piece.

➢ Fringe with β is measured using micrometer eyepiece

➢ The distance 2d between the sources S and S' is measured using lens displacement method

➢ The distance D is measured and wavelength of light is calculated using the formula

$$\boxed{\lambda = \frac{\beta(2d)}{D}} \qquad \because \beta = \frac{\lambda D}{2D}$$

Differentiate Lloyd's and Biprism mirror fringes

1) In biprism arrangement, the complete pattern of interference fringes on both sides of central fringes is obtained. In Lloyd's mirror arrangement, only few fringes on one side of the central fringe are observed and the central fringe itself invisible.
2) In biprism arrangement, the central fringe is bright while that of Lloyd's mirror is dark
3) The central fringe is biprism is less sharp than that in Lloyd's mirror.
4) The conditions of constructive and destructive interference in Lloyd's mirror are opposite to that in biprism due to a phase change of Π (or a path difference of $\lambda/2$) in reflected beam.
5) In biprism experiment, the fringe width is same for all pairs of coherent sources. In Lloyd's mirror d is different for different pairs of coherent sources and hence fringe width is different pairs of coherent sources.

2.4 Interference By Division of Amplitude

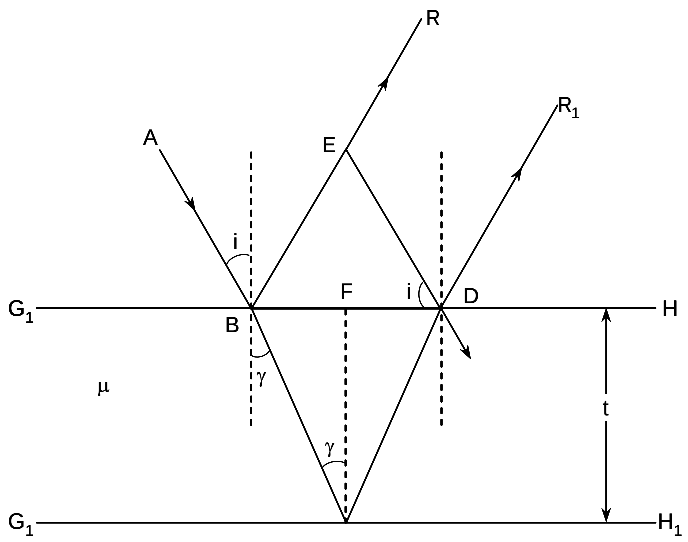
2.4.1 Oblique Incidence of a Plane Wave on a thin Film due to Reflected and Transmitted Light (Cosine Law)

Q9. Explain oblique incidence of plane wave on a thin film due to reflected light (cosine law).

Ans : **(Imp.)**

Consider a transparent film of uniform thickness

➢ Let GH and G_1H_1 be the two surfaces of a transparent film of uniform thickness t and effective index μ suppose a ray AB of monochromatic light is incident on its upper surface. This ray is partly reflected along BR & refracted along BC.



- After one reflection at C, we obtain the ray CD. after refraction at D, the ray finally emerges out along DR_1 in air
- Obviously, DR_1 is parallel to BR. Our aim is to find out the effective path difference between the rays BR & DR_1. For this purpose we draw a normal DE & BR.

The path difference Δ is given by

$$\Delta = \mu\,(BC + CD) - BE \qquad \text{... (1)}$$

From diagram

$$\frac{CF}{BC} = Cosr$$

$$\frac{t}{BC} = Cosr$$

$$BC = CD = \frac{t}{Cosr} \qquad \text{... (2)}$$

- In order to calculate BE, we first find BD which is equal to (BF + FD). We consider triangle BFC.

$$\frac{BF}{FC} = tanr$$

(or) $$\frac{BF}{t} = tanr$$

$$BF = t\,tanr$$

Now, $BD = BF + FD = 2BF = 2t\,tanr$

from triangle BED, $\frac{BE}{BD} = Sini$

$$BE = BD\,Sini = 2t\,tanr\,sini$$

(or) We know that

$$\frac{Sini}{Sinr} = \mu \text{ (or) } sini = \mu\,sinr$$

$$BE = 2t\,tanr\,(\mu Sinr)$$

$$BE = 2\mu t\,tanr\,sinr \qquad \text{... (3)}$$

From equations (2) and (3), substituting the values of BC & BE in (1) equation, we get

$$\Delta = \mu - \left(\frac{2t}{\cos r}\right) 2\mu t.tanr.sinr$$

$$= \frac{2\mu t}{Cosr} - 2\mu t\,\frac{Sin^2 r}{Cosr}$$

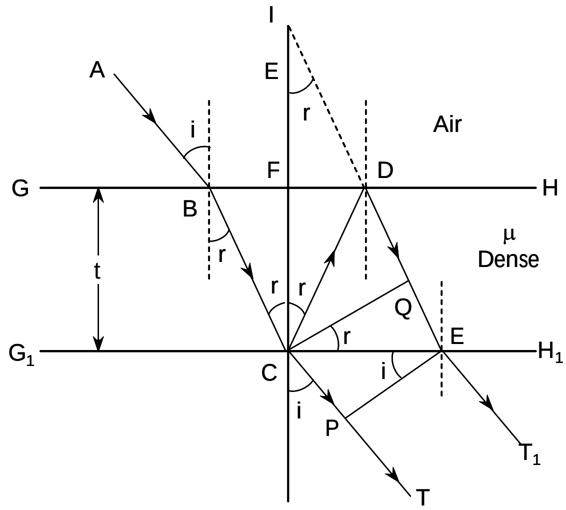
$$= \frac{2\mu t}{Cosr}[1 - \sin^2 r] = = \frac{2\mu t}{Cosr}\cos^2 r$$

$\therefore$ $\boxed{\Delta = 2\mu t \cos r}$ This is also known as "cosine law"

Q10. Explain Interference due to transmitted light.

Ans :

Let us consider a transparent film of uniform thickness 't' bounded by two parallel surfaces and refractive index 'μ"



- The Diagram shows the geometry of the transmitted light. Due to simultaneous reflection and refraction we obtain two transmitted rays CT and εT_1. These rays originated from the same point source, hence they have a constant phase difference and are in a position to produce sustained interference when combined. In order to calculate the path difference between the two transmitted rays. We draw normal CQ and EP DE and CT respectively
- We also produce ED in the backward direction which meets produced CF at I

The effective path difference is given by

$$\Delta = \mu\,(CD + DE) - CP \qquad \text{... (1)}$$

$$m = \frac{Sini}{Sinr} = \frac{CP/CE}{QE/CE} = \frac{CP}{Q\varepsilon}$$

$$CP = \mu\,(QE) \qquad \text{... (2)}$$

From equations (1) & (2)

$$\Delta = \mu\,(CD + DQ + QE) - \mu\,(QE)$$

$$= \mu\,(CD + DQ) = \mu\,(QI) \qquad (\because CD = ID)$$

$$= 2\,\mu t \cos r$$

- Here, it should be remembered that inside the film, reflection at different points takes place at the surface backed by rarer medium (air) thus, no abrupt change of Π takes in this case

The maxima occurs when effective path

difference $\Delta = n\lambda$

i.e., $\therefore$ $\boxed{2\mu t \cos r = n\lambda}$ ... (A)

- If this condition is fulfilled, the film appears bright in transmitted light
- Thus, the conditions is fulfilled, the film appears bright in transmitted light
- The minima occurs when the effective path difference is $(2n \pm 1)\lambda/2$, i.e.,

$$\boxed{2\mu t \cos r = (2n \pm 1)\lambda/2}$$

Where n = 0,1,2,3........etc, when this condition is fulfilled, the film will appear dark.

Thus, the conditions of maxima and minima in transmitted light are just reverse of the condition for reflected light

2.4.2 Colours of thin films, Non – Reflecting Films

Q11. Explain colors of thin films.

Ans : **(Jan.-21)**

When a thin film is illuminated by monochromatic light and seen the reflected light, it will appear bright if $2\mu t \cos r = (2m+1)\lambda/2$ and dark if $2\mu t \cos r = m\lambda$.

- If, however the film is illuminated by white light, the film shows different colours.
- The colours exhibited in reflection by thin films of oil, soap bubbles and coatings of oxides on heated metals etc are due to interference of light from an extended source such as sky. It may be understood as follows.

The films are usually observed by reflected light. The eye looking at the thin film receives light waves reflected from the top and bottom surfaces of the film. For a think film these rays are very close to each other and in a position to interefere. The optical path difference between the interfering ray is $\boxed{\Delta = 2\mu t \cos r - \lambda/2}$.

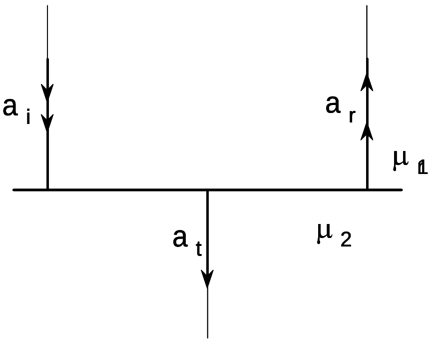
- It is seen that the path difference depends upon the thickness "t" of the film, the wavelength l and the angle r, which is related to the angle of incidence of light on the film.
- White light consists of a range of wavelengths and for specific values of t and r, waves of only certain wavelengths (colors) satisfy the condition of maxima. Therefore, only those colours are present in the reflected light.
- The other wavelengths interfere destructively and hence are absent from the reflected light. Hence, the film at a particular point appears coloured.
- As the thickness and the angle of incidence vary from point to point, different colours are intensified at different places. The colours seen are not isolated colours, as at each place there is a mixture of colours.

As the conditions of maxima and minima for transmitted light are opposite to those for reflected light, therefore with the use of white light, the colours visible in the reflected light will be complementary to the colours visible in the transmitted light.

Q12. Describe the principle and application of non–reflecting films.

Ans :

An important application of the thin film interference phenomenon is the coating of optical lens surfaces with non–reflecting film. Let us discuss the need of non–reflecting film in detail.



- When a light beam propagating in a medium of refractive index m$_1$ is incident normally on a transparent surface of refractive index μ_2, then the amplitude of reflected and transmitted beams are given by

$$\left.\begin{aligned} a_r &= \left(\frac{\mu_1-\mu_2}{\mu_1+\mu_2}\right)a_1 \\ &\text{and} \\ a_t &= \left(\frac{2\mu_1}{\mu_1+\mu_2}\right)a_1 \end{aligned}\right\} \text{respectively}$$

- If the intensity of the incidenting light beam is I $(=a_i^2)$ then intensity of the reflected is transmitted light beam are

$$\left.\begin{aligned} I_r &= a_r^2\left(\frac{\mu_1-\mu_2}{\mu_1+\mu_2}\right)^2 I \\ &\text{and} \\ I_t &= a_t^2\left(\frac{2\mu_1}{\mu_1+\mu_2}\right)^2 I \end{aligned}\right\} \text{respectively}$$

At air – glass surface [μ_1 = 1 & μ_2 = 1.5], the intensity of the reflected beam

$$Ir = \left(\frac{1-1.5}{1+1.5}\right)^2 I = \frac{1}{25} I$$

$$= 0.04\ I$$

$$\frac{I_r}{I} = 0.04 \text{ or } 4\%$$

i.e., 4% of the incident light is reflected. For a dense flint glass m $\simeq$ 1.67 and about 6% of light is reflected. Thus, if we have a large number of surfaces, the losses at the interfaces can be considerable. In order to reduce these losses, lens surfaces are often coated with a $\dfrac{\lambda}{4\mu_f}$ "non-reflecting film".

- The refractive index of the film ($=\mu_f$) being less than that of the lens

A film having a thickness of $\frac{\lambda}{4\mu_f}$ and having refractive index less than that of glass is coated on glass, then waves reflected from the upper surface of the film distructively interfere with the waves reflected from the lower surface of the film. Such a film is known as a non–reflecting film. For example, glass (μ = 1.5) may be coated with an MgF_2 film. Let the film thickness be t (since the refractive index of the non – reflecting film is greater than that of air and less than that of the glass, abrupt phase change of Π occurs at both the reflections and when $2\mu t \cos r = (2m+1)^{\lambda}/_{2}$, there would be destructive interference).

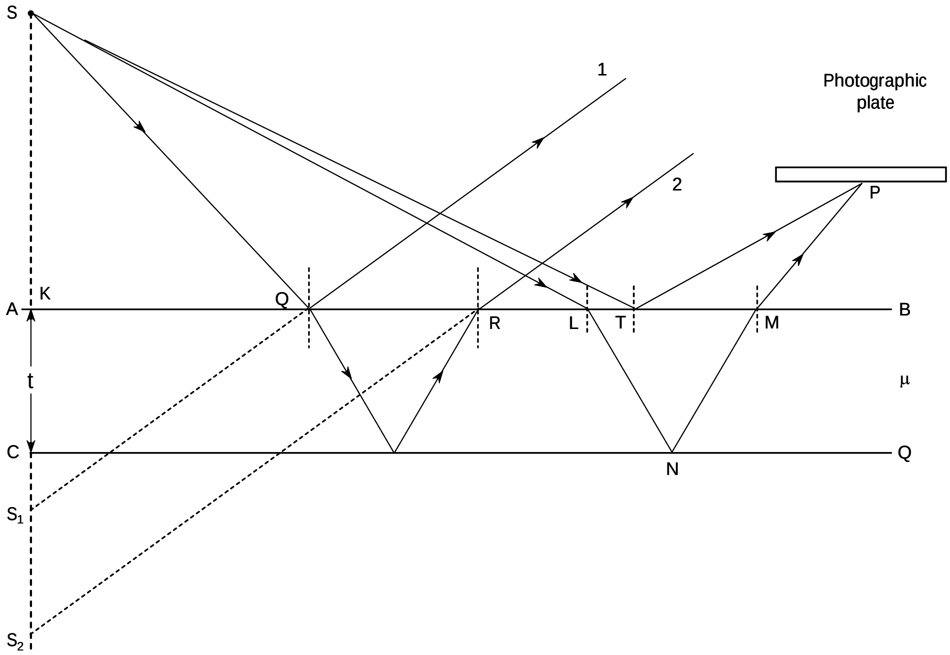
The film thickness "t" is such that $2\mu_f t = {}^{\lambda}/_{2}$ or $t = \frac{\lambda}{4\mu_f}$

2.4.3 Interference by a Plane Parallel Film Illuminated by a Point Source

Q13. Discuss the phenomenon of interference by a plane parallel film when illuminated by a point source.

Ans :

Consider a plane parallel film of uniform thickness 't' and refractive index μ. Suppose light from a point source S is allowed to fall on the transparent film. The distance of the source S from the upper surface AB is SK



- The wave SQ reflected from the upper surface appears to emerge from S_1 such that $SK = KS_1$. Further the wave reflected from the lower surface will appear to emerge from S_2 such that $KS_2 = KS_1 + S_1S_2 = KS + S_1S_2$. Thus, for very nearly normal incidence, S_1 and S_2 act as two coherent sources and produce interference pattern. The interference pattern will be same as observed in young's double slit experiment. If a photographic plate P is placed as shown in diagram.
- Then the interference pattern will be registered on it. The path difference between the waves reaching at P is given by

Path difference $\Delta = SL + \mu\,(LN + NM) + MP - (ST + TP)$

The conditions for maximum or minimum intensities are

$$\Delta = (2n + 1) \lambda/2$$

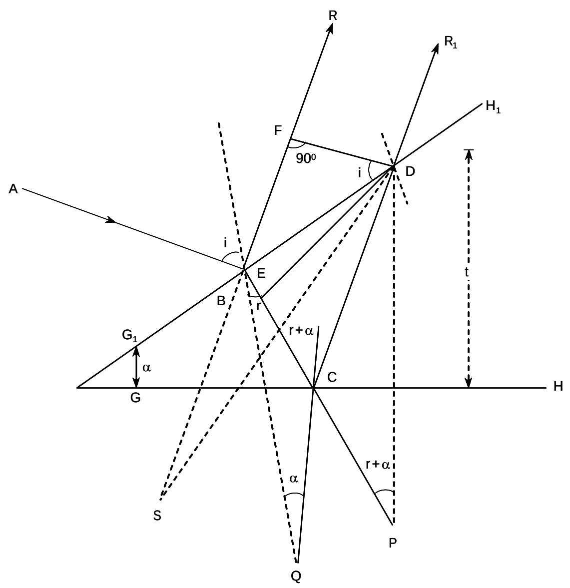
$$\Delta = n\lambda$$

When the photographic plate is parallel to the reflecting surface, dark & bright rings are observed.

2.4.4 Interference by a Film with Two Nonparallel Reflecting Surfaces (Wedge Shaped Film)

Q14. What is wedge shaped film? Describe the fringes observed when a wedge shaped film illuminated by normally reflected light.

Ans :



- Consider two plane surfaces GH and G_1H_1 inclined at an angle α and enclosing a wedge shaped air film. The thickness of air film increasing from G to H as shown in diagram. Let μ be the refractive index of the material of this film. When this film is illuminated by sodium light, then the interference between two system of rays one reflected from the front surface and the other obtained by internal reflection at the back surface and consequent transmission at the first surface, takes place.

- It is observed from the figure that the interfering waves BR & DR_1 are not parallel but appear to diverge from a point S. Thus, the interferences takes place at S which is virtual. In order to consider the interferences between these two waves we will first calculate the path difference between them. Clearly the optical path difference Δ is given by

$$\Delta = \mu (BC + CD) - BF \qquad (BF = \mu BE)$$

$$= \mu (BE + EC + CD) - \mu BE$$

$$= \mu (EC + CD) = \mu (EP)$$

$$= 2 - \mu t \cos (r + \alpha) \qquad \text{... (1)}$$

- Due to reflection an additional phase change of $\lambda/2$ is introduced

hence $\Delta = 2\mu t \cos (r + \alpha) \pm \lambda/2$ ... (2)

for constructive interference, $\Delta = n\lambda$

$$2\mu t \cos(r + \alpha) \pm \lambda/2 = n\lambda$$

(or) $2\mu t \cos(r + \alpha)\ (2n \pm 1)\ \lambda/2$ = (for maxima)... (3)

for destructive interferences

$$D = (2n \pm 1)\ \lambda/2$$

$\therefore\ 2\mu t \cos(r + \alpha) \pm \lambda/2 = n\lambda = (2n \pm 1)\ \lambda/2$

(or) $\boxed{2\mu t \cos(r + \alpha) = n\lambda}$ (for minima) ... (4)

Where n = 0, 1, 2, 3 ..... etc.

Nature of Interference pattern

If the light illuminating the film is parallel then 'i' is constant everywhere and so is 'r', the angle of refraction. In addition, if the light used is monochromatic, the path change will occur only due to t. In this case the fringes will be characteristic of equal optical thickness. Since, in this case of wedge – shaped film, 't' remains constant only in direction parallel to the thin edge of wedge, hence the straight fringes parallel to the edge of the wedge are obtained.

Thus, bright or dark fringes are obtained, here as the condition for the thickness 't' is satisfied according to equations (3) and (4) respectively.

Spacing between two consecutive bright bands :

For n^{th} maxima, we have

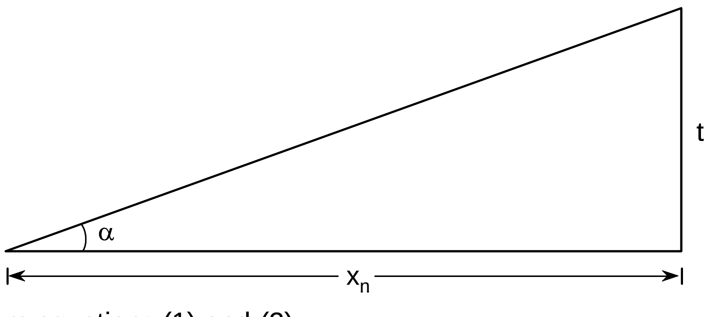
$$2\mu t \cos(r + \alpha) = (2n + 1)\ \lambda/2 \quad \text{... (1)}$$

For normal incidence and air film,

$r = 0$ and $\mu = 1$

$$2t \cos\alpha = (2n + 1)\ \lambda/2 \quad \text{... (2)}$$

➢ Let this band be obtained at a distance x_n from the thin edge as shown diagram



➢ From the diagram equations (1) and (2)

$$2x_n \tan\alpha \cos\alpha = (2n + 1)\ \lambda/2$$

$$2x_n \sin\alpha = (2n + 1)\, \lambda/2 \qquad \ldots (3)$$

$\Rightarrow$ If the $(n + 1)^{th}$ maximum is obtained at a distance

x_{n+1} from the thin edge,

$$2x_{n+1} \sin\alpha = [2\,[n + 1] + 1]\, \lambda/2$$

$$= (2n + 3)\, \lambda/2 \qquad \ldots (4)$$

Subtracting equation (4) from equation (3), we get

$$2\,(x_{n+1} - x_n) \sin\alpha = \lambda$$

spacing $\beta = x_{n+1} - x_n$

$$\beta = \frac{\lambda}{2\sin\alpha} = \frac{\lambda}{2\alpha}$$

Where α is small and measured in radians

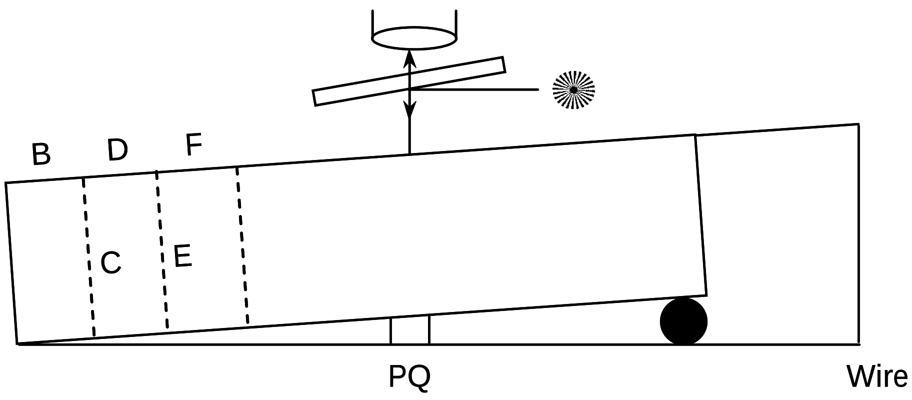
As the fringe width is independent of n, all bright fringes are equally spaced

2.4.5 Determination of Diameter of Wire

Q15. Explain the Determination of Diameter of wire.

Ans :

Two ordinary rectangular optical plane glass plates are taken and a wire is placed between the two plates at one end and the two plates are tied together to form an air wedge. Light from a monochromatic source ‘S’ is allowed to incident normally on the combination by a glass plate G inclined at an angle 45° with the horizontal. A microscope M is focussed on the top of the wedge as shown in diagram



- Light rays reflected from the wedge are brought to focus at the centre of the cross–wire. The reflected ray from the bottom surface of the top plate and partly reflected ray from the top surface of the lower plate are in a condition to interfere. Hence alternate dark and bright fringes of equal thickness like AB, CD, EF etc are observed. Let P and Q correspond to the centre of n^{th} and (n+1)th dark fringes.

➢ Let us suppose that the thickness of the film at these points be t_1 and t_2 respectively. For air film and for normal incidence we have

$$2t_1 = n\lambda \text{ (Path difference } = 2\mu t \cos r\text{, here } \mu = 1 \text{ and } r = 0)$$

and $2t_2 = (n + 1)\lambda$

$$t_2 - t_1 = \lambda/2$$

from diagram we have

$$\tan\theta = \frac{t_1}{AP} = \frac{t_2}{AQ}$$

So $t_1 = AP \tan\theta$ and $t_2 = AQ \tan\theta$

$$t_2 - t_1 = (AQ - AP)\tan\theta$$

$$\tan\theta = \frac{t_2 - t_1}{AQ - AP}$$

But $(AQ - AP)$ = fringe width = β

$$(t_2 - t_1) = \beta \tan\theta$$

Also we have, $(t_2 - t_1) = \lambda/2$

Thus $\beta \tan\theta = \lambda/2$

$$\beta = \frac{\lambda}{2\tan\theta}$$

➢ Since β is independent of n, all the dark fringes are equally spaced. If “d” is the diameter of the wire which is at a distance x from the line of contact of the two plates, then

$$\tan\theta = d/x$$

But $\tan\theta = \lambda/2\beta$

$$\therefore\ d/x = \frac{\lambda}{2\beta}$$

$$d = \frac{\lambda x}{2\beta}$$

Using the above equation, the diameter of wire can be calculated.

2.5 Newton s Rings in Reflected Light with and without contact between Lens and Glass Plate

Q16. Explain Newton Rings in reflected light with contact between lens and Glass plate.

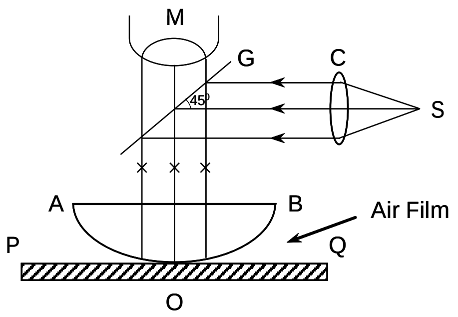
(OR)

What are Newton Rings? Derive and expression for the diameter of bright rings.

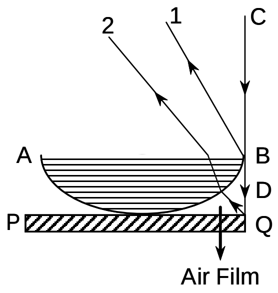
Ans : **(Jan.-21, June-18, Imp.)**

If we place a plan convex lens of large radius of curvature on optically plane glass surface; a thin film of air is formed between the curved surface of lens (AOB) and the plane glass plate (POQ). the thickness of the air film is zero at the point of contact 'O' is increase gradually as we move away from 'O' on either side. When this film is viewed in the reflected mono – chromatic light; due to interference, alternate bright and dark concentric rings are observed in the film. These are called Newton's rings

- Let us discuss the phenomena in detail



(a)



(b)

- Light from a monochromatic source 's' is rendered parallel by a convex lens C. This parallel beam falls on thin partially reflecting glass plate G. The reflected light from G falls normally on the air film formed between the curved surface of the plano convex lens L and plane glass plate P. Light reflected from air film enters the microscope M which is focused on air film at the point of contact "O" of lens and glass plate

  In diagram (b) the rays 1 and 2 are the reflected rays from upper and lower surfaces of the film corresponding to the incident ray CD. The rays 1 and 2 interfere.

- Thus the effective path difference between the interfering rays is given by

$$\Delta = 2\mu t \cos(\alpha + r) + \lambda/2 \quad \text{--- (1)}$$

μ = refractive index

r = angle of refraction in the film

λ = wavelength

t = thickness

- For a film normal incidence of light r = 0 and when radius of curvature of the plane convex lens in very large the α is small, So $\cos\alpha \simeq 1$

$$\therefore \text{ from (1) } \Delta = 2t + \lambda/2$$

At the point of contact O of the lens and the plate A = 0

$$\therefore \Delta = \lambda/2$$

This is the condition for minimum intensity. Hence the central spot is dark. The condition for maximum intensity is $\Delta = n\lambda$

$$\text{i.e., } 2t + \lambda/2 = n\lambda$$

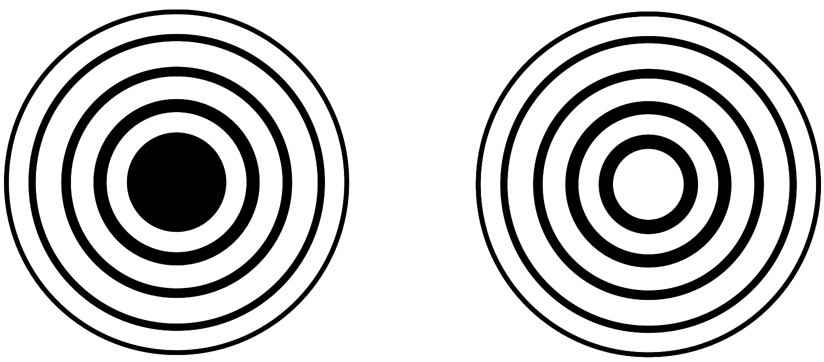
(or) $\boxed{2t\ (2n - 1)\ \lambda/2}$ for brightness in reflected light condition for minimum intensity is

$$\Delta = (2n + 1)\ \lambda/2 \qquad \text{-- (2)}$$

$$\text{i.e., } 2t + \lambda/2 = (2n + 1)\ \lambda/2$$

or $\boxed{2t = n\lambda}$ for darkness in reflected light -- (3)

- It is clear from the equation (3) and (4) that a bright or a dark fringe of any particular order will occur for a constant value of t.
- In the air film formed between the lens and the glass plate 't' remain constant along a circle with its centre at the point of contact O. So interference pattern consists of concentric rings about O as shown in following diagram



a b

Radius of the Circular Bright Fringe

Since t is small t^2 can be neglected when compared to R

Let R be the radius of the plano – convex lens L and O is the point of contact of lens L and glass plate 'P' as in diagram.

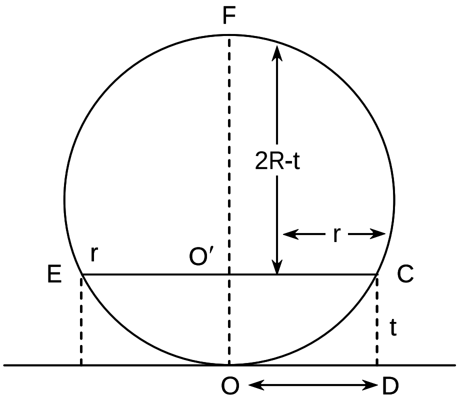


Diagram - I

Since t is small t^2 can be Neglected when compared to R

From diagram

CD = t is the thickness of the air film at a distance r from 0.

From geometry we can write

EO' × O'C = F O' × O' O

$r \times r = (2R - t)\,t$

$r^2 = 2Rt - t^2$

$\therefore\ r^2 = 2Rt$

$2t = r^2/R$

Sub $2t = r^2/R$ in (1) we get

$$\frac{r^2}{R} = (2n-1)\,\lambda/2$$

$$\boxed{r^2 = (2n-1)\,\lambda/2\;R}$$ for bright rings

- On substituting n = 1, 2, 3 ..... n etc in above equation we will get radii of 1, 2, 3 ......etc bright rings respectively

from the above equation we can write $r^2 \propto (2n-1)$ for bright rings

($\because$ λ, R are constants)

$\boxed{r \propto \sqrt{2n-1}}$ for bright rings -- (A)

- We know (2n – 1) is an odd number. So radii of the bright rings are proportional to the square root of food numbers similarly for dark ring

$$\frac{r^2}{R} = n\lambda$$

$$\boxed{r^2 = n\lambda R} \text{ for bright rings} \quad \text{-- (I)}$$

On substituting n = 1, 2, 3 ..... etc in the above equation we will get radii of 1, 2, 3.....etc dark rings respectively

From the equation (1)

$$r^2 \propto n$$

$$\boxed{r \propto \sqrt{n}} \text{ for dark rings} \quad \text{-- (B)}$$

So radii of the dark rings are proportional to square root of natural numbers

From equations (A) and (B) we can understand that separation between successive bringht rings or dark rings decreases as we move from the central dark ring.

2.5.1 Newtons Rings in Transmitted Light (Haidinger Fringes)

Q17. Explain Newton's Rings in reflected light without contact between lens and glass plate, and explain Haidinger fringes.

Ans :

We noticed from the discussion about Newton's rings in reflected light that when the glass plate and lens are in, contact at the point O as in diagram – I

1. The condition for a dark fringes is that $2t = n\lambda$ where 't' is the thickness of the air column between the lens and the glass plate and λ is the wavelength of the monochromatic light used

   with n = 0, we get the central dark spot

   n = 1, we get the first dark ring

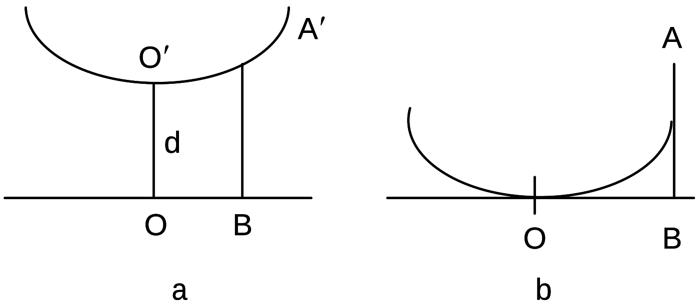
   n = 2, we get the second dark ring and so on

2. The radius of the first dark ring is given by $r_1 = \sqrt{\lambda R}$, of second dark ring, $r_2 = \sqrt{2\lambda R}$, of third dark ring by $r_3 = \sqrt{3\lambda R}$, and in general the radius of the nth dark ring as

$$r_n = \sqrt{n\lambda R}$$

These values correspond to the path difference $2t = \lambda$, $2t = 2\lambda$, 3λ ..... and $2t = n\lambda$ with $2t = 0$ giving central dark spot.

Now let us consider that the lens and the glass plate are not in contact. Let the lens be at a distance $d = \lambda/4$ above the glass plate. This is shown in diagram



- Now, the path difference at the centre 'O' is no longer 2t = 0, but is 2t = 2d = $\lambda/2$ as d = $\lambda/4$. This corresponds to a bright spot.

- Let us consider that a point A on the lens is at a distance AB = $\lambda/4$. When the lens is in content with the glass plate as in diagram (b)

  Then 2t = 2AB

  $= 2\,\lambda/4 = \lambda/2$

  and this corresponds to a bright ring. Now, when the lens is raised a distance d = $\lambda/4$ above O, then A goes to A' and A'B = $\lambda/4 + \lambda/4 = \lambda/2$ and 2A'B = 2 $\lambda/2 = \lambda$. This path difference corresponds to a dark ring. That is, the previous bright ring has now turned and to be a dark ring at the same place.

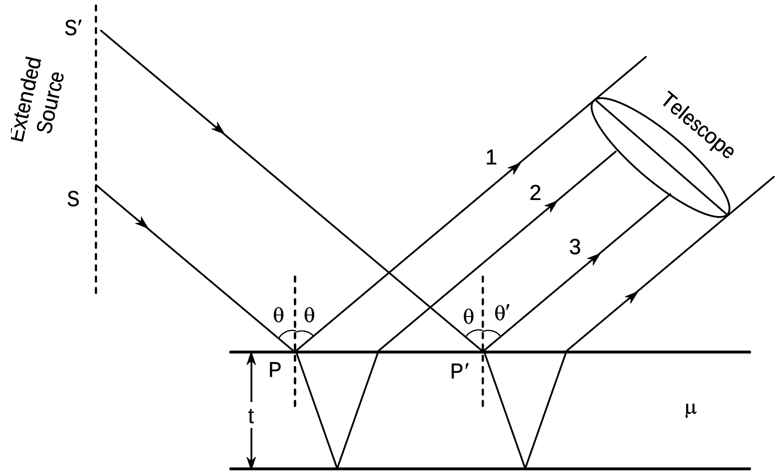
  Thus when d = $\lambda/4$, we get the dark rings in place of bright rings and bright rings in place of dark rings.

- Next, let us consider the lens to be raised a distance d = $\lambda/2$ above the glass plate. Now, at the centre the path difference is 2.$\lambda/2$ = λ. This corresponds to the previous first dark ring. That is, the previous first dark ring now collapse to the centre and the central spot will be again dark. Similarly, the previous 2nd dark ring now becomes 1st dark ring previous 3rd dark ring becomes 2nd dark ring etc.

  Same shifting will take place in the case of bright rings also with d = $\lambda/2$

Haidinger Fringes

The interference fringes with thin films are caused due to a change in the path difference 2μt cosr between the interfering rays. Obviously for a given film the path difference changes with the change of thickness or the angle of refraction r inside the film. If the film is of uniform thickness, then the path difference. 2μt cosr between the coherent beams can change only with r. The path difference will change appreciably even with a very small change in r, if the thickness t is large. In this case fringes are produced due to the super position of rays, which are equally inclined to the normal. These fringes are called fringes of equal inclination or Haidinger fringes.



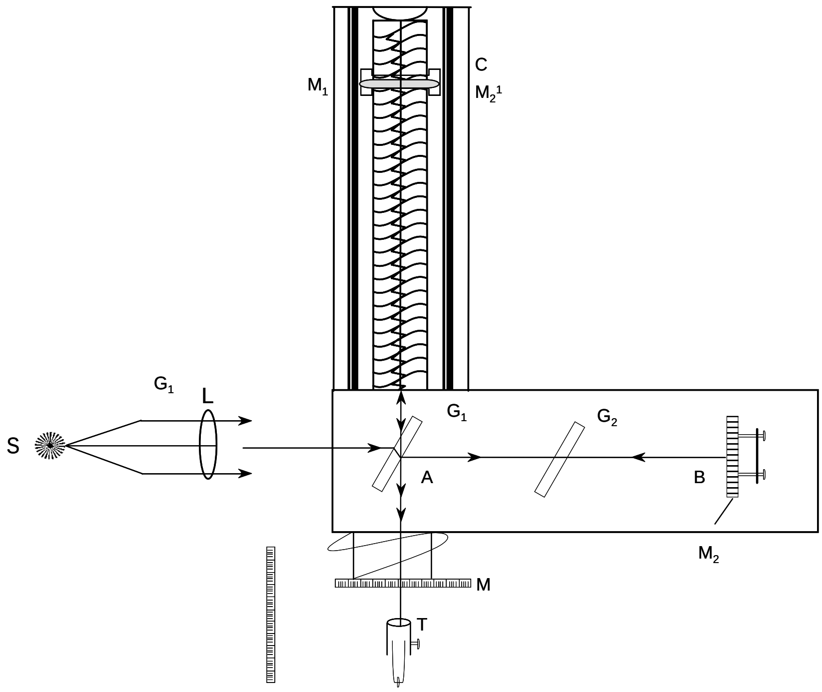
In this case all the pairs of interfering coherent rays of equal inclination pass through the plate as a parallel beam and hence meet at a point at infinity. Thus they can be located with a telescope focussed at infinity. In this case the fringes are said to be at infinity or the fringes localised at infinity. To produce Haidinger fringes, the source must be an extended source, the film thickness must be appreciably large and the observing instrument is to be focussed for parallel rays.

2.6 Michelson's Interferometer

2.6.1 Determination of Wavelength of Monochromatic Light

Q18. Describe Michelson's interferometer with a neat diagram.

Ans : **(June-19, June-18, Imp.)**



- An interferometer is a device that can be used to measure lengths or changes in length with great accuracy by means of interference phenomenon.

- A schematic diagram of the Michelson interferometer is shown in diagram. It consists of a beam splitter G_1, a compensating plate G_2 and two plane mirrors M_1 and M_2. The beam splitter G_1 is a partially silvered plane glass plate and the compensating plate G_2 is a simple plane glass plate having the thickness as G_1. The two plates G_1 and G_2 are held parallel to each other and are inclined at an angle 45° with respect to the mirror M_2. The mirror M_1 is capable of moving away or towards the glass plate G_1 with the help of micrometer screw. The distance through which the mirror M_1 is moved can be read with the help of a graduated drum attached to the screw. In the normal adjustment of the intereferometer, the mirrors M_1 and M_2 are perpendicular to each other and G_1 is at 45° to the mirror M_2. The interference bands are observed in the field of view of telecsope T.

  Monochromatic light from an extended source S is rendered parallel by means of collimating lens L and is made to incident on the beam splitter G_1. It is partly reflected at the back surface of G_1 along AC and partly transmitted along AB. The beam AC undergoes a further reflected at M_1 and this reflected beam gets transmitted through G_1 which is shown as AT in diagram

- The transmitted beam AB travels towards the mirror M_2 and is reflected along the same path. It is reflected at the back surface of G_1 and proceeds along AT. Thus two beams along AT produced from a single source through division of amplitude and are hence coherent. They interfere and produces interference fringes.

- From the diagram it is clearly seen that a light ray starting from the source S and undergoing reflection at M_1 passes through the glass plate G_1 three times. On the other hand in the absence of plate G_2, the ray reflected at M_2 travels through the glass plate G_1 only once. For compensating this path difference, a compensating plate G_2 fo the same thickness is inserted into the path AB and is held exactly parallel to G_1.

  The interferences pattern may be regarded to be due to the light reflected from the surfaces M_1 and M_2' respectively. Thus the arrangement is equivalent to an air film enclosed between the reflecting surfaces M_1 and M_2'.

2.6.2 Types of Fringes

Q19. Explain different types of fringes.

Ans :

1. Circular Fringes

The type of the fringes formed in Michelson interferometer depends upon the inclination of M_1 and M_2' be the image of M_2 formed by reflection at the half silvered surface of the plate G_1 so that $AM_1 = AM_2'$

The phase changes on reflection at M_1 and M_2 are similar and Edser has shown that for thin coating of silver on plate G_1 the phase changes due to reflection from it, in air and glass are also similar, each being equal to Π. the optical path difference between the two beams is therefore simply due to different paths traversed in air before reaching the observer. Consequently the two waves will interfere constructively or destructively, according to path difference Δ between them is even or odd multiple of $\lambda/2$.

The condition for maxima is

$$\Delta = 2m \cdot \lambda/2 = m\lambda$$

i.e., $2d \cos q = m\lambda$

and the condition for destructive interference (Minima)

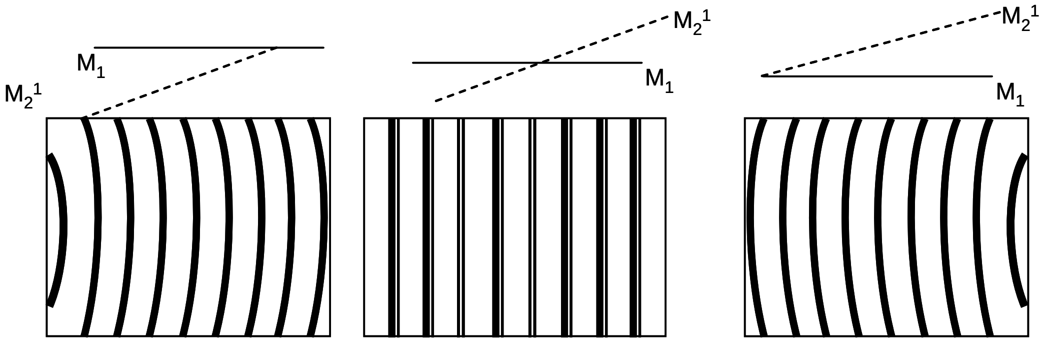
$$\Delta = (2m+1)\lambda/2$$

Where m is an integer and $d = x_1 \sim x_2$

i.e., $2d\cos\theta = (2m + 1)\lambda/2$

2. Localized Fringes

When the two mirror are tilted, they are not exactly perpendicular to each other and therefore the mirror M_1 and the virtual image M_2' are not parallel. In this case the air path between them is wedge shaped and with such a film, the locus of points of equal thickness is a straight line parallel to the edge of the wedge and therefore fringers appear to be straight. When M_1 actually intersects M_2' in the middle, we will get perfect straight lines. In the other positions, the shape of the fringes is shown in diagram (a) and (c) they are curved and are always convex towards the thin edge of the wedge. This type of fringes are not observed for large path differences.



(a) (b) (c)

3. White light Fringes

White light source is used only when path difference is small with white light the central fringe will be dark and around it there will be few coloured fringes.

If mirrors M_1 and M_2 are slightly tilted, the wedge shaped air film will form between M_1 & M_2'.

- If we use white light in this case there will be a distinct bright straight fringe and on either side of it there will be a few coloured fringes. These fringes are useful for the determination of zero path difference.

2.6.3 Difference in Wavelength of Sodium D_1, D_2 Lines and thickness of a thin Transparent plate

Q20. Determine the difference in wavelength of sodium D_1, D_2 lines and Find the thickness of thin transparent plate.

Ans : **(June-19, June-18, Imp.)**

Let the sodium source of light emits two lines namely D_1 and D_2 of wavelength λ_1 and λ_2. Let $\lambda_1 > \lambda_2$. the apparatus is adjusted for the circular fringes. Now the mirror M_1 is moved and the position is found where the fringes are very bright. In this position the bright fringe down to λ_1 coincides with the bright fringe due to λ_2 and the fringes are very distinct and well defined.

When the mirror M_1 is further moved, the visibility of the fringes decreases and finally the fringes disappear. Now the field appears uniformly illuminated. This is the position of the maximum indistinctness. In this position the dark fringes of one set fall on the bright fringes of the other. On further moving the mirror M_1 the visibility increases and again a position comes where the fringes are well defined and very distinct.

- When "X" is the distance moved by the mirror for two consecutive positions of maximum distinctness, the path difference is 2x. During this movement if m is the change in order of the longer wavelength λ_1 at the centre of the field then (m + 1) will be change in the order of wavelength λ_2 at the centre, so that we have

$$2x = m\lambda_1 = (m+1)\lambda_2$$

$$m\lambda_1 = m\lambda_2 + \lambda_2$$

or $$m = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

substituting m value in equation $\lambda = \frac{2x}{N}$ = in place of N

we get $$2x = \left(\frac{\lambda_2}{\lambda_1 - \lambda_2}\right)\lambda_1$$

$$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2x}$$

If λ_1 and λ_2 are very nearly equal, then $\lambda_1\lambda_2 = \lambda_2$ and

$$\lambda = \lambda_1 + \frac{\lambda_2}{2}$$, and therefore we have

$$\Delta\lambda = \lambda_1 - \lambda_2 + \frac{\lambda_2}{2x}$$

From the above equation the difference in wave length can be calculated

Thickness of a thin Transparent Plate

- Michelson's interferometer is set for localised fringes of white light. The cross wires are set on central fringe. Now, the thin plate is introduced in one of the interfering rays, we know that the introduction of a plate of thickness "t" and refractive index 'μ' increases the path by $2(\mu - 1)t$. Thus, the fringes shift from their positions. The mirror M_1 is now moved either backward or forward till the central fringe coincides with the cross – wire. the distance x moved by mirror M_1 is noted with the help of micrometer screw. Hence,

$$2x = 2(\mu - 1)\,t \text{ (or) } t = \frac{x}{(\mu - 1)}$$

Using this equation, t can be calculated.

Problems

1. In the young's double slit experiment, the distance between the two slits is 0.5mm, λ = 5 × 10^{-5} cm and D = 50 cm what will be the fringe width.

Sol :

Given

D = 0.5 mm = 0.05 cm

$\lambda = 5 \times 10^{-5}$ cm

D = 50 cm

β = ?

$\beta = \frac{\lambda D}{d} = 5 \times 10^{-5} \times = 5 \times 10^{-2}$ cm

$\therefore$ Fringe width b = 5 × 10^{-5} cm

2. Light of wavelength 6000A$^\circ$ falls normally on a thing wedge film of refractive index 1.4, forming fringes taht are 2mm apart. Find the angle of the wedge.

Sol :

Given

$\mu = 1.4$; $\lambda = 6 \times 10^{-5}$ cm

$$\theta = \frac{\lambda}{2\mu\beta}$$

$$= \frac{(6 \times 10^{-5}\ \text{cm})}{2 \times 1.4 \times (0.2\ \text{cm})}$$

$= 1.07 \times 10^{-4}$ radians

$= 1.07 \times 10^{-4} \times 57.3^0 = (6.13 \times 10^{-3})^\circ$

3. Find the thickness of a soap fil (μ = 1.33) which gives constructive second order interference of reflected red light of λ = 700 mm (milli Microns).

Sol :

$2\mu t \cos r = (n + \frac{1}{2})\lambda$, n = 0, 1, 2 ..... (maxima)

For second order, n = 1, for normal incidence r = 0

$$t = \frac{\left(1+\frac{1}{2}\right)\times 700\ \text{nm}}{2\ \times 1.33} = 394.7\ \text{nm}$$

4. In an interference pattern, at a point we observe the 18th order maximum for light of 500A, what order will be visible at this point if the source is replaced by a light of 4500A.

Sol :

For nth order maximum, $\Delta = n\lambda$

or $n_1\lambda_1 = n_2\lambda_2$

Here, $n_1 = 18$, $n_2 = ?$, $\lambda_1 = 5000^0A$; $\lambda_2 = 4500^0A$

$$n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{15\times 5000}{4500} = 20$$

5. Two glass plates enclose a wedge – shaped air film touching at one edge and separated by a wire at a distance of 1.5 cm from the edge. If l = 6000°A and the fringe width is 910^{-3} cm, determine the diameter of the wire.

Sol :

We know that

diameter of a wire $\boxed{\beta = \lambda/2\theta}$

$D = l\,\theta$

$$\theta = \frac{\lambda}{2\beta}$$

Given $\lambda = 6 \times 10^{-5}$

$\beta = 9 \times 10^{-3}$

$$\theta = \frac{\lambda}{2\beta} = \frac{6\times 10^{-5}}{2\times 9\times 10^{-3}} = 0.333 \times 10^{-2}$$

$D = l\theta = 1.5 \times 0.333 \times 10^{-2} = 0.005$ cm

6. A biprism forms interference fringes with monochromatic light of wavelength 5450°A On introducing a thin glass plate (m = 1.5) in the path of one of the interfering beams the central bright band shifts to the position previously occupied by third bright fringe. Find the thickness of the plate.

Sol :

Given

$\mu = 1.5$; $\lambda = 5450\ A^0 = 5450 \times 10^{-10}$ m n = 3

$$t = \frac{n\lambda}{(\mu-1)}$$

$$= \frac{3\times5450\times10^{-10}}{2.5-1}$$

$= 3.27 \times 10^{-6}$ m

7. Using sodium light (l = 5893 A°) interference fringes are formed from a think our wedge, when viewed normally 10 fringes are observed in a distance of 1cm. Calculate the angle of the wedge.

Sol :

We have $\beta = \frac{\lambda}{2\mu\theta}$

$$\Rightarrow \theta = \frac{\lambda}{2\mu\beta}$$

Here $\lambda = 5893A^0 = 5893 \times 10^{-8}$ cm

$\mu = 1, \beta = \frac{1}{10}$ cm = 0.1 cm

$\theta = \frac{5893\times10^{-8}}{2\times1\times0.1} = 2.946 \times 10^{-4}$ radian

8. Two optically plane glass strips of length 10cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light used has wavelength 5900°A, find the fringe width.

Sol :

The angle of the widge $\theta = t/x$

Where 'x' is the length of the film and t is the thickness of the foil

Fringe width $\beta = \frac{\lambda}{2\mu\theta} \quad \frac{\lambda x}{2\mu t}$

$= \frac{5900\times10^{-8}\times10}{2\times1\times0.001} = 0.295$ cm

9. In Newton's rings experiment a plano – convex lens of radius 400 cm is used. the wavelength of light used is 5320°A. If fringes are fromed with water (μ = 1.33) what is the radius of the 10th dark ring.

Sol :

Given n = 10, $r_n^1 = 50$; $\mu = 1.33$

$rn = \sqrt{\frac{n\lambda R}{\mu}} = \sqrt{\frac{10\times5.32\times10^{-5}\times400}{1.33}} = 0.4$ cm

10. A parallel beam of light of wavelength 6000A^0 is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction is 60^0. find the least thickness of the plate for which is appears totally dark in reflected light.

Sol :

$n = 1, \lambda = 6000 \times 10^{-8}$ cm ; $\mu = 1.5, r = 60^\circ$

For darkness $2\mu t \cos r = n\lambda$

$$t = \frac{n\lambda}{2.4 \cos r}$$

$$t = \frac{1 \times 600 \times 10^{-8}}{2 \times 1.5 \times \cos 60^\circ} = \frac{6000 \times 10^{-8}}{2 \times 1.5 \times 5} = 4 \times 10^{-5} \text{ cm}$$

---

11. In a Newton's rings experiment the diameter of 3rd and 23rd dark rings are 0.2 cm and 0.56 cm respectively. If the radius of curvature of plano converx lens is 92 cm, find the wave length of light.

Sol : **(Jan.-21)**

Given that,

In Newton's rings experiment,

Diameter of 3rd dark ring, $D_3 = D_n = 0.2$ cm

Diameter of 23rd dark ring, $D_{23} = D_{n+20} = 0.56$ cm

Radius of curvature of plano convex lens, $R = 92$ cm

The expression for wavelength of light in Newton rings experiment is given as,

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Here, $D_n = D_3$

$D_{n+p} = D_{23} = D_{3+20}$

$\Rightarrow$ $p = 20$

Substituting corresponding values in equation (1),

$$\lambda = \frac{(0.56)^2 - (0.2)^2}{4 \times 20 \times 92}$$

$= 0.0000372$ cm

$= 37.2 \times 10^{-6}$ cm

$\therefore$ $\lambda = 37.2$ μcm

12. The diameter of a one of the dark ring in the Newton's rings experiment is 6 mm. Find the diameter of same ring when the experiment is conducted in the liquid of refractive index is 1.5.

Sol : **(Jan.-21)**

Given that,

In Newton's rings experiment,

Refractive index, $\mu = 15$

Diameter of one dark ring, $D_3 = 6 \text{ mm} = 0.6 \text{ cm}$

The expression for diameter of dark ring is given as,

$$D_1^2 = 4n\lambda R$$

The expression for diameter of dark ring in liquid is given as,

$$D_1^2 = \frac{4n\lambda R}{\frac{4n\lambda R}{\mu}}$$

$$\Rightarrow \frac{D_1^2}{D_1^2} = \mu$$

$$\Rightarrow D_1^2 = \frac{D_1^2}{\mu}$$

$$\Rightarrow D_1 = \frac{D_1}{\sqrt{\mu}}$$

Substituting the corresponding values in above equation,

$$D_1 = \frac{0.6}{\sqrt{1.5}}$$

$$= 0.489$$

Therefore, the diameter of same ring when the experiment is conducted in liquid is 0.489 cm.

Short Question and Answers

1. Explain the terms temporal and spatial coherence.

Ans :

Coherence is two types

i) Temporal coherence

ii) Spatial coherence

(i) Temporal Coherence

Definition

If two waves maintain a definite relationship between their phases at a given time and at certain time later, then the waves are said to be temporally coherent, this phenomenon is known as temporal coherence.

- Consider a point source of quasi monochromatic lights, which emit light in all directions as shown in diagram
- Consider the light travelling along the path SP_1P_2.
- The phase relationship between the points P_1 and P_2 depends on the distance P_1P_2 and the coherence length of the light beam. The coherence length L, of the light is defined by the equation.

$$L = C\,T_C$$

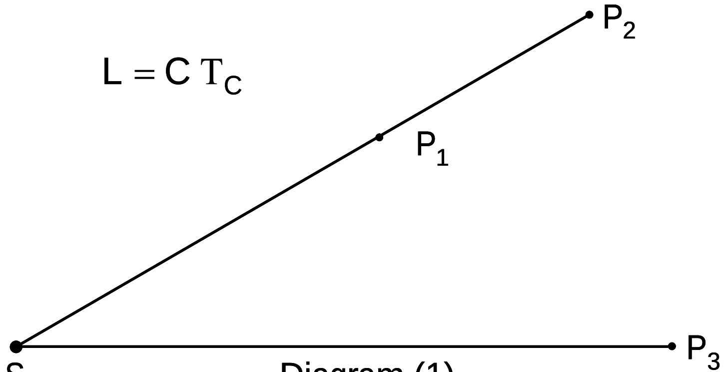


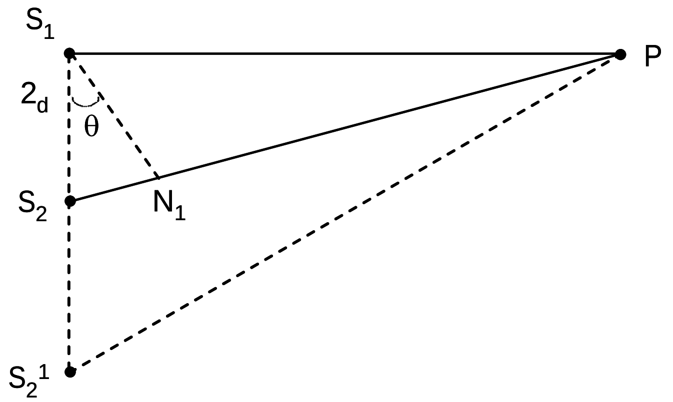
Diagram (1)

Where τ_c is called the coherence time and c is the speed of light.

- Temporal coherence is characterized by two parameters namely coherence length (L) and coherence time (τ_c)

(ii) Spatial Coherence

Spatial coherence refers to the continuity and uniformity of a wave in a direction perpendicular to the direction of propagation. If the phase difference for any two fixed points in a plane normal to the wave propagation does not vary with time, then the wave is said to exhibit spatial coherence



- From diagram (1) $SP_1 = SP_3$ so, the fields at points P_1 & P_2 have same phase. Thus an ideal point source exhibits spatial coherence if the waves produced by it are likely to have the same phase at points in space which are equidistant from the source.

2. Explain colors of thin films.

Ans :

When a thin film is illuminated by monochromatic light and seen the reflected light, it will appear bright if $2\mu t \cos r = (2m+1)\,\lambda/_2$ and dark if $2\mu t \cos r = m\lambda$.

- If, however the film is illuminated by white light, the film shows different colours.
- The colours exhibited in reflection by thin films of oil, soap bubbles and coatings of oxides on heated metals etc are due to interference of light from an extended source such as sky. It may be understood as follows.

The films are usually observed by reflected light. The eye looking at the thin film receives light waves reflected from the top and bottom surfaces of the film. For a think film these rays are very close to each other and in a position to interefere. The optical path difference between the interfering ray is $\boxed{\Delta = 2\mu t \cos r - \lambda/2}$.

- It is seen that the path difference depends upon the thickness "t" of the film, the wavelength l and the angle r, which is related to the angle of incidence of light on the film.
- White light consists of a range of wavelengths and for specific values of t and r, waves of only certain wavelengths (colors) satisfy the condition of maxima. Therefore, only those colours are present in the reflected light.
- The other wavelengths interfere destructively and hence are absent from the reflected light. Hence, the film at a particular point appears coloured.
- As the thickness and the angle of incidence vary from point to point, different colours are intensified at different places. The colours seen are not isolated colours, as at each place there is a mixture of colours.

3. Explain the conditions for sustainable Interference of Light.

Sol :

The conditions for obtaining a distinct well-defined interference pattern are,

(i) Conditions for Sustained Interference

- The frequency of light waves from two sources must be same
- The two light waves must be coherent to each other
- The coherence length (l_{coh}) must be greater than the path difference (Δ) between the overlapping waves. i.e., $l_{coh} > \Delta$.
- The polarization planes of two sets of waves must be same.

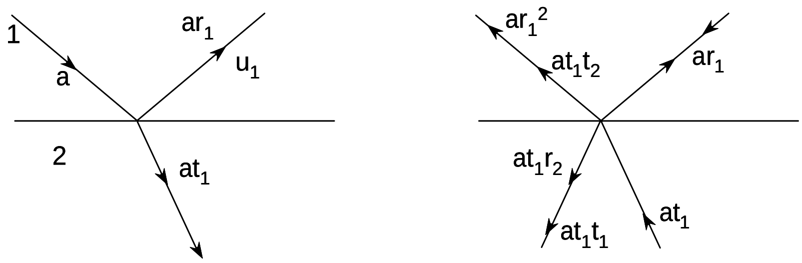
(ii) Conditions for the Formation of Distinct Fringe Pattern

- The clearly observe fringes, the distance between two coherent sources must be small.
- The screen must be placed at a large distance from the two sources.
- In dark regions, the vector sum of overlapping electric field vectors must be zero.

4. What is meant by phase change on reflection?

Ans :

Consider a light ray incident on an interface of two media of refractive indices μ_1 and μ_2. Let medium 2 be optically denser than medium 1 ($\mu_2 > \mu_1$)



- The ray is partly reflected and partly refracted. Let r_1 is the reflection coefficient (i.e., the fraction of the amplitude reflected) and 't_1' is the transmission coefficient. Thus, if the amplitude of the incident ray is a, then the amplitudes of the reflected and refracted beam would be ar_1 and at_1 respectively.
- If the paths of the reflected ray and the refracted ray are reversed, then by the principle of optical reversibility they should give rise to the wave of original amplitude a, provided there is no dissipation of energy by absorption.
- We know reverse the rays and consider a ray of amplitude at_1 incident on medium 1 and a ray of amplitude ar_1 incident on medium 2 as shown in diagram. The ray of amplitude at_1 will rive rise to a reflected ray of amplitude at_1r_2 and a transmitted ray of amplitude at_1t_2, where r_2 and t_2 are the reflection and transmission coefficients when a ray is incident from medium 2 to medium 1. Similarly, the ray of amplitude ar_1 will give rise to a ray of amplitude ar_1^2 and a refracted ray of amplitude ar_1t_1.

  According to the principle of optical reversibility, we have

$$ar_1^2 + at_1t_2 = a$$

or $$t_1t_2 = 1 - r_1^2$$

Further the two rays of amplitudes at_1r_2 and ar_1t_1 must cancel each other;

i.e., $at_1 r_2 + ar_1 t_1 = 0$

(or) $r_2 = - r_1$

From the equation $t_1t_2 = 1 - r_1^2$, we have

$r_1^2 = 1 - t_1t_2$. Also we have $r_1^2 + t_1^2 = 1$

From these two equations we get $t_1 = t_2$

- Whereas in the equation $r_2 = - r_1$, the negative sign represents that an abrupt phase change of Π occurs when light gets reflected by a denser medium and that no such abrupt phase change occurs when light gets reflected by a rarer medium.

5. Write short a note on Principle of Superposition.

Ans :

If two or more light waves of the same frequency overlap at a point, the resultant effect depends on the phases of the waves as well as their amplitudes. The resultant wave at any point, at any instant of time is governed by the principle of superposition.

- In simple way, we can say that the principle of superposition of waves states that the resultant displacement at a point is equal to the vector sum of the displacements of different waves states that the resultant displacement at a point is equal to the vector sum of the displacements of different waves at that point.
- The principle of superposition applies to electromagnetic waves also and is the most important principle in wave optics
- Interference is an important consequence of superposition of coherent waves

6. Explain the features of Interference pattern produced with a Biprism.

Ans :

1. In a biprims experiment, the interference pattern consists of coloured fringes on both sides of white fringe when a white light is illuminated from the slit.
2. In this, monochromatic light produces only bright fringes of same colour.
3. The thickness of a thin sheet of transparent material can be determined through biprism experiment, when it is introduced in the path of two interfering beams.

7. What are Newton Rings?

Ans :

If we place a plan convex lens of large radius of curvature on optically plane glass surface; a thin film of air is formed between the curved surface of lens (AOB) and the plane glass plate (POQ). the thickness of the air film is zero at the point of contact 'O' is increase gradually as we move away from 'O' on either side. When this film is viewed in the reflected mono – chromatic light; due to interference, alternate bright and dark concentric rings are observed in the film. These are called Newton's rings

8. Explain different types of fringes.

Ans :

(i) Circular Fringes

The type of the fringes formed in Michelson interferometer depends upon the inclination of M_1 and M_2' be the image of M_2 formed by reflection at the half silvered surface of the plate G_1 so that $AM_1 = AM_2'$

The phase changes on reflection at M_1 and M_2 are similar and Edser has shown that for thin coating of silver on plate G_1 the phase changes due to reflection from it, in air and glass are also similar, each being equal to Π. the optical path difference between the two beams is therefore simply due to different paths traversed in air before reaching the observer. Consequently the two waves will interfere constructively or destructively, according to path difference Δ between them is even or odd multiple of $\lambda/2$.

The condition for maxima is

$$\Delta = 2m \cdot \lambda/2 = m\lambda$$

i.e., $2d \cos q = m\lambda$

and the condition for destructive interference (Minima)

$$\Delta = (2m + 1)\, \lambda/2$$

Where m is an integer and $d = x_1 \sim x_2$

i.e., $2d \cos\theta = (2m + 1)\, \lambda/2$

(ii) Localized Fringes

When the two mirror are tilted, they are not exactly perpendicular to each other and therefore the mirror M_1 and the virtual image M_2' are not parallel. In this case the air path between them is wedge shaped and with such a film, the locus of points of equal thickness is a straight line parallel to the edge of the wedge and therefore fringers appear to be straight. When M_1 actually intersects M_2' in the middle, we will get perfect straight lines.

9. Differentiate Lloyd's and Biprism mirror fringes

Ans :

(i) In biprism arrangement, the complete pattern of interference fringes on both sides of central fringes is obtained. In Lloyd's mirror arrangement, only few fringes on one side of the central fringe are observed and the central fringe itself invisible.

(ii) In biprism arrangement, the central fringe is bright while that of Lloyd's mirror is dark

(iii) The central fringe is biprism is less sharp than that in Lloyd's mirror.

(iv) The conditions of constructive and destructive interference in Lloyd's mirror are opposite to that in biprism due to a phase change of Π (or a path difference of $\lambda/2$) in reflected beam.

(v) In biprism experiment, the fringe width is same for all pairs of coherent sources. In Lloyd's mirror d is different for different pairs of coherent sources and hence fringe width is different pairs of coherent sources.

10. Determination of Wavelength.

Ans :

- For this purpose Lloyd's mirror is mounted vertically on an upright two of an optical bench with its surface parallel to the length of the bench. On first upright, a narrow vertical slit is mounted and is illuminated with mono chromatic light whose wave length is to be determined. Micrometer eye piece is mounted on upright four. Now the mirror is rotated about an axis parallel to the length of the bench until distinct fringes are observed throughout eye piece.
- Fringe with β is measured using micrometer eyepiece
- The distance 2d between the sources S and S' is measured using lens displacement method
- The distance D is measured and wavelength of light is calculated using the formula

$$\boxed{\lambda = \frac{\beta(2d)}{D}} \qquad \because \beta = \frac{\lambda D}{2D}$$

Choose the Correct Answer

1. In an interference pattern, energy is [c]
   (a) Created at bright band (b) destroyed at dark band
   (c) conserved but redistributed (d) zero
2. In young's double slit experiment the distance between any two consecutive dark fringes is [a]
   (a) $\beta = \frac{\lambda D}{d}$ (b) $\beta = \frac{\lambda d}{D}$
   (c) $\beta = \frac{Dd}{\lambda}$ (d) $\beta = \lambda d$
3. If the distance between the two coherent sources is large, the fringes will become [b]
   (a) Wide (b) narrow
   (c) disappear (d) move closer
4. When light gets reflected by a rarer medium phase change occurs is [b]
   (a) Π (b) O
   (c) $\Pi/2$ (d) $\Pi/6$
5. In the case of interference due to transmitted light, dark fringe occurs when path difference is [a]
   (a) $2\mu t$ cosr $(2m + 1) \lambda/2$ (b) $2\mu t$ cosr $= \lambda/2$
   (c) $2\mu t$ cor $= \lambda$ (d) $2\mu t = \frac{\lambda}{2 \cos r}$
6. If the wedge angle θ is increased, the fring seperation [c]
   (a) decreases (b) increases
   (c) remains same (d) disappears
7. In reflected light, the central fringe of newton's ring pattern is [c]
   (a) Non – Uniform (b) bright
   (c) dark (d) colored
8. When a thin film of oil or soap bubble is illuniated with white light, multiple colours appear. This is due to [c]
   (a) diffraction (b) polarization
   (c) interference (d) total internal reflection
9. In which of the following the interference is produced by the duivision of wavelength [d]
   (a) Fabry-perot interferometer (b) Michelson's interferometer
   (c) Newton's Ring (d) Fresnel's Biprism
10. In newton ring expriment properties than can be measured [d]
    (a) wavelength of light (b) thickness of the film
    (c) Refraction index of liquid (d) All of the above

Fill in the blanks

1. The relation between coherence length (L_C) and coherence time (τ_C) is given by ________.
2. A phase change of P occurs, when light gets reflected by ________ medium.
3. To produce Haidinger fringes, the thickness of the film must be ________.
4. The technique of reducing the reflectivity is known as ________.
5. The refractive index of non-reflecting film is ________ than that of lens on which it is coated.
6. A phase difference n between two interfering beams is equivalent to ________ path difference.
7. In transmitted light the central fringe of Newton's ring is ________.
8. In interference with two coherent sources, the fringe width varies ________ with wavelength.
9. The thickness of antireflection coating is of the order of ________ wavelength of light used.
10. When the lens is moved upwards from the glass plate,the newton's rings ________ to the centre.

ANSWERS

1. $L_C = C\tau_c$
2. Denser
3. Large
4. Blooming
5. Less
6. $\pi/2$
7. Bright
8. Directly
9. $\pi/4$
10. Collapse

One Mark Answers

1. Define temporal coherence.

Ans :

If two waves maintain a definite relationship between their phases at a given time and at contain, time later, then the waves are said to be temporally coherent phenomenon is known as temporal coherence.

2. Define coherence time.

Ans :

The average time during which the ideal sinusodial charmonic wave emission exists is called coherence time (τ_c)

3. Define coherence length.

Ans :

It is the length of the wave packet over which it may be assumed to be sinusoidal and has predicable phase.

4. Define Interferences.

Ans :

Interference is a phenomenon in which two waves superpose to form a resultant wave of greater, lower (or) the same amplitude

5. What is meant by biprism?

Ans :

A triangular prism with vertex angle of nearly 1800 used to obtain images of a single source in observing the interferences of light.

6. What are Newton rings?

Ans :

Newton's rings is a phenomenon in which an interference pattern is created by the reflection of light between two surfaces a spherical surface and an adjacent touching flat surface

UNIT - III

Diffraction

Introduction - Distinction between Fresnel and Fraunhofer diffraction, Fraunhofer diffraction:-

Diffraction due to single slit and circular aperture - Limit of resolution - Fraunhofer diffraction due to double slit - Fraunhofer diffraction pattern with N slits (diffraction grating).

Resolving Power of grating - Determination of wave length of light in normal and oblique incidence methods using diffraction grating.

Fresnel diffraction - Fresnel's half period zones - area of the half period zones -zone plate - Comparison of zone plate with convex lens - Phase reversal zone plate - diffraction at a straight edge - difference between interference and diffraction.

3.1 Diffraction

3.1.1 Introduction

Q1. Define diffraction. What are the types of diffractions?

(OR)

What is diffraction?

Ans : **(Jan.-21, June -18, Imp.)**

Diffraction refers to various phenomenon that occur when a wave encounters an obstacle or a slit. It is defined as the bending of light around the corners of an obstacle or aperture into the region of geometrical shadow of the obstacle.

Diffraction phenomenon can be divided into following two general cases :

1. **Fraunhofer's Diffraction**

   In this class of diffraction, source and the screen or telescope are placed at infinity or effectively at infinity from aperture. In this case the wavefront which is incident on the aperture or obstacle is plane.

2. **Fresnel's Diffraction**

   In this class of diffraction, source and screen are placed at finite distances from the aperture or obstacle having sharp edges. In this case no lenses are used for making the rays parallel or convergent. The incident wavefronts are either spherical or cylindrical.

Q2. What are Fresnel's Diffraction assumptions?

Ans : **(June-18)**

1. The complete wavefront is divided into a large number of elements known as Fressel's strips or zones of small area such that each of these elements acts as a source of secondary wages.
2. The resultant effect at any point is the combination effect of all secondary waves reacting at that point.
3. The effect at any point due to a particular zone depends on the,
   (i) Distance of point from the zone.
   (ii) Inclination of the point with reference to zone under consideration.
   (iii) Area of the zone.

3.1.2 Distinction between Fresnel and Fraunhofer Diffraction

Q3. What are the differences between Fraunhofer and Fresnel Diffraction?

Ans :

S.No.	Nature	Fraunhofer Diffraction	Fresnel Diffraction
1.	**Wave Fronts**	Planar wave fronts	Cylindrical wave fronts
2.	**Observation distance**	Observation distance is infinite. In practice, often at focal point of lends.	Source of screen at finite distance from the obstacle.
3.	**Movement of diffraction pattern**	Fixed in position	Move in a way that directly corresponds with any shift in the object.
4.	**Surface of calculation**	Fraunhofer diffraction patterns on spherical surfaces	Fresnel diffraction patterns on flat surfaces
5.	**Diffraction patterns**	Shape & intensity of a Fraunhofer diffraction pattern stay constant	Change as we propagate them further "downstream" of the source of scattering.

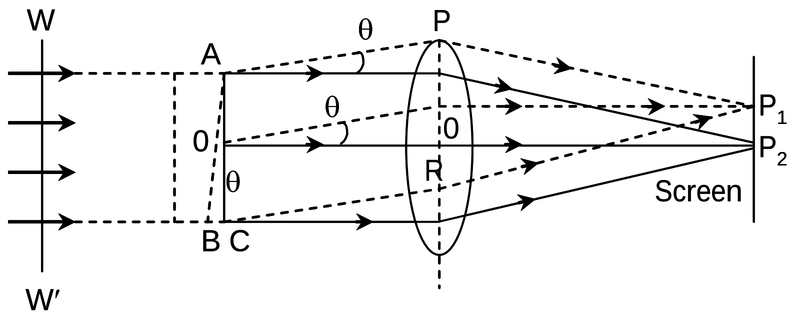
3.2 FRAUNHOFER DIFFRACTION

3.2.1 Diffraction Due to Single Slit

Q4. Discuss Fraunhofer diffraction due to a single slit. Explain the distribution of intensity of light in the diffraction pattern.

Ans : **(June-19, June-18)**

The diagram (I) represents a section AB of a narrow slit of width e perpendicular to the plane of the paper. Let a plane wavefront ww' of monochromatic light of wavelength λ.



Propagating normally to the slit be incident on it. Let the diffracted light be focused by means of a convex lens on a screen placed in the focal plane of the lens. According to Huygens - Fresnel, every point of the wavefront in the plane of the slit is a source of secondary spherical wavelets, which spread out to the right in all direction OP_0, are brought to focus at P_0 by the lens. Thus, P_0 is a bright central image. The secondary wavelets travelling at an angle θ with the normal are focussed at a point P_1 on the screen. The point P_1 is of the minimum intensity or maximum intensity depending upon the path difference between the secondary waves originating from the corresponding points of the wavefront.

In order to findout intensity at P_1, draw a perpendicular AC on BR, the path difference between secondary wavelets from A and B in direction θ.

$$= BC = AB \sin\theta - e\sin\theta$$

and corresponding phase difference = $\frac{2\Pi}{\lambda}$. esinθ

Let us consider that the width of the still is divided into n equal pants and the amplitude of the wave from each part is “a”. The phase difference between any two consecutive waves from these parts would be

$$\frac{1}{n}\text{ (Total phase) } = \frac{1}{n}\left(\frac{2\Pi}{\lambda}e\sin\theta\right) = d\text{ (say)}$$

Using the method of vector addition of amplitudes as discussed in the previous article, the resultant amplitude R is given by

$$R = a\frac{\sin \frac{nd}{2}}{\sin \frac{d}{2}} = a\frac{\sin\left(\Pi e\sin\theta/\lambda\right)}{\sin\left(\Pi e\sin\theta/n\lambda\right)}$$

$$= a\frac{\sin\alpha}{\sin\alpha/n} \qquad \text{(where } \alpha = \Pi \text{ esin } \theta/\lambda)$$

$$= a\frac{\sin\alpha}{\alpha/n} = na\frac{\sin\alpha}{\alpha} = \frac{A\sin\alpha}{\alpha}$$

Thus the resultant amplitude is given by

$$R = A\frac{\sin\alpha}{\alpha}$$

When $n \to \infty$, $a \to 0$, but product $na = A$

Now the intensity is given by

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

(or) $$\boxed{I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2} \qquad \text{... (I)}$$

Intensity Distribution in Single Slit

Principal maximum

The expression for resultant amplitude ‘R’ can be written in accending powers of α as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right]$$

If the negative terms vanish, the value of R will be maximum, i.e., $\alpha = 0$

$$\alpha = \frac{\pi e \sin\theta}{\lambda} = 0 \quad \text{(or)} \quad \sin\theta = 0$$

$$\theta = 0$$

or

Now, maximum value of R is A and intensity is proportional to A^2. The condition $\theta = 0$ means that this maximum is formed by those secondary wavelets which travel normally to the slit. The maximum is known as principal maximum.

Minimum intensity positions

The intensity will be minimum when $\sin\alpha = 0$, the value of α which satisfy this equation are

$$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \ldots \text{etc}, = \pm m\pi$$

(or) $$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

(or) $e\sin\theta = \pm m\lambda$ where $m = 1, 2, 3, \ldots$ etc.

In this way we obtain the points of minimum intensity on either side of the principal maximum. The value of $m = 0$ is not admissible, because for this value, $\theta = 0$ and this corresponds to principal maximum.

Secondary maxima

In addition to principal maximum at $\alpha = 0$, there are weak secondary maxima between equally spaced minima. The positions can be obtained with the rule of finding maxima and minima of a given function in a calculus. Differentiating the expression of I with respect to α and equating to zero, we have

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

(or) $$A^2 \times \frac{2\sin\alpha}{\alpha} \times \frac{(\alpha\cos\alpha - \sin\alpha)}{\alpha^2} = 0$$

either $\sin\alpha = 0$

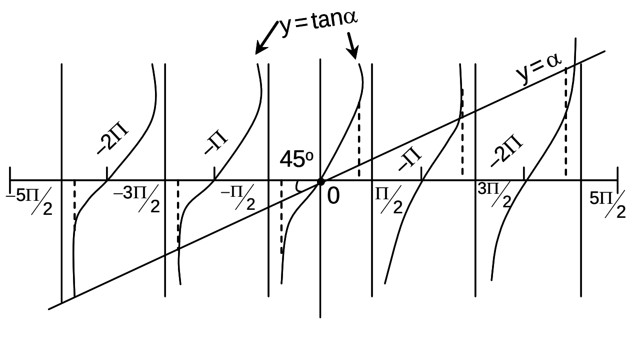
(or)

$(\alpha\cos\alpha - \sin\alpha) = 0$

The equation $\alpha = 0$ gives the values of α for which the intensity is zero on the screen. Hence, the positions of maxima are given by the roots of the equation $\alpha\cos\alpha - \sin\alpha = 0$ (or) $\boxed{\alpha = \tan\alpha}$ – (A) the values of α satisfying the above equation are obtained graphically by plotting the curves.

$y = \alpha$ and $y = \tan\alpha$ on the same graph

The point of intersection of two points shown in graph.



The points of intersections are

$$\alpha = 0, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \text{etc.}$$

(or) more exactly to $\alpha = 0, \pm 1.430\,\pi, \pm 2.462\,\pi, \pm 3.471\,\pi$, etc

$\alpha = 0$ gives principal maximum

Substituting approximate values of α in (I), we get the intensities in various maxima.

$I_0 = A^2$ (principal maxima)

$$I_1 = A^2\left[\frac{\sin\left(3\pi/2\right)}{\left(3\pi/2\right)}\right]^2 = \frac{A^2}{22} \text{ (Approximately)}$$

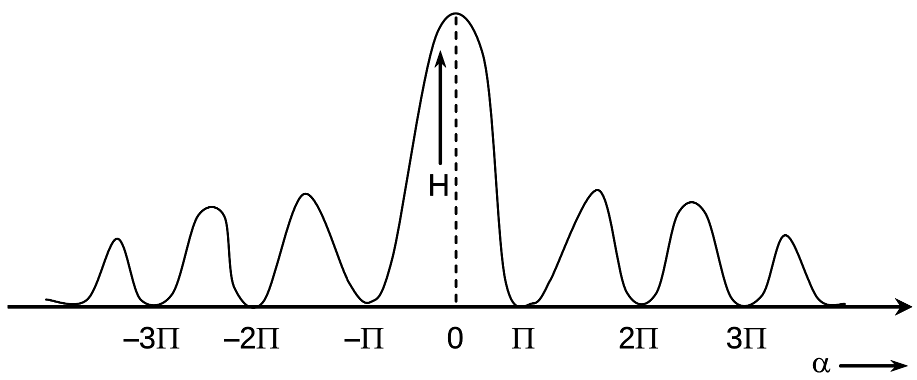
$$I_2 = A^2\left[\frac{\sin\left(5\pi/2\right)}{\left(5\pi/2\right)}\right]^2 = \frac{A^2}{62} \text{ (Apply)}$$

and so on.

From the expressions of I_0, I_1, I_2 it is evident that most of the incident light is concentrated in the principal maximum.

Intensity distribution graph

A graph showing the variation of intensity with α.

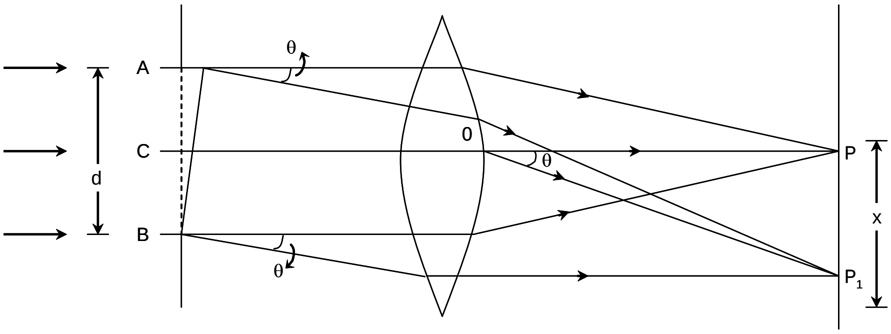


3.2.2 Fraunhofer Diffraction at Circular Aperture

Q5. Explain Fraunhofer diffraction at circular aperture.

Ans :

The following arrangement for shows single slit Fraunhofer diffraction. The slit aperture is AB and its axis is CP. If the diagram is rotated about the axis CP, we get AB as circular aperture with source at infinity. A plane wavefront is incident on the circular aperture.



The secondary wave travelling in the direction co comes to focus at P. Therefore, P corresponds to the position of the central maximum. Here, all the secondary waves emanating from points equidistant from '0' travel the same distance before reaching P and hence they all reinforce one another.

Now let us consider the secondary waves travelling in a direction inclined at an angle θ with the direction CP. All these secondary waves meet at P_1 on the screen. Let the distance PP_1 be x.

The path difference between the secondary waves emanating from the points B and A is AD

From the ΔABD, AD = d sinθ

The point P_1 will be of minimum intensity if this path difference is equal to integral multiples of λ as in the case of single slit.

i.e., $d \sin \theta_n = n\lambda$ ... (1)

The point P_1 will be of maximum intensity if the path difference is equal to odd multiples of λ/2.

i.e., $d \sin \theta_n = \dfrac{(2n+1)\lambda}{2}$ ... (2)

If P_1 is the point of minimum intensity, then all the points at the same distance from P and P_1 and lying on a circle of radius x will be of minimum intensity.

The intensity of the dark rings is zero and that of the bright rings decreases gradually outwards from P. Further if the collecting lens is very near the slit or when the screen is at a large distance from the lens,

$$\sin \theta = \theta = \frac{X}{f}$$ ... (a)

where f is the focal length of the lens.

Also for the first secondary minimum, d sinθ = λ

or $\sin\theta \simeq \theta = \lambda/d$ ... (b)

From (a) and (b)

$$\frac{X}{f} = \lambda/d \quad \text{(or)} \quad X = \frac{f\lambda}{d}$$ ... (I)

Where X is the radius of the Airy's dise. But actually the radius of the first dark ring is slightly more than that given by equation (I). According Airy, it is given by

$$X = \frac{1.22\,\lambda}{d}$$

With increase in diameter of the aperture, the radius of the central bright ring decreases.

3.2.3 Limit of Resolution

Q6. Define limit of resolution and obtain an expression for Rayleigh's criterion.

Ans : **(Jan.-21, June-18, May-18)**

The ability of the instrument to produce their separate patterns is known as resolving power.

The limit of resolution of an optical instrument is defined as the smallest angle subtended at its objective by two point objects which can just be distinguished as separate.

The reciprocal of the limit of resolution is called the resolving power.

Rayleigh's Criterion :

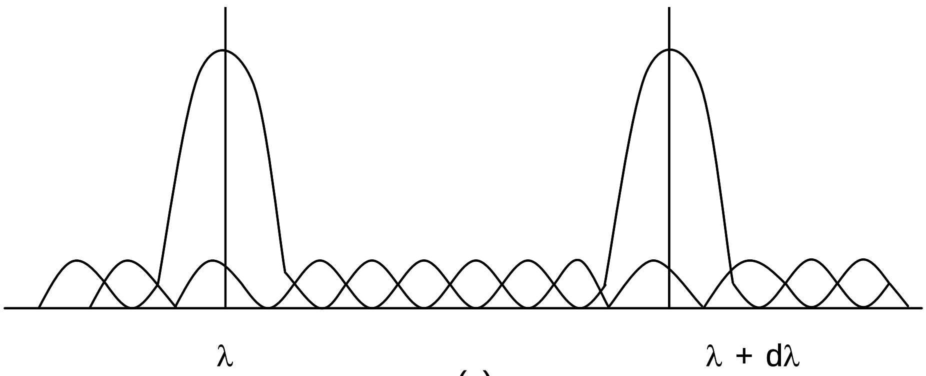
According to Rayleigh's criterion, the two point sources or two spectral lines of equal intensity are just resolved by an optical instrument when the central maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other.

To illustrate the above criterion, consider the intensity distribution curves of two wavelengths λ and $\lambda + d\lambda$. The separation between their central maximum will depend upon the value of $d\lambda$. If $d\lambda$ is sufficiently large, the central maxima due to two wavelength are quite separate and the two spectral lines appear well resolved.

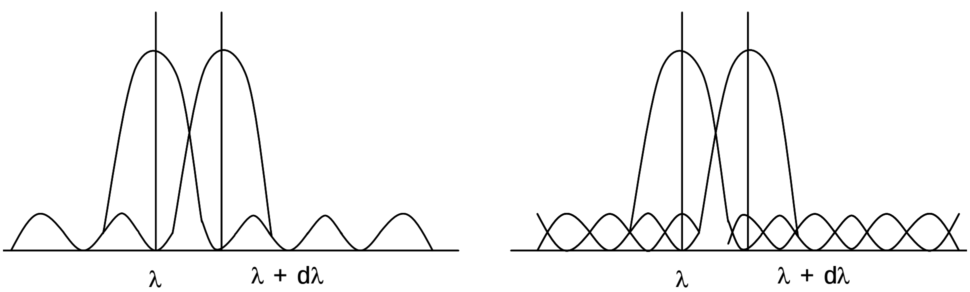
However $d\lambda$ will have a limiting value under this condition, according to Rayleigh, the two spectral lines are just resolved.

The intensity distribution is of the form

$$I = I_0 \frac{\sin^2 x}{x^2}$$



(a)



(b)

The first minimum occurs at x = π. At the middle of the two maxima the intensity due to each is given by putting x = $\Pi/2$.

Hence, the total intensity at the middle of the maxima (b) is given by putting

$$I_{mid} = \frac{2I_0 \sin^2(\Pi/2)}{(\Pi/2)^2}$$

$$= \left(\frac{8}{\Pi^2}\right) I_0$$

$$\frac{I_{mid}}{I_0} = \frac{8}{\Pi^2} = 0.81$$

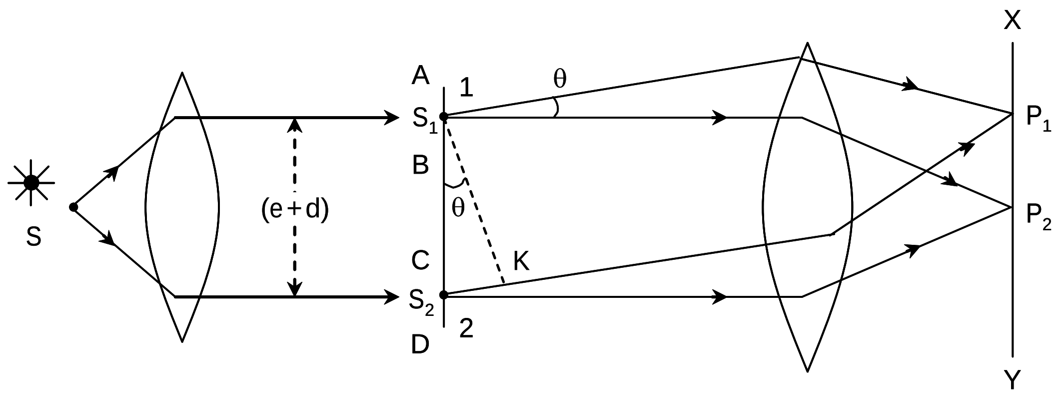
Thus the Rayleigh criterion may be stated as the two images of equal intensities are just resolved if the intensity at the dip in the middle is $\frac{8}{\Pi^2}$ times the intensity at either of the maximum.

3.3 Fraunhofer's Diffraction due to a Double Slit

Q7. Describe the features of a double slit Fraunhofer's diffraction pattern. What is the effect of increasing the slit separation and wavelength?

Ans : **(Jan.-21)**

Let AB and CD be two parallel slits of equal width e and separated by an opaque distanced. The distance between the corresponding middle points of the two slits is (etd).



Let a parallel beam of monochromatic light of wavelength "A" be incident normally upon the two slits. The light diffracted from these slits is focused by a lens on the screen XY placed in the focal plane of lens.

The diffraction at two slits is the combination of diffraction as well as interference, i.e., the pattern on the screen is the diffraction pattern due to a single slit on which a system of interference fringes is superposed.

According to the theory of diffraction at a single slit, the resultant amplitude R due to all wavelets diffracted from each slit in a direction θ is given by

$$R = \frac{A\sin\alpha}{\alpha}$$

where A is a constant being equal to the amplitude due to a single slit when $\theta = 0$ and $\alpha = (\Pi e \sin\theta/\lambda)$.

Thus, for simplicity we can consider the two slits as equivalent to two coherent sources S_1 and S_2 arranged at mid-points of the slits, and each source sending a wavelet of amplitude (Asin α/α) in a direction θ.

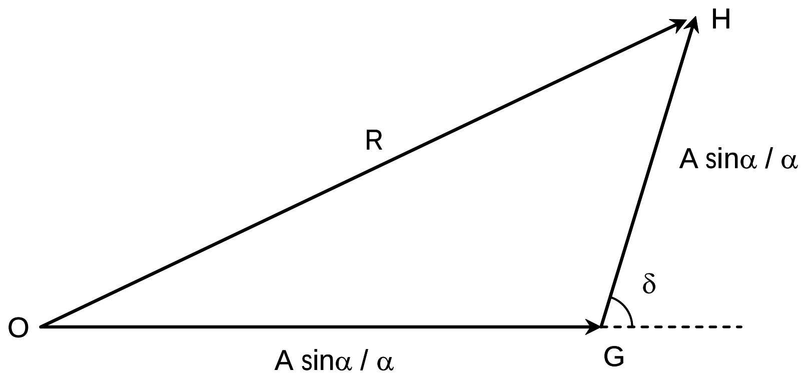
Therefore, the resultant amplitude at a point P_1 on the screen will be a result of interference between two waves of amplitude (Asin α/α) and having a phase difference δ (say). To calculate δ, we draw a perpendicular S_1K or S_2K. The path difference between the wavelets from S_1 and S_2 in the direction θ.

$$= S_2K$$
$$= (e + d)\ \text{Sin}\theta$$

$$\therefore \quad \text{Phase difference} = \frac{2\Pi}{\lambda} \times (\text{path difference})$$

(or) $$\delta = \frac{2\Pi}{\lambda}(e + d)\sin\theta \qquad \text{... (1)}$$

The resultant amplitude R at P_1 can be obtained with the help of triangle of diagram (a)



From figure,

$$(OH)^2 = (OG)^2 + (GH)^2 + 2\,(OG)\,(GH)\,\text{Cos}\delta$$

$$\therefore \quad R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{A\sin\alpha}{\alpha}\right)\cos\delta$$

or $$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 [1 + 1 + 2\cos\delta]$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 [2 + 2 + \cos\delta]$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 2\,[1 + \cos\delta]$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 2\left[1 + 2\cos^2\frac{\delta}{2} - 1\right]$$

$$R^2 = 4\left(\frac{A\sin\alpha}{\alpha}\right)^2 \cos^2 \delta/2$$

$$= 4\,\frac{A^2\sin^2\alpha}{\alpha^2}\cos^2\left[\frac{\Pi}{\lambda}(e+d)\sin\theta\right]$$

$$R^2 = 4\,\frac{A^2\sin^2\alpha}{\alpha^2}\cos^2\beta \text{ where } \beta = \Pi/\lambda\ (e + d)\sin\theta$$

Therefore, the resultant intensity at p_0 is given by

$$I = R^2 = 4\,\frac{A^2\sin^2\alpha}{\alpha^2}\cos^2\beta = 4\,I_0\cos^2\beta\left(\frac{\sin^2\alpha}{\alpha^2}\right]$$

Important Results

(i) Effect of Increasing Slit Width

Let the slit width e be increased, As a result, the envelope of the fringe pattern changes such that the central peak is sharper of course, the fringe spacing does not change because it depends on slit separation (d).

(ii) Effect of increasing the distance between the slits

Let the slit width e is kept constant and the separation d between the slits is increased. In this case, the fringes become closer together and the envelope of the pattern remains unchanged. Hence, more interference maxima fall within the central envelope.

(iii) Effect of Increasing the Wavelength A

When the wavelength of monochromatic light falling on the slit increases, the envelope become broader. As a result, the fringes move farther apart.

Q8. If white light is used in young's double slit experiment, what will happen to the interference bands.

Ans : **(June-18)**

When white light is used in young's double slit experiment, waves of each wavelength forms their separate interference pattern because white light consists of waves of innumerable wavelengths starting from violet to red colour.

3.3.1 Fraunhofer Diffraction Pattern with N Slits

Q9. Explain diffraction of grating by fraunhofer diffraction pattern with N slits.

Ans : **(May-18, Imp.)**

Let a parallel beam of light from monochromatic source incident normally on N parallel slits as shown in diagram. Let 'a' be the width of each slit and b is the opaque spacing between any two consecutive slits. Thus the distance between corresponding points of two consecutive slits is (a + b) = d, where d is called grating element or grating constant. The diffracted light through N slits is focussed by lens L on the screen XY, lying in the focal plane of lens L. The pattern obtained on the screen is called Fraunhofer diffraction pattern due to N parallel slits.

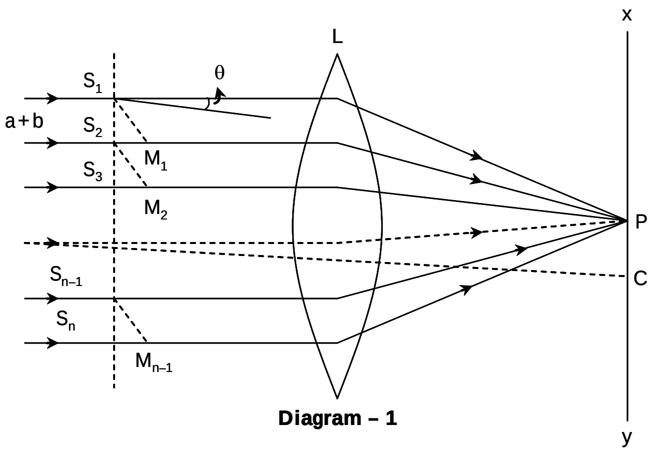


Diagram – 1

We have seen that the resultant amplitude and intensity of light diffracted by angle θ due to the single slit are given by equations.

$$A_\theta = A_0 \left(\frac{\sin\alpha}{\alpha}\right) \quad \text{... (1)}$$

and $$I_\theta = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{... (2)}$$

where $$\alpha = \Pi d \frac{\sin\theta}{\lambda}$$

Since all the slits are identical, the intensity at a particular direction θ due to all individual slits will be same as given in equation (2).

Now the amplitudes A_θ from all the N slits will interfere among themselves and resultant intensity is obtained on screen. Consider the middle points of any two consecutive slits. Let they are denoted as S_1, S_2 as shown in diagram. The distance between them is (a + b) = d. Thus the path difference between wavelets from S_1 & S_2 is d sinθ and phase difference will be

$$\theta = \frac{2\Pi}{\lambda} d\sin\theta.$$

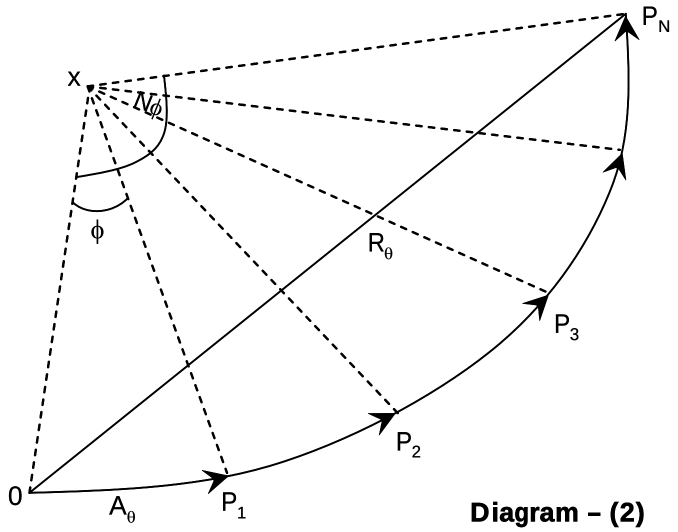


Diagram – (2)

Phase difference between corresponding wavelets from first and second slit is θ, that between first and third is 2φ and so on.

Thus we have N waves each of equal amplitude "A_θ" but differing in phase by θ from its proceeding one.

The resultant amplitude can be obtained graphically. In diagram (2) OP_1, P_1P_2, ... each representing a length A_0 and angle between any two consecutive vectors is ϕ. The N vectors from the side of a polygon whose centre is X. The resultant amplitude R_θ is represented by OP_N.

From diagram

$OP_1 = A_\theta = 20 \times \sin(\phi/2)$ and

$OP_N = R_\theta = 20 \times \sin(N\phi/2)$ because the

Total angle $\angle O \times P_N = N\phi$

$$\frac{R_\theta}{A_\theta} = \frac{\sin(N\phi/2)}{\sin(\phi/2)}$$

$$R_\theta = A_\theta \left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin(N\phi/2)}{\sin(\phi/2)}\right)$$

Thus the intensity at an angle "θ" from incident direction for the diffraction due to the N slits is given by

$$I_\theta = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}\right) \quad \text{... (I)}$$

If N = q, then we get

$I_\theta = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2$ which is same as obtained for single slit diffraction pattern.

It is seen that the first factor in the equation (I) is simply the intensity distribution due to diffraction by single slit. The second factor arises due to interference between all such diffracted wavelets from N slits.

Formation of Spectrum with a Grating :

The direction of principal maxima are given by

$$(a + b)\sin\theta = \pm n\lambda$$

This equation show that

(i) For a particular value of λ, the directions of principal maxima of different orders are different.

(ii) The angle of diffraction increases as wavelength increases, for a given value of n (order). Therefore the angle of diffraction for red colour is greater than that for violet colour. Hence if white light is incident normally on a grating, each order will contain principal maxima of different wavelengths in different directions.

For $n = 0$, $\theta = 0$ for all values of λ i.e., zero order principal maxima for all wavelengths lie in the same direction. Thus the zero order principal maxima will be white and first from the first one and similarly the remaining orders will be formed. Most of the light is concentrated in the principal maximum of zero order and the intensity goes on diminishing gradually as we go to the higher orders.

Thus the spectrum consists of white maximum of zero order having on either side of it the first order spectra, the second order spectra and so on.

The spectra of each order consists of spectral colours in the order from violet to red.

Condition for absent spectra with a diffraction grating

The directions for principal maxima with a diffraction grating is

$$(a + b)\sin\theta = n\lambda$$

where n is the order of the principal maximum.

The directions of minima in the diffraction pattern due to single slit is

$$a\sin\theta = m\lambda,\ m = 1, 2, 3, \ldots$$

If the above two equations are simultaneously satisfied, the principal maxima of order "n" will not be present in the grating spectrum.

Dividing above two equations

$$\frac{a+b}{a} = \frac{n}{m}$$

or $$n = \frac{a+b}{a} \cdot m$$

This condition is the condition for n^{th} order to be absent in the granting spectrum.

when $b = a$ then $n = \frac{a+a}{a} m = 2m = 2, 4, 6 \quad (\because m = 1, 2, 3, \ldots)$

i.e., when $b = a$, 2^{nd}, 4^{th}, 6^{th} order spectra will be absent.

when $b = 2a$, 3^{rd}, 6^{th}, 9^{th}, ... order spectra will be absent.

Maximum number of orders with a diffraction grating

The direction of n^{th} order principal maxima for wavelength λ is given by

$(a + b) \sin\theta = n\lambda$ (for normal incident)

This can also be written as,

$$n = \frac{(a+b)\sin\theta}{\lambda}$$

The maximum possible value of $\theta = 90^o$. Therefore the maximum number of possible order is

$$n_{max} = \frac{(a+b)\sin 90^o}{\lambda} = \frac{(a+b)}{\lambda}$$

If the grating element is less than twice the wavelength of light used, then

$(a + b) < 2\lambda$

$$\therefore \quad n_{max} < \frac{2\lambda}{\lambda} < 2$$

i.e., only the first order is possible.

3.4 Resolving Power of Grating

Q10. "Resolving power of grating is independent of grating element". Explain.

OR

Explain Resolving power of grating.

Ans :

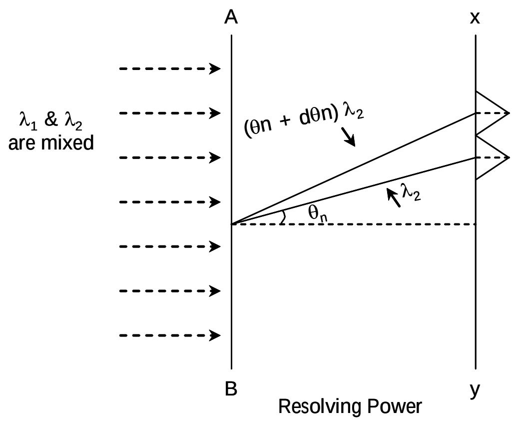
The resolving power of a grating is defined as its ability to just resolve two spectral lines very close to each other and it is given by the equation $R = \lambda/\Delta\lambda$

Let the direction of n^{th} principal maxima for wavelength λ_1 is given by

$(a + b) \sin \theta_n = n\lambda_1$ ... (1)

(or)

$N(a + b) \sin \theta_n = Nn\lambda_1$



Resolving Power

and the first minima will be in the direction given by

$$N(a + b) \sin (\theta_n + d\theta_n) = m\lambda_1$$

Where M is an integer except 0, N, 2N, ..., because at these values condition of maxima will be satisfied. The first minima adjacent to the nth maxima will be in the direction (en + den) only

when $m = (nN + 1)$. Thus

$$N(a + b) \sin (\theta_n + d\theta_n) = (nN + 1)\lambda_1$$

For just resolution, the principal maxima for the wavelength λ_2. Must be formed in the direction (e11 + de11)

$$\therefore \quad (a + b) \sin (\theta_n + d\theta_n) = n\lambda_2$$

$$\text{(or)} \quad N(a + b) \sin (\theta_n + d\theta_n) = Nn\lambda_2$$

Now equating the two equations.

$$(nN + 1)\lambda.1 = Nn.\lambda_2$$

$$(nN + 1)\lambda = Nn(\lambda + d\lambda)$$

$$\lambda = Nnd\lambda$$

$$\lambda_1 = \lambda, \ \lambda_2 - \lambda_1 = d\lambda, \ \lambda_2 = \lambda + d\lambda$$

The resolving power of grating is found as

$$R.P = \lambda / d\lambda = nN$$

Resolving power = Order of spectrum × Total number of lines on grating

which can also be written as

$$N(a + b) \sin \theta/\lambda = w\sin \theta_n/\lambda$$

$$\therefore \quad R.P = \frac{\lambda}{\Delta\lambda} = nN \quad \text{... (2)}$$

It appears that the resolving power would increase if the value of N is increased. However this is not quite true because the resolving power is essentially determined by the width of the grating. This can be easily seen by substituting the value of n in equation (1) and (2).

$$n = (a + b) \sin \theta_n / \lambda_n$$

$$R = \frac{\lambda}{\Delta\lambda} = N(a + b) \sin \theta_n / \lambda = D \sin \theta_n / \lambda$$

where $D = N(a + b)$ represent the width of grating

3.4.1 Wavelength of Light in Normal and Oblique Incidence

Q11. Determine the wavelength of light in normal and oblique incidence methods using diffraction grating.

Ans :

Normal Incidence

The condition for principal maxima for normal incidence of light on grating surface is given by

$$(a + b) \sin\theta = n\lambda, \ n = 0, 1, 2, 3 \quad \text{... (1)}$$

where "a" is the width of the slit, b is the width of the opaque portion, (a + b) is called grating element. θ is the angle of diffraction for a particular order n for wavelength λ.

Knowing grating element (a + b), angle of diffraction θ for order n, the wavelength of light λ can be find.

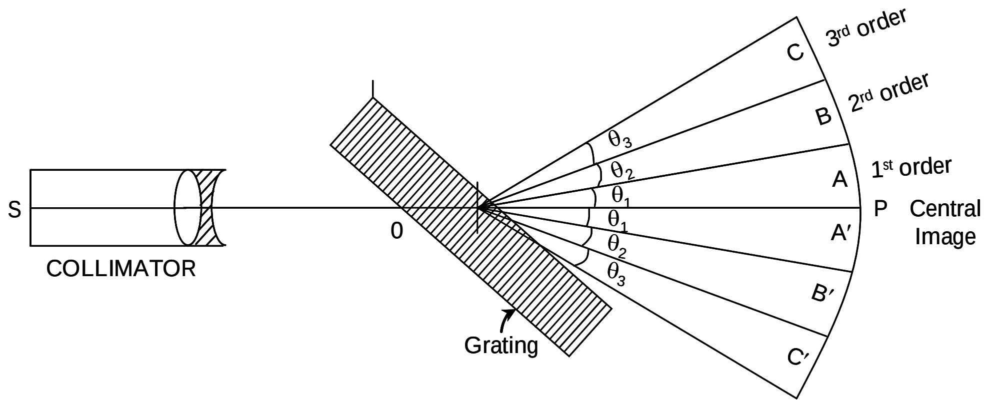
Grating element (a + b) can be determined as follows

On every grating the number of rulings per inch is written. Therefore, if N is the number of rulings per inch, then

$$N(a + b) = 1" = 1.54 \text{ cm}$$

$$(a + b) = (2.54 / N) \text{ cm}$$

Next the angle of diffraction θ is determined using the spectrometer.

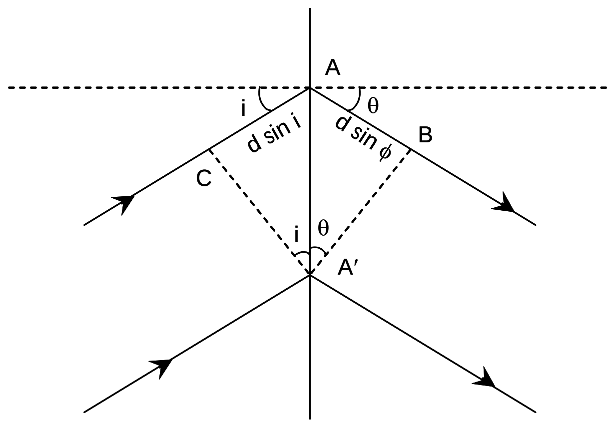


If the source of light emits radiations of different wavelengths, then the beam gets dispersed by the grating and in each order a spectrum of the constituent wavelengths is observed. By noting the diffracting angles in the first and second orders and using the equation (1), the wavelength of the spectral line can be calculated.

Wavelength of any spectral line can be determined very accurately with the diffraction grating as the method does not involve measurements of very small distances.

Oblique Incidence

The experimental setting to achieve the condition of normal incidence to a great precision is quite difficult and it is seen that slight deviations from normal incidence will introduce considerable errors. It is, therefore, mor practical to consider the general oblique incidence case. Let the light incidence at an angle "i" with the normal on the grating surfaces as show in diagram. Let "θ" be the angle of diffraction.



Let A and A' represent two consecutive slits. Since AA' = (a + b) = d, path difference of the diffracted rays from two corresponding points in adjacent slits will be

$$CA + AB = d\sin i + d\sin\theta$$

Thus the principal maxima will occur when

$$d\sin i + d\sin\theta = n\lambda$$

$$d(\sin i + \sin\theta) = n\lambda \qquad \text{... (2)}$$

From diagram we see that total deviation is the sum of i and θ.

$$\therefore \quad D = i + \theta$$

Rewriting equation (2)

$$d(\sin i + \sin\theta) = n\lambda$$

$$d\left(2\sin\left(\frac{i+\theta}{2}\right)\cos\left(\frac{i-\theta}{2}\right)\right) = n\lambda$$

$$2d\sin\frac{D}{2}\cos\left(\frac{i-\theta}{2}\right) = n\lambda$$

$$(\because \; D = i + \theta)$$

In the above equation D is maximum when $\sin\left(D/2\right)$ is minimum

Therefore, for fixed values of n and λ, $\cos\left(\frac{i-\theta}{2}\right)$ should be maximum.

$$\text{i.e.,} \quad \cos\left(\frac{i-\theta}{2}\right) = 1$$

$$\text{or} \quad i - \theta = \theta \;\text{(or)}\; i = \theta$$

Thus we have $2d\sin\left(D/2\right) = n\lambda$

The minimum deviation position is obtained as follows :

By Looking through the telescope rotate the grating table in such a way that angle of incidence is decreased. A position will come where the spectrum starts retracing its path. This is the minimum deviation position.

This position is noted for particular order and particular wavelength. Then the direct image of the source is seen through the telescope. Angle between these two positions gives the minimum deviation angle D. Substituting the values of D, d, n in the above equation, the value of λ can be calculated. Since the adjustments are relatively simpler, this provides a more accurate method for the determination of λ.

3.5 Fresnel Diffraction

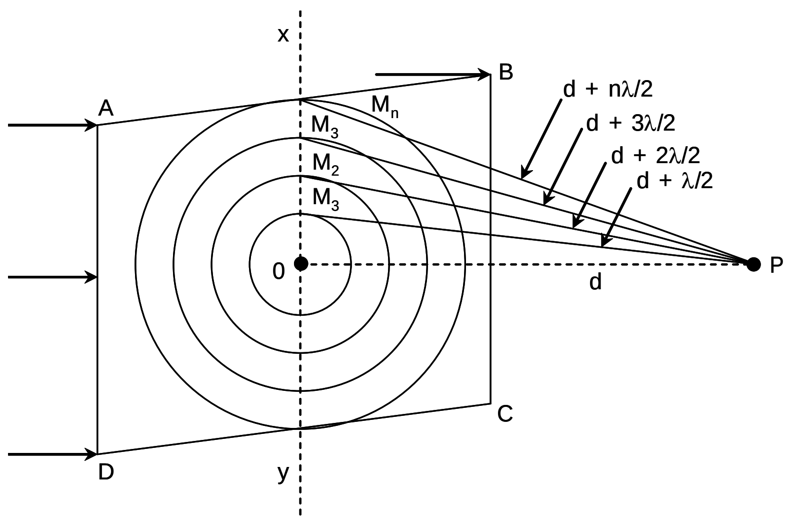
3.5.1 Fresnel s Half Period Zones, Area of the Half Period Zones

Q12. Explain fresnel's half period zones.

Ans : **(May-18)**

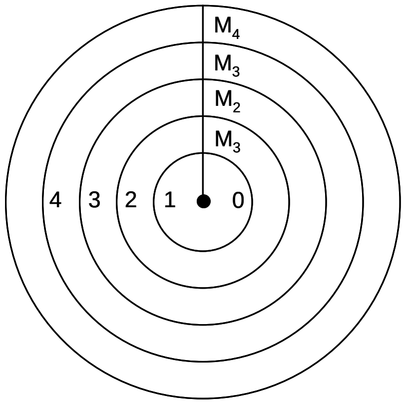
Consider a plane wavefront ABCD perpendicular to the plane of the paper as shown diagram. Let "P" is an external point at a distance a perpendicular to ABCD. In order to find the resultant intensity at P due to the wavefront, by Fresnel's method, the wavefront is divided into a number of half period elements or zones called Fresnel's zones and then the effect of all the zones at point P is found with "P" as centre and radii equal to b + $\lambda/2$, b + $2\lambda/2$, ... etc. spheres are constructed. These spheres will cut out circular areas of radii OM_1, OM_2, OM_3, etc., on the wavefront.

- These circular zones are called half period zones or half period elements.
- The secondary wavelets from any two consecutive zones reach P with a path difference $\lambda/2$ or time difference half period.
- Thus the zones are called half period zones. A Fresnel half period zone with respect to an external point "P" is a thin annular zone of the primary wavefront in which the secondary waves from any two corresponding points of neighbouring zones differ in path by $\lambda/2$.
- The point "θ" is called the pole of the wavefront XY with respect to point P, OP is perpendicular to XY.



Area of Half Period Zones

In the diagram 1, 2, 3 etc. are the half period zones constructed on the primary wavefront XY.



OM_1 is the radius of the first zone.

OM_2 is the radius of the second zone and so on.

The radius of first half period

Zone :

$$OM_1 = \sqrt{((M_1P)^2 - (OP)^2)}$$

$$= \sqrt{\left(\left(b + \lambda/2\right)^2 - b^2\right)}$$

$$= \sqrt{\left(b^2 + \lambda^2/4 + b\lambda - b^2\right)}$$

$$= \sqrt{b\lambda}\text{ , as “b” is very much greater than } \lambda.$$

The radius of second half period zone,

$$OM_2 = \sqrt{((M_2P)^2 - (OP)^2)}$$

$$= \sqrt{\left(\left(b + 2\lambda/2\right)^2 - b^2\right)}$$

$$= \sqrt{(b^2 + \lambda^2 + 2b\lambda - b^2)}$$

$$= \sqrt{(2b\lambda)}$$

Similarly the radius of n^{th} half period zone.

$$OM_n = \sqrt{(nb\lambda)}$$

Thus we see that the radii of half period zones are proportional to the square roots of the natural numbers.

The area of the first half period zone is $= \Pi (OM_1)^2$

$= \Pi (b\lambda)$

The area of the second half period zone is

$= \Pi ((OM_2)^2 - (OM_1)^2)$

$= \Pi (2b\lambda - b\lambda)$

$= \Pi b\lambda$

Thus it is seen that the area of each half period zone is equal to $\Pi b\lambda$

However, it should be remember that area of the zones are not constant they are dependent on

(i) The wavelength of light "λ"

(ii) The distance of point from the wavefront "b".

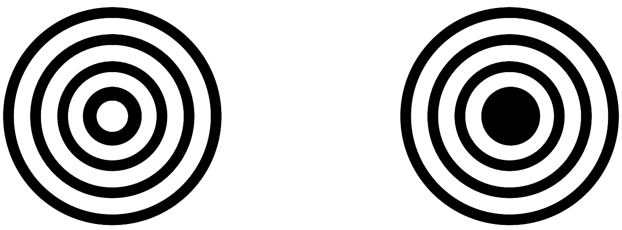
3.5.2 Zone Plate

Q13. Explain the concept of Fresnel half period zone plates.

Ans : **(Jan.-21, June-19)**

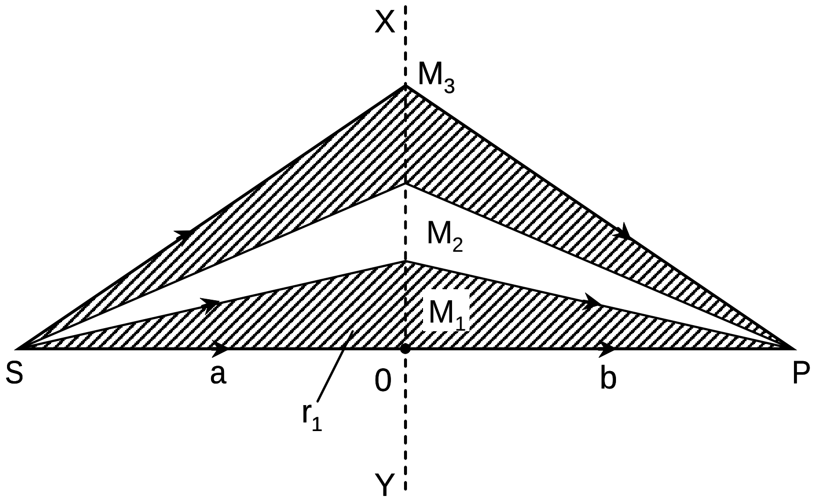
The interesting application of the concept of Fresnel half period zones lies in the construction of the zone-plate.

A zone plate consists of a large number of concentric circles whose radii are proportional to the square root of natural numbers and the alternate annuals regions of which are blackened as shown in diagram (a). It can be designed so as to cut off light due to the over numbered zones or that due to the odd numbered zones.



(a) (b)

- To construct a zone plate, concentric circles are drawn on white paper such that the radii are proportional to the square roots of the natural numbers.
- The odd numbered zones are covered with black ink and a reduced but accurate photograph is taken. In the negative of photograph, the odd zones are transparent and the even zones will out off light.
- If the even zones are transparent and odd zones are opaque, it is said to be a negative zone plate.
- Here in diagram, XY represents the edgewise section of a zone plate. S is a point source of light.
- Let P be the point on the screen at a distance b from "0" at which intensity is to be found.



The distance of source from "O" be "a". Let OM_1, OM_2, OM_3 etc., are the radii of the first, second third etc., half period zones. Let these radii are represented by r_1, r_2, r_3 .... etc. The position of the screen is such that from one zone to the next there is an increasing path difference of $\lambda/2$.

Thus, $SO + OP = a + b$

$$SM_1 + M_1P = a + b + \lambda/2$$

$$SM_2 + M_2P = a + b + 2\lambda/2 \text{ and so on.}$$

From

ΔSM_1O

$$SM_1 = \left(SO^2 + OM_1^{\,2}\right)^{1/2} = \left(a^2 + r_1^{\,2}\right)^{1/2}$$

Similarly from the ΔOM_1P

$$M_1P = \left(OP^2 + OM_1^{\,2}\right)^{1/2}$$

$$= \left(b^2 + r_1^{\,2}\right)^{1/2}$$

Thus, we have

$$\left(a^2 + r_1^{\,2}\right)^{1/2} + \left(b^2 + r_1^{\,2}\right)^{1/2} = a + b + \lambda/2$$

$$a\left(1 + \frac{r_1^{\,2}}{a^2}\right)^{1/2} + b\left(1 + \frac{r_1^{\,2}}{b^2}\right)^{1/2} = a + b + \lambda/2$$

$$a + \frac{r_1^{\,2}}{2a} + b + \frac{r_1^{\,2}}{2b} = a + b + \lambda/2$$

$$\frac{r_1^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{\lambda}{2}$$

$$r_1^2\left(\frac{1}{a}+\frac{1}{b}\right) = \lambda;$$

$$\therefore\ r_1 = \sqrt{\frac{\lambda ab}{(a+b)}}$$

Similarly for r_n i.e., the radius of the n^{th} zone, the relation can be written as

$$r_n^2\left(\frac{1}{a}+\frac{1}{b}\right) = n\lambda;$$

$$\therefore\ r_n = \sqrt{\frac{\lambda nab}{(a+b)}}$$

Applying the sing convention,

$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n}$$

where, $f_n = \frac{r_n^2}{n\lambda}$

The above equation is similar to the equation $\left(\frac{1}{f} = \frac{1}{v} - \frac{1}{4}\right)$ in case of lenses with a and b as the object and image distances and f_n is the focal length. Thus a zone plate acts as a converging lens, further, from the equation,

$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2}$$

or $r_n^2 = \frac{n\lambda ab}{(a-b)}$

Since a, b and λ are constants, we have $r_n \alpha \sqrt{n}$

The area of the n^{th} zone is given by

$$\Pi\left(r_n^2 - r_{n-1}^2\right) - \Pi\left(\frac{n\lambda ab}{(a-b)} - \frac{(n-1)\lambda ab}{a-b}\right) = \frac{\Pi\lambda ab}{(a-b)}$$

Thus the independent of "n", hence for a given object and image, the area of all the zones remain the same.

Further, the area diminishes as a or b decreases i.e., as the plate is approached by the object as the image.

3.5.3 Comparison of Zone Plate with Convex Lens

Q14. Compare a zone plate and a convex lens.

Ans : **(Jan-21)**

Zone Plate	Convex Lens
(A) Similarities	
1) $\frac{1}{b} = \frac{1}{a} = \frac{n\lambda}{r_n^2} = 1/fn$ Both form a real image an object on the other side.	$1/v - 1/u = 1/f$ The relations connecting the conjugate distances are similar.
2) Focal lengths of both depend upon the wavelength λ as $fn = \frac{r_n^2}{n\lambda}$ and hence both suffer from chromatic aberration.	$1/f = (u-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ But the chromatic aberration in a zone plate is much more severe than the chromatic aberration in a lens.
(B) Dissimilarities	
1) A zone plate simultaneously acts as a convex lens and a concave lens. In addition to a real image, a virtual image is also formed simultaneously.	1) A convex lens can form only a real image on the other side.
2) The image is formed by the diffraction phenomenon.	2) The formation of image is due to refraction of light.
3) The zone plate has got multiple foci as f, f/3, f/5, f/7 ... $\rightarrow \frac{r_n^2}{n\lambda}, \frac{r_n^2}{3n\lambda}, \frac{r_n^2}{5n\lambda}$ ... on either side of the plate. Hence, the intensity of the image formed will be much less	3) The convex lens has got only one focus on the right side. (w.r.t to object or) (incident light) as all the light is focussed at one point, the intensity of the image will be more.
4) For a zone plate $f_{red} < f_{voilet}$	4) For a lens $f_{red} > f_{voilet}$
5) In a zone plate, waves reaching the image point through any two alternate zones differ in path by λ and by phase of 2Π.	5) All the rays reaching the image point have the same optical path (the path difference or phase difference is zero)
6) A zone plate can be used with X-rays and microwaves also – over a wide range of wavelengths.	6) Glasses lenses can not be used beyong the optically visible region of wavelengths.

3.5.4 Phase Reversal Zone Plate

Q15. Explain phase reversal zone plate.

Ans :

Rayleigh suggested that if the alternate zones of zone plate, instead of being blocked, are coated with a thin film of some transparent material which introduces an additional optical path difference $\lambda/2$, then the secondary disturbances from all the zones both odd or even, would reach the image point in the same phase and so the amplitude from successive zones will help each other consequently the intensity of the image would be four - fold. Such zone plates are called phase reversal zone plates. A phase reversal zone plate can be prepared by the following method.

A chemically cleaned thin glass plate is coated with a thin layer of gelatine solution and dried. It is then immersed in a weak solution of potassium dichromate for a few seconds and dried again in dark. It is then placed in contact with an ordinary zone plate and exposed to sun light for several minutes. Light passing through transparent zones acts on gelatine and makes it insoluble in water while the gelatine in contact with opaque zones remains soluble in water. Finally the glass plate is immersed in water where the gelatine of the unexposed parts is dissolved to such a depth that an additional optical path difference of half a wavelength $\left(\lambda/2\right)$ is introduced between the waves from successive zones. Thus the phase reversal zone plate is formed.

3.5.5 Diffraction at a Straight Edge

Q16. Describe and explain the phenomenon of diffraction due to a straight edge. Explain why the bands are neither equidistant nor equally illuminated.

Ans : **(Jan-21, June-19, June-18)**

Let "S" be narrow slit illuminated by a monochromatic light of wavelength λ. AD is a straight edge and length of the edge is parallel to the length of the slit. The edge A and the slit are perpendicular to the plane of the paper. PQ is the incident cylindrical wave front. Joint SA and produce it to meet the screen MN at c as shown in diagram.

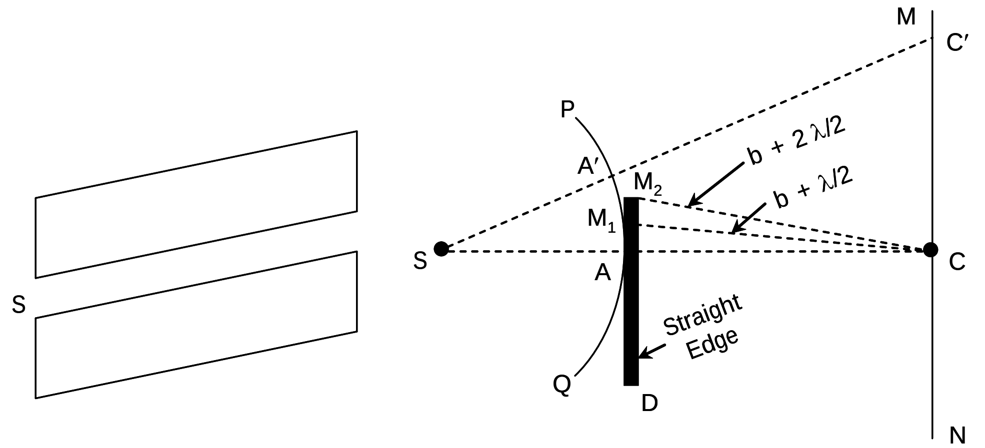


Diagram - 1

According to the laws of geometrical optics we should get uniform illumination above C and geometrical shadow below C. But it is observed that there are a few unequally spaced diffraction fringes in the illuminated region and the intensity doesn't become zero in geometrical shadow region, but it falls rapidly and becomes zero at a small finite distant point from C.

Let the distance AC be "b" with the reference to point "C", the wavefront is divided into a number of half period strips as shown in diagram (2). PQ is the wavefront, A is the pole of wave front and AM_1, M_1M_2, M_2M_3 etc., measure of the thickness of the first, second, third etc., half period strips. From figure $CM_1 = b + \lambda/2$ and $CM_2 = b + 2\lambda/2$ etc.

Let c' be a point on the screen in the illuminated portion as shown in diagram (3).

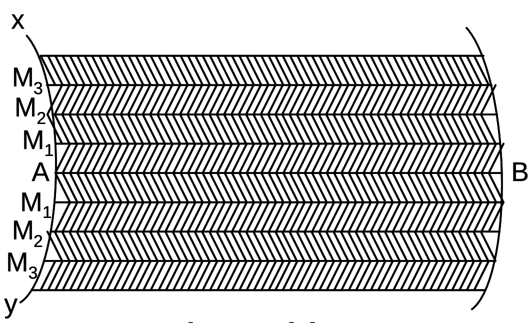


Diagram (3)

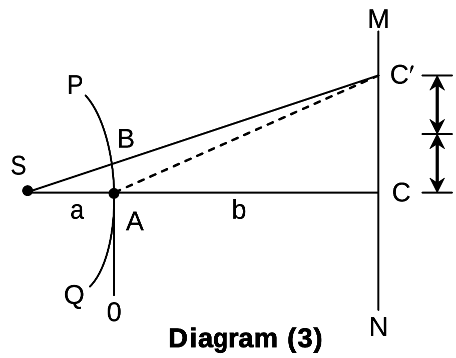


Diagram (3)

To calculate the intensity act c' due to the wavefront PQ, joint SC'. This line meets the wavefront at B. B is the pole of the wavefront with reference to the point c' and the intensity at c' will depend mainly on the number of half period strips enclosed between the points A and B.

The effect at c' due to the wavefront above B is same at all points on the screen where as it is different at different points due to the wave - front between B and A. The point c' will be of maximum intensity, if the number of half period strips enclosed between B and A is odd and the intensity at c' will be minimum if the number of half period strips enclosed between B and A is even.

Let "a" be the distance between the slit and straight edge and b is the distance between the straight edge and screen. Let cc' be x.

Path difference,

$$\delta = AC' - BC'$$

$$= AC' - (SC' - SB)$$

$$= \left(b^2 + x^2\right)^{1/2} - \left[\left((1+b)^2 + x^2\right)^{1/2} - a\right]$$

$$= b\left(1 + \frac{x^2}{2b^2}\right] - (a + b)\left[1 + \frac{x^2}{2(a+b)^2}\right] + a$$

$$= b + \frac{x^2}{2b} - a - b - \frac{x^2}{2(a+b)} + a$$

$$= \frac{x^2}{2}\left(\frac{1}{b} - \frac{1}{a+b}\right)$$

$$= \frac{x^2}{2}\left(\frac{a+b-b}{b(a+b)}\right) = \frac{x^2}{2} \cdot \frac{a}{b(a+b)}$$

C' will be of maximum intensity if $\delta = (2n + 1)\, \lambda/2$.

$$\therefore \quad (2n + 1)\, \lambda/2 = \frac{ax_n^{\,2}}{2b(a+b)}$$

$$\text{(or)} \quad x_n^{\,2} = \frac{(2n+1)(a+b)b\lambda}{a} \quad \text{(or)} \quad x_n = \sqrt{\frac{(2n+1)(a+b)b\lambda}{a}}$$

Where x_n is the distance of n^{th} bright band from "C".

Similarly C' will be of minimum intensity.

$$\text{if } \delta = 2n\, \lambda/2$$

$$\therefore \quad X_n = \left(\frac{(2n)(a+b)b\lambda}{a}\right)^{1/2}$$

Where "x_n" is the distance of n^{th} dark band from C.

Thus, diffraction bands of varying intensity are observed above the geometrical shadow i.e., above 'C' and the bands disappear and uniform illumination occurs if C' is far away from C.

If C' is a point below C and B is the new pore of the wave front with reference to the point C', then the half period strips below B are cut off by the obstacle and only the uncovered half period strips above "B" will be effective in producing the illumination at C'.

As " C' " moves farther from C, more number of half period strips above B is also cut off and the intensity gradually falls. Thus within the geometrical shadow, the intensity gradually fall off depending on the position of C' with respective C.

Intensity at the edge of geometrical shadow

As the lower half of the wavefront is completely blocked by the obstacle, the intensity at C is entirely due to the upper half of the wavefront. Thus the resultant amplitude at C is $\frac{R_1}{2}$ and hence the resultant amplitude at C had been $\frac{R_1}{2} + \frac{R_1}{2} = R_1$ and hence the resultant intensity at C had been $R_1^{\,2}$.

Thus the intensity at C is one quarter of the intensity of that produced by the complete exposed wavefront.

The intensity distribution due to a straight edge is shown in diagram (4) is the source, AD is the straight edge and MN is the screen. In the illuminated portion $\rho\lambda 9$, alternate bright and dark bands of gradually in the region of the geometrical shadow.

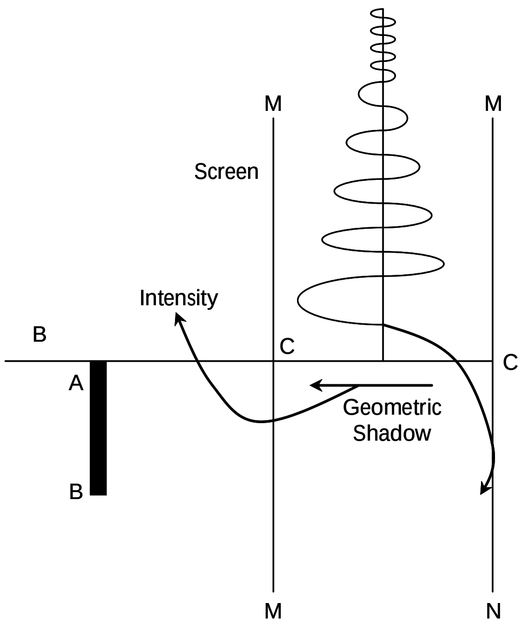


Diagram (4)

In general there is gradual fading of intensity in the region of geometrical shadow and with monochromatic light bright and dark bands of unequal widths are observed in the illuminated portion.

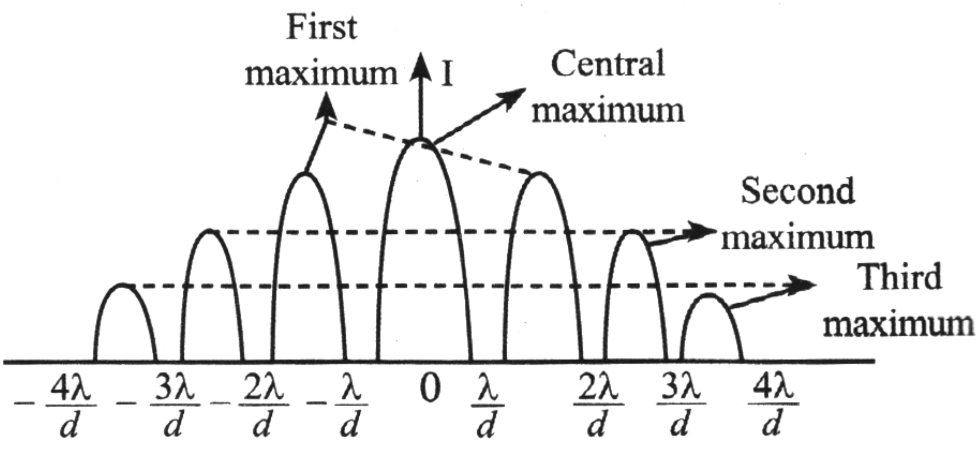
The bands become narrower as distance from “C” increases till there is general illumination.

Q17. Describe and explain the phenomenon of diffraction due to straight edge. Explain why the bands are neither equidistant not equality illuminated.

Ans : **(June-18, Imp.)**

The diffraction pattern of a monochromatic light as a source consists of unequal width of alternate bright and dark bands.

The intensity distribution pattern of diffraction of a single slit is shown in figure.



The central maximum possess maximum intensity, as the order of the maximum decreases, the intensity pattern also decreases. First secondary maximum is due to wavelets of $\frac{1}{3}^{rd}$ part of slit (i.e.. two parts send wavelets in opposite direction), second secondary maximum is due to 5^{th} part of the slit (i.e.. four parts send wavelets in opposite direction) and soon. It is observed that, the intensity of secondary maximum is decreases with distance from the central maximum Hence, the pattern consists of unequal spacings or separations.

The diffraction pattern of white light as a source consists of colours. As the bandwidth is directly proportional to 'λ', the central maximum will be white and other bands will be coloured. Since, the wavelength of red band is large and highly illuminates than the violet band of low illumination (smaller wavelength). Hence, the pattern consists of unequal illuminations.

Therefore, the diffraction bands are neither equidistant nor equally illuminated.

3.5.6 Difference between Interference and Diffraction

Q18. What are the difference between interference and diffraction?

Ans :

The difference between interference and diffraction is given below.

Interference	Diffraction
1. Interference is the phenomenon of interaction among two distinct wavefronts propagating from two coherent sources.	1. Diffraction is the phenomenon of interaction among secondary wavelets propagating from distinct points of same wavefront.
2. The width of interference fringes may or may not be constant.	2. The width of diffraction fringes always varies (i.e., as the order increases, width decreases).
3. The intensity of all the bright fringes is equal.	3. The intensity of each bright fringe varies from one another
4. The minimum intensity fringes are completely dark.	4. The minimum intensity fringes are partially dark.

Problems

1. Calculate the angles at which the first dark bnand and then next bright band are farmed in the fraunhofer diffraction pattern of slit 0.3 mm wide.

Sol :

For the first dark band, we have, $\sin\theta_1 = \lambda/a$

$$\sin\theta_1 = \frac{5890\times10^{-8}}{0.3\times10^{-1}} = \frac{5890\times10^{-6}}{3} \simeq 0.0019$$

θ_1 = Sin^{-1} (0.0019) = 6'.

For the position of next bright band, we have,

$$\sin\theta_2 = \frac{3}{2}\lambda/a \qquad (\because m = 1 \ \& \ a\sin\theta = \left(\frac{2m+1}{2}\right)\lambda)$$

$$\text{Sin}\theta_2 = \frac{3\times5890\times10^{-8}}{2\times0.3\times10^{-1}} = 0.0030$$

θ_2 = sin^{-1} (0.0030) = 10' = 10'.

2. In a grating spectrum, which spectral line in fourth order will overlap with third order line of 5461Å.

Sol : **(June-18)**

The wavelength 'λ' of mth order will overlap with wavelength λ of nth order, if

$(a + b)$ sinq = $m\lambda' = n\lambda$

or $m\lambda' = n\lambda$

$$\lambda' = \frac{n}{m}\lambda$$

Here n = 3; λ = 5461 A°; m = 4

$$\lambda' = \frac{3\times5461}{4}A^0 = 4095.75\ A^\circ$$

3. A plane transmission grating has 80000 lines in all. Find in the wavelength region of 6000 Å, in the second order

a) The resolving power of the grating.

b) The smallest wavelength difference that can be resolved.

Sol :

Number of lines on the grating = 80000

Wavelength, λ = 6000 A° = 6000 × 10^{-8} cm, n = 2

Resolving power = N_n = 80000 × 2 = 160000

The smallest resolvable wavelength difference dλ is given by

$$\frac{\lambda}{d\lambda} = Nn = 160000$$

$$d\lambda = \frac{\lambda}{160000} = \frac{6000\times10^{-8}}{160000} = 375 \times 10^{-4} \text{ cm} = 0.0375 \text{ A}^\circ.$$

4. What is the highest order of spectrum which may be observed with monochromatic light of wavelength 5000 A° by means of grating with 5000 lines/cm ?

Sol :

Order of spectra, $n = \frac{d\sin\theta}{\lambda}$

Highest order occurs when sinq = 1

$$\therefore\ n = \frac{d}{\lambda} = \frac{1}{N\lambda} = \frac{1}{5000\times5000\times10^{-8}} = \frac{1}{25\times10^{-2}}$$

$$= \frac{100}{25} = 4$$

$\therefore$ n = 4

5. A zone plate has the radius of the first ring 0.05 cm. If plane waves (l = 5000Å) fall on the plate, where should the screen be placed so that the light is focussed to a bright

Sol :

The focal length of the zone plate is given by

$$f = \frac{r_n^2}{n\lambda}$$

Given, radius of first ring r_1 = 0.05 cm &

λ = 5000 A° = 5000$10^{-8}$ cm

$$f = \frac{(0.05)^2}{1\times5000\times10^{-8}} = 50 \text{ cm}$$

f = 50 cm

6. Find the angular width of the central bright maximum in the fraunhofer diffraction pattern of a slit of width 12 × 10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.

Sol : **(May-18)**

$$\text{Sin }\theta = \frac{6\times10-5}{12\times10^{-5}} = 0.5$$

$\theta = 30^\circ$

Angular width of the central maximum

$2\theta = 60^\circ$

7. What is the highest order spectrum which may be seen with monochromatic light of wavelength 6000Å by means of a diffraction grating with 5000 lines / cm?

Sol :

$$b/\lambda = 5000$$

$$n_{max} = b/\lambda = \frac{1}{5000} \times \frac{1}{6000 \times 10^{-8}} = 3.33 \text{ or } 3.$$

8. How many orders will be visible if the incident radiation is 500Å and the number of lines on the grating is 2620 in one inch?

Sol :

$$\text{Number of lines / cm} = \frac{2620}{2.54} = 1031$$

Maximum number of orders

$$n_{max} = b/\lambda = \frac{1}{(\lambda/b)} \times \frac{1}{5 \times 10^{-5} \times 1031} = 19.4$$

$$n_{max} = 19$$

9. With respect to a point 50 cm distance for a wavelength of 6000Å, calculate the number of half period zones in a circular hole of radium 1 cm.

Sol :

Given

$\lambda = 6000^\circ$ A, $r_n = 1$cm; $b = 50$ cm

$$b = \frac{r_n^2}{n\lambda} = \frac{1^2}{50 \times 6 \times 10^{-5}} = 333$$

10. What is maximum number of lines of grating which will reserve the third order spectrum of two lines having wavelengths 5890Å and 5896Å?

Sol :

Given $n = 3$; $\lambda = 5890A^\circ$; $d\lambda = 6^\circ A$

We know that the resolving power of grating is

$$\frac{\lambda}{d\lambda} = nN$$

$$N = \frac{1}{n}\frac{\lambda}{d\lambda} = \frac{1}{3} \cdot \frac{5890\ A^{\circ}}{6A^{\circ}} = 327.22 \cong 327$$

11. Find the possible order of diffraction with a grating of element 0.12 × 10^{-5} m and wavelength is 6000Å.

Sol : **(June -19)**

Given that,

For a diffraction

grating element, $(a + b) = 0.12 \times 10^{-5}$ m

Wavelength, $\lambda = 6000\ \mathring{A} = 6000 \times 10^{-10}$ m

The expression for grating is given as,

$(a + b) \sin\theta = n\lambda$

For higher order spectrum, $\theta = 90^{o}$

$\Rightarrow$ $\sin\theta = 1$

$$\Rightarrow n = \frac{(a+b)}{\lambda}$$

$$= \frac{0.12 \times 10^{-5}}{6000 \times 10^{-10}}$$

$$= \frac{0.12}{0.06}$$

$$= 2$$

Therefore, the possible order of diffraction is 2.

Short Question and Answers

1. Rayleigh's Criterion for resoltion.

Ans :

According to Rayleigh's criterion, the two point sources or two spectral lines of equal intensity are just resolved by an optical instrument when the central maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other.

To illustrate the above criterion, consider the intensity distribution curves of two wavelengths λ and $\lambda + d\lambda$. The separation between their central maximum will depend upon the value of $d\lambda$. If $d\lambda$ is sufficiently large, the central maxima due to two wavelength are quite separate and the two spectral lines appear well resolved.

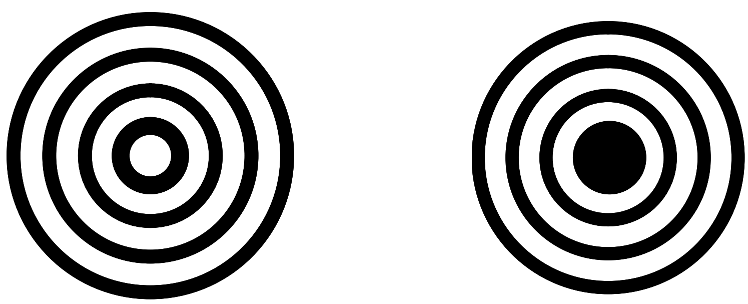
However $d\lambda$ will have a limiting value under this condition, according to Rayleigh, the two spectral lines are just resolved.

2. Describe the construction of Zone plates.

Ans :

The interesting application of the concept of Fresnel half period zones lies in the construction of the zone-plate.

A zone plate consists of a large number of concentric circles whose radii are proportional to the square root of natural numbers and the alternate annuals regions of which are blackened as shown in diagram (a). It can be designed so as to cut off light due to the over numbered zones or that due to the odd numbered zones.



(a) (b)

- To construct a zone plate, concentric circles are drawn on white paper such that the radii are proportional to the square roots of the natural numbers.

- The odd numbered zones are covered with black ink and a reduced but accurate photograph is taken. In the negative of photograph, the odd zones are transparent and the even zones will out off light.
- If the even zones are transparent and odd zones are opaque, it is said to be a negative zone plate.
- Here in diagram, XY represents the edgewise section of a zone plate. S is a point source of light.
- Let P be the point on the screen at a distance b from "0" at which intensity is to be found.

3. Define diffraction.

Ans :

Diffraction refers to various phenomenon that occur when a wave encounters an obstacle or a slit. It is defined as the bending of light around the corners of an obstacle or aperture into the region of geometrical shadow of the obstacle.

4. What are Fresnel's Diffraction assumptions?

Ans :

1. The complete wavefront is divided into a large number of elements known as Fressel's strips or zones of small area such that each of these elements acts as a source of secondary wages.
2. The resultant effect at any point is the combination effect of all secondary waves reacting at that point.
3. The effect at any point due to a particular zone depends on the,
   (i) Distance of point from the zone.
   (ii) Inclination of the point with reference to zone under consideration.
   (iii) Area of the zone.

5. Explain fresnel's half period zones.

Ans :

Consider a plane wavefront ABCD perpendicular to the plane of the paper as shown diagram. Let "P" is an external point at a distance a perpendicular to ABCD. In order to find the resultant intensity at P due to the wavefront, by Fresnel's method, the wavefront is divided into a number of half period elements or zones called Fresnel's zones and then the effect of all the zones at point P is found with "P" as centre and radii equal to b + $\lambda/2$, b + $2\lambda/2$, ... etc. spheres are constructed. These spheres will cut out circular areas of radii OM_1, OM_2, OM_3, etc., on the wavefront.

- These circular zones are called half period zones or half period elements.
- The secondary wavelets from any two consecutive zones reach P with a path difference $\lambda/2$ or time difference half period.

- Thus the zones are called half period zones. A Fresnel half period zone with respect to an external point "P" is a thin annular zone of the primary wavefront in which the secondary waves from any two corresponding points of neighbouring zones differ in path by $\lambda/2$.
- The point "θ" is called the pole of the wavefront XY with respect to point P, OP is perpendicular to XY.

6. Compare a zone plate and a convex lens.

Ans :

S.No.	Zoneplate	Convex lens
1)	A zone plate simultaneously acts as a convex lens and a concave lens. In addition to a real image, a virtual image is also formed simultaneously.	1) A convex lens can form only a real image on the other side.
2)	The image is formed by the diffraction phenomenon.	2) The formation of image is due to refraction of light.
3)	The zone plate has got multiple foci as f, f/3, f/5, f/7 ... $\rightarrow \frac{r_n^2}{n\lambda}, \frac{r_n^2}{3n\lambda}, \frac{r_n^2}{5n\lambda}$ ... on either side of the plate. Hence, the intensity of the image formed will be much less	3) The convex lens has got only one focus on the right side. (w.r.t to object or) (incident light) as all the light is focussed at one point, the intensity of the image will be more.

7. Explain phase reversal zone plate.

Ans :

Rayleigh suggested that if the alternate zones of zone plate, instead of being blocked, a re coated with a thin film of some transparent material which introduces an additional optical path difference $\lambda/2$, then the secondary disturbances from all the zones both odd or even, would reach the image point in the same phase and so the amplitude from successive zones will help each other consequently the intensity of the image would be four - fold. Such zone plates are called phase reversal zone plates. A phase reversal zone plate can be prepared by the following method.

A chemically cleaned thin glass plate is coated with a thin layer of gelatine solution and dried. It is then immersed in a weak solution of potassium dichromate for a few seconds and dried again in dark. It is then placed in contact with an ordinary zone plate and exposed to sun light for several minutes. Light passing through transparent zones acts on gelatine and makes it insoluble in water while the gelatine in contact with opaque zones remains soluble in water. Finally the glass plate is immersed in water where the gelatine of the unexposed parts is dissolved to such a depth that an additional optical path difference of half a wavelength $\left(\lambda/2\right)$ is introduced between the waves from successive zones. Thus the phase reversal zone plate is formed.

8. What are the difference between interference and diffraction?

Ans :

The difference between interference and diffraction is given below.

Interference	Diffraction
1. Interference is the phenomenon of interaction among two distinct wavefronts propagating from two coherent sources.	1. Diffraction is the phenomenon of interaction among secondary wavelets propagating from distinct points of same wavefront.
2. The width of interference fringes may or may not be constant.	2. The width of diffraction fringes always varies (i.e., as the order increases, width decreases).
3. The intensity of all the bright fringes is equal.	3. The intensity of each bright fringe varies from one another
4. The minimum intensity fringes are completely dark.	4. The minimum intensity fringes are partially dark.

9. If white light is used in young's double slit experiment, what will happen to the interference bands.

Ans :

When white light is used in young's double slit experiment, waves of each wavelength forms their separate interference pattern because white light consists of waves of innumerable wavelength starting from violet to red colour.

Choose the Correct Answers

1. Effect of diffraction is greatest if waves pass through a gap with width equal to [b]
   (a) Frequency (b) Wavelength
   (c) Amplitude (d) Wavefront
2. Grating element is equal to [a]
   (a) $\frac{n\lambda}{\sin\theta}$ (b) $n\lambda$
   (c) $\sin\theta$ (d) $\cos\theta$
3. In fraunhofer diffraction the incident wavefront is [a]
   (a) Plane (b) Circular
   (c) Cylindrical (d) Elliptical
4. Fresnel diffraction arises when the source of light is effectively at [b]
   (a) Infinite distance (b) Finite distance
   (c) Both (a) and (b) (d) Neither (a) and (b)
5. Fraunhofer diffraction arises when the source of light is effectively at [a]
   (a) Infinite distance (b) Finite distance
   (c) Both (a) and (b) (d) Neither (a) and (b)
6. In a single slit Fraunhofer diffraction if the slit width is increased the width of central maximum. [c]
   (a) Increases (b) Remains same
   (c) Decreases (d) Becomes zero
7. The intensity of the central maximum in double slit diffraction is times of single slit diffraction. [b]
   (a) Two times (b) Four times
   (c) Three times (d) Intensity does not change
8. When a white light is incident on plane diffraction grating the zero order maximum becomes. [b]
   (a) Coloured (b) Dark
   (c) White colour (d) Disappear
9. With the increase of number of lines per centimeter of a plane transmission grating the resolving. [a]
   (a) Increases (b) Remains same
   (c) Decreases (d) Becomes zero
10. When white light is incident on the zone plate and if f_v, f_R are the focal lengths of violet and red colours respectively then [c]
    (a) $f_V = f_R$ (b) $f_V < f_R$
    (c) $f_V > f_R$ (d) $f_V = \infty$, $f_R = 0$

Fill in the Blanks

1. The images of two objects close to each other are said to be resolved, if the ________ of one falls on the first minima if the other.
2. When source of light and screen are at infinite distance from the slit, the diffraction produced is called ________ diffraction.
3. When the source of light and screen are at finite distance from the slit, the diffraction produced is called ________ diffraction.
4. The ability of the optical instrument to produce distinct and separate images is called ________ of that instrument.
5. The ________ of the limit of resolution is known as “Resolving power” of the instrument.
6. Light waves are diffracted only when the size of obstacle or aperture is ________ to the wavelength of light.
7. If “?” is the intensity of light due to first half period zone, then the intensity at any point is ________.
8. In the positive zone plate ________ zones are transparent and in the negative zone plate ________ zones are transparent.
9. Zone plate forms ________ image of object.
10. In a zone plate the path difference and phase difference between any two alternative zones are respectively.

ANSWERS

1. Central maxima
2. Fraunhofer
3. Fresnel
4. Resolving power
5. Reciprocal
6. Comparable
7. $I/4$
8. Odd, even
9. Real
10. λ, 2Π

One Mark Answers

Q1. Define diffraction.

Ans :

It is defined as the bending of light around the carners of an obstacle or aperture into the region of geometrical shadow of the obstacle.

Q2. Define limit of resolution.

Ans :

The capacity of an optical system to resolve point objects as separate images.

Q3. Define resolving power of grating.

Ans :

It is defined as its ability to form separate diffraction maxima of two wavelengths which are very close to each other.

Q4. What is zone plate ?

Ans :

A some plate is a device used to focus light or other things exhibiting wave character

Q5. Write some examples for Fraunhoffer diffraction.

Ans :

i) Diffraction by a slit of infinite depth

ii) Diffraction by a rectangular aperture

iii) Diffraction by a circular aperture

iv) Diffraction by a double slit

v) Diffraction by a grating

UNIT - IV

Polarization

Polarized light : Methods of Polarization, Polarization by reflection, refraction, Double refraction, selective absorption, scattering of light - Brewster's law - Nicol prism polarizer and analyzer - Refraction of plane wave incident on negative and positive crystals (Huygen's explanation) - Quarter wave plate, Half wave plate - Babinet's compensator - Optical activity, analysis of light by Laurent's half shade polarimeter.

4.1 Polarized Light

4.1.1 Methods of Polarization

Q1. Define polarized light and polarization. What are the various methods of polarization?

Ans :

- A light wave that is vibrating in more than one plane is referred to as unpolarized light.
- The waves in which the vibrations occur in a single plane called polarized light waves.
- The process of transforming unpolarized light into polarized light is known as polarization.

Methods of Polarization

Following are the methods used for producing plane polarized light

1. Polarization by reflection
2. Polarization by refraction
3. Polarization by double refraction
4. Polarization by selective absorption
5. Polarization by scattering

4.1.2 Polarization By Reflection and Refraction Double refraction, selective absorption , scattering of light

Q2. Explain various methods to produce the plane polarized light.

Ans : **(June-19)**

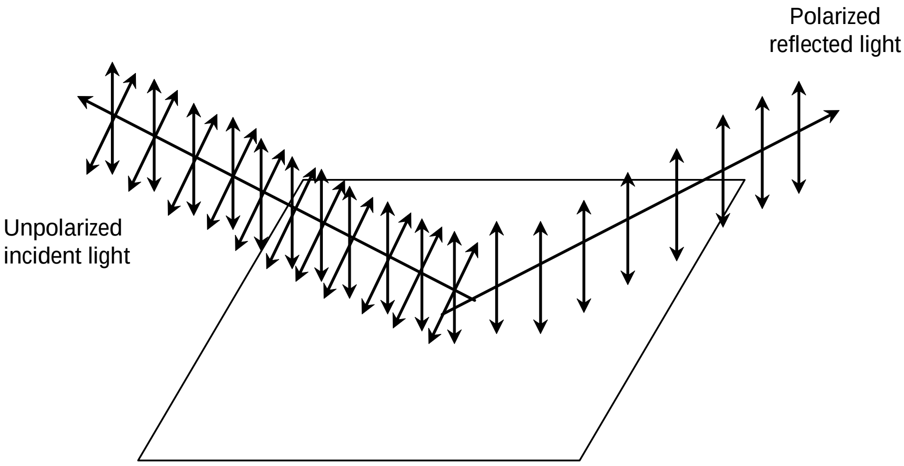
1. Polarization by Reflection

Polarization of Light by reflection from the surface of glass was discovered by Malus in 1808. He found that polarized light is obtained when an ordinary light reflected by a plane sheet of glass. The extent to which polarization.

Occurs depends on the angle at which polarization occurs depends on the angle at which the light approaches the plane surface and the material which the surface i made of.

- Metallic surfaces reflect light in different directions, such reflection light is unpolarized. However, non metallic surfaces like snow fields and water reflect light such that there is a large concentration of vibration in a plane parallel to the reflecting surface.
- A person viewing objects by means of light reflected from non-metallic surfaces will often perceive glare if the extent of polarization is large.

Consider the light incident on the glass surface, light is reflected with the same angle of incidence. In the path of reflected ray, place a tourmaline crystal and rotate is slowly. It will be observed that light is completely extinguished only at one particular angle of incidence. This angle of incidence is equal to 57.5° for a glass surface and is known as the polarizing anlage. The production of polarized lights by glass is explained as follows.

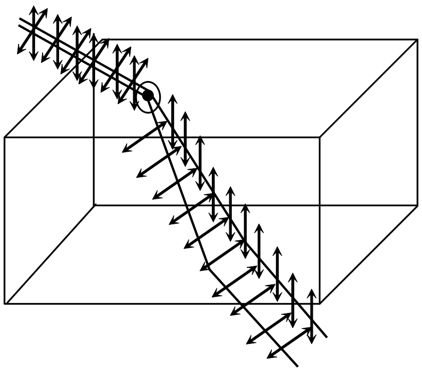


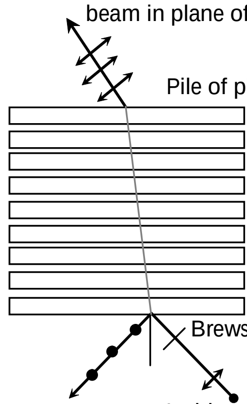
- The vibrations of the incident light can be resolved into two components, one parallel to the glass surface and the other perpendicular to the glass surface.
- Light due to the components parallel to the glass surface is reflected whereas light due to components perpendicular to the glass surface is transmitted. Thus, the light reflected by glass is plane polarized and can be detected by a tourmaline crystal.

2. Polarization by Refraction

1. Polarization can also occur by the refraction of light. Refraction occur when a beam of light passes from one material into another material.
2. At the surface of the two materials, the path of the beam changes its direction. The refracted beam acquires some degree of polarization. Most often, the polarization occurs in a plane perpendicular to the surface. The polarization of refracted light is often demonstrated using a unique crystal Iceland spar rare from of the mineral calcite, refracts incident light into two different path as soon as the light enters the crystal.
3. Subsequently, if an object is viewed looking through an Ice land spar crystal, two images will be seen. The two images are the result of the double refraction of light.
4. Both refracted light beams are polarized one in a direction parallel to the surface and the other in a direction perpendicular to the surface and the other in a direction perpendicular to the surface.
5. Since these two refracted rays are polarized with a perpendicular orientation. a polarizing filter can be placed in the path to completely block one of the images.
6. If the polarization axis of the filter is aligned perpendicular to the plane of polarized light is completely blocked by the filter, mean while the second image is as bright as it can be and if the filter is then turned 90° in either direction, the second image reappears and the first image disappears. This confirms the wave nature of the light.

To get the maximum polarization by refraction we use the following method.





- For a light incident on glass (μ = 1.5) at the polarization angle, 100% of the light vibrating parallel to the plane of incidence is transmitted whereas for the perpendicular to the plane of incidence are reflected by the first plate and the rest are transmitted.
- The process continues and when the beam has traversed about 15 or 20 plates, the transmitted light is completely tree from the vibrations at right angles to the plane of incidence and is having vibrations only in the plane of incidence.
- Thus we get plane polarized light by refraction with the help of a pile of plates, the vibrations being in the plane of incidence are shown in diagram.
- The pile of plate consists of number of glass plates and are supported in a tube of suitable size and are inclined at an angle of 32.5° to the axis of the tube. A beam of monochromatic light is allowed to fall on the pile of plates at the polarizing angle. The transmitted light is polarized perpendicular to the plane of incidence and can be examined by a similar pile of plates works as an cylinder.

  If light is polarized perpendicular to the plane of incidence, it means vibrations are in the plane of incidence. If light is polarized in the plane of incidence it means vibrations are perpendicular to the plane of incidence.

3. Polarization by Double Refraction

The process of producing two refracted rays by a Crystal is known as double refraction or birefringent. The rays obtained through this method are linearly polarized in perpendicular directions.

When an ordinary unpolarized light is allowed pass through calcite crystal, it splits into two refracted beams instead of one as in glass. This double bending of beam is called double refraction or birefringence. When these two emerging beams are analyzed with polarized sheet it is found that beams are plane or linearly polarized with their planes of vibration perpendicular to each other. Both refracted beams travels with different velocities. The refracted ray which satisfies ordinary laws of refraction, travels with same velocity in all directions and have oscillations in perpendicular direction to the principle section is called ordinary ray (O-ray). The refracted ray which does not satisfies ordinary laws of refractions, travels with different velocities in all directions and have oscillations in principal section is called extraordinary ray (E-ray). Crystals exhibiting this property are called doubly refracting crystals.

Based on the number of directions through which the refracted rays propagate in the case of crystal, doubly refracting crystals are classified into two types, They are

(i) Uniaxial Crystals

The crystals in which the two refracted rays will propagate along a single direction (optic axis) with equal velocity are known as uniaxial crystals.

Example: Calcite, tourmaline and quartz.

(ii) Biaxial Crystals

The crystals in which the two refracted rays will propagate along two directions with equal velocities are known as biaxial crystals.

Example: Topaz, aragonite etc.

Consider a beam PQ of unpolarized light passed through calcite crystal. Consider that an angle of incidence of beam be V as shown in figure.

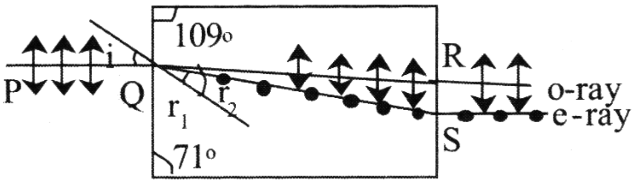


Fig. (c): Double Refraction in Calcite Crystal

The beam inside the crystal is doubly refracted into O-ray and e-ray. O-ray and E-ray along QR and QS makes angle of refractions r_1 and r_2 respectively. As the two faces of crystal are always parallel both O-ray and E-ray emerge parallel to incident ray. Then, the expressions for refractive indices of O-ray and E-ray are,

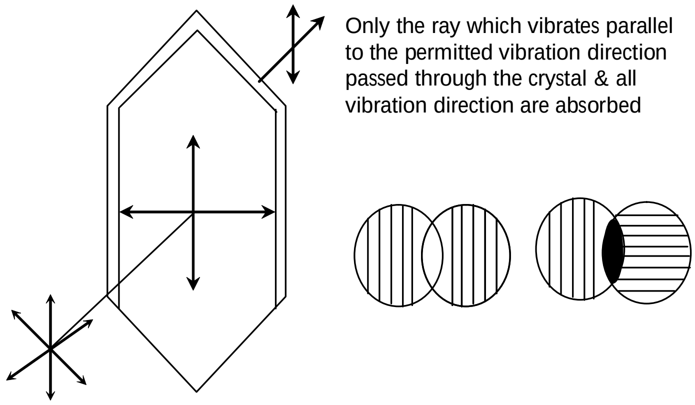
$$\mu_o = \frac{\sin i}{\sin r_1}, \mu_e = \frac{\sin i}{\sin r_2}$$

For calcite crystal $\mu_o > \mu_e$ since $r_1 < r_2$.

Thus, the light velocity for o-ray will be lesser than the light velocity for e-ray within the crystal. However, the velocity of e-ray will be vary from direction to direction unlike o-ray due to the variation in 'μ_e' value for different incident angles.

4. Polarization by selective absorption

When ordinary light enters a crystal of tourmaline, double refraction takes place in calcite, but with a difference. The O-ray is entirely absorbed in the crystal while the E-ray passes through as shown in diagram. This phenomenon is called selective absorption because the crystal absorbs light waves vibrating in one plane and not those vibrating in the other. Tourmaline crystals are therefore like Nicol prisms as they absorb unpolarized light and transmit only the plane polarized light. When two such crystals are lined un parallel, the plane polarized light from the first crystal passes through the second with less intensity. If the crystals are turned 90° with respect to each other, i.e., in the crossed position the light is completely absorbed no light passes through.



A tourmaline crystals are not used in optical instruments because they are yellow in colour and do not transmit white light. A more satisfactory substance for this purpose is newly manufactured material called polarized. This material is made in the form of thin films. They are made form small needle shaped crystals of an organic compound such as iodo sulphate of quinine. All these crystals acts like tourmaline by absorbing one component of polarized light and transmitting the other. In crossed position no light passes through two films. Whereas in parallel position light vibrating in plane indicated by parallel lines is transmitted.

5. Scattering of Light

1. When a light wave travelling in space strikes an extremely small particle (compared to the wavelength of light) such as dust particle, water particle or molecules of a substance, a portion of the light is scattered by the particle.
2. When the light passes through a number of particles, its intensity goes on decreasing due to scattering.

   According Lord Rayleigh, the intensity of scattered light is,

   (i) Proportional to the intensity of incident light,

   (ii) Proportional to the square of the volume of scattering particles and

   (iii) Inversely proportional to the fourth power of the wavelength of light used, i.e.,

   $$I \; \alpha \; \left(\frac{1}{\lambda^4}\right)$$

It has been observed that the scattered light is polarized fully or partially depending on the size of scattering particle, when the size of the particles are sufficiently small, the scattered light is fully polarized while when the particles are larger, the scattered light is partially polarized.

Q3. Explain different types of polarization.

(OR)

Define plane, circular and elliptical polarized light.

Ans : **(June-19, June-18)**

Generally, a polarized light specifies the shape and locus of the tip of the electric field (E) vector at a particular point in space as a function of time. Based on the locus of the tip of the electric field (E) vector, light exhibit three types of polarization. They are,

1. Plane or linear polarization
2. Circular polarization
3. Elliptical polarization

1. Plane or linear polarization

The polarization in which the occurrence of oscillations will be restricted to a single plane perpendicular to the direction of propagation is referred to as plane polarization. The direction of field vector are along a plane perpendicular to the direction of propagation at each point in space and time. Hence, this type of polarization is also referred as linear polarization.

In plane polarization , the location of E-vector is fixed at a point in space (i.e., direction of 'E' is constant with respect to time and magnitude of 'E' is continuously varies with time). The plane polarized light wave, which is polarized in an arbitrary direction is shown in figure (1).

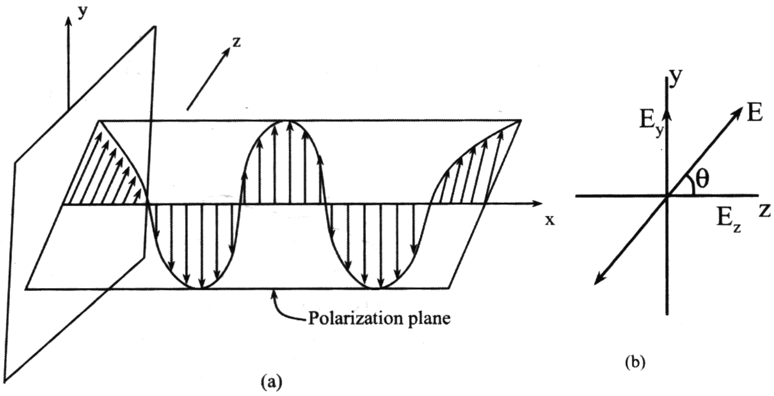


Fig. (1)

Depending on the direction of field, the plane polarization is classified into two types. They are,

(i) Horizontal Plane Polarization

The plane polarization in which the variation of electric field vector will be in horizontal direction i.e., perpendicular to the plane of paper is referred as horizontal plane polarization.

(ii) Vertical Plane Polarization

The plane polarization in which the variation of electric field vector will be in vertical direction with reference to the plane of paper is referred to as vertical plane polarization.

The diagrammatic representation of horizontal and vertical plane polarization is shown in figure (2) and figure (3) respectively.

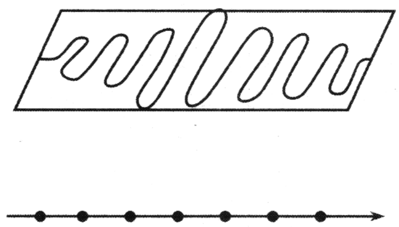


Fig. (2)

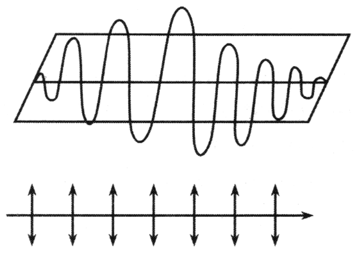


Fig. (3)

When a light wave plane polarized at any specific arbitrary angle, which results in an integration of horizontally and vertically polarized light with necessary amplitude and phase (i.e., in phase or 180° out of phase) as shown in figure (4).

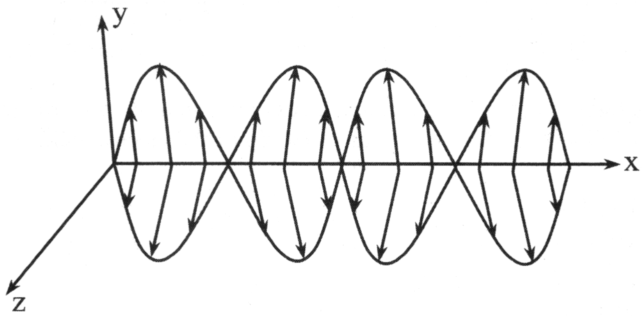


Fig. (4)

2. Circular Polarization

The polarization in which the amplitude of electric field vector *'E'* remains fixed and revolves at a constant rate around the direction of propagation and also forms a circular helix in space is referred to as circular polarization, which is shown in figure (5)

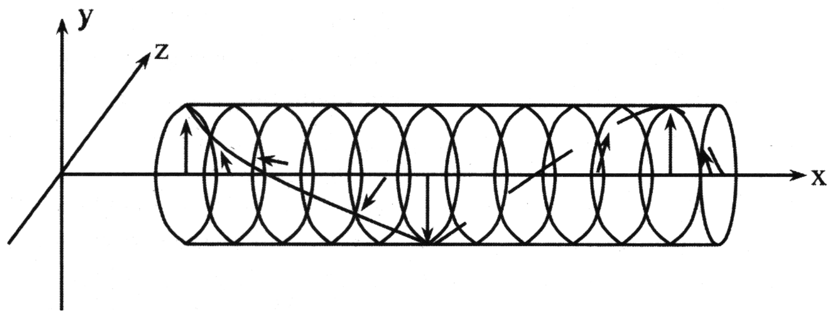


Fig. (5)

In circular polarization, the tip of electric field vector 'E' constitute a circular path in space and finishes one revolution within one wavelength. The direction of oscillation for circular polarization is distributed in all the planes. The circularly polarized light can be also referred as the wave resulted by the superposition of two coherent plane polarized waves of same amplitude and a phase difference of 90° vibrate in mutually perpendicular planes.

A circularly polarized wave, formed by combining horizontally and vertically polarized waves with equal magnitudes and a phase difference of 90° is shown in figure (6).

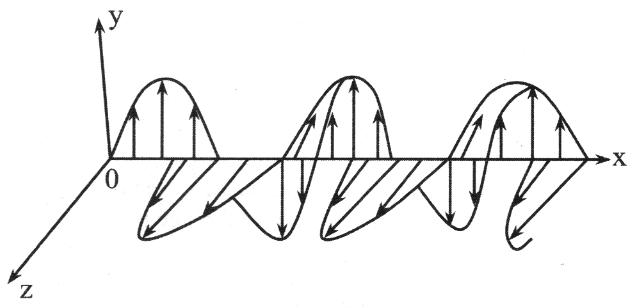


Fig. (6)

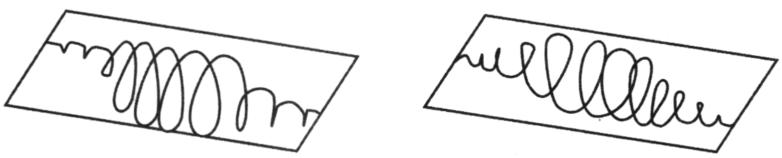
The observations made from figure (6) are,

(i) The instant at which .E-field vector of y-polarized wave will be at maximum amplitude and the E-field vector of Z-polarized wave will be at zero, the polarization is said to be vertical.

(ii) The instant at which E-field vector of y-polarized wave will be at zero and the E-field vector of Z-polarized wave will be at maximum amplitude, the polarization is said to be horizontal.

(iii) In the same way, the instant at which E-field vector of γ-polarized and Z-polarized waves will be at some equal amplitude, the polarization is said to be circular.

The wave is said to be left circularly polarized, if the tip of electric field vector 'E' is rotated in anti-clockwise direction. Similarly, the wave is said to be right circularly polarized, if the tip of electric field vector 'E' is rotated in clockwise direction as shown in figure (7).



(a) Right Circular **(b) Left Circular**

Figure (7)

3. Elliptical Polarization

The polarization in which the amplitude of electric field vector 'E' varies time and revolves around the direction of propagation and also forms a flattened helix in space is referred to as elliptical polarization, which is shown in figure (8).

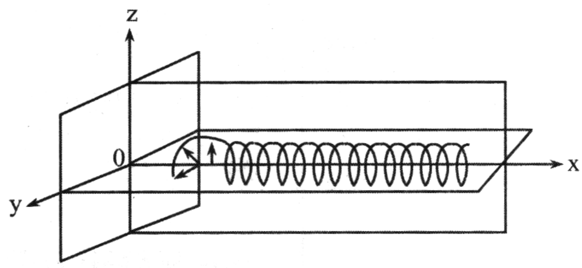


Fig. (8)

In elliptical polarization, the tip of electric field vector 'E' constitute a elliptical path in space. However, the elliptically polarized light can be also referred as the wave resulted by the superposition of two coherent plane polarized waves of distinct amplitudes and a phase difference of other than 90^o vibrate in mutually perpendicular planes.

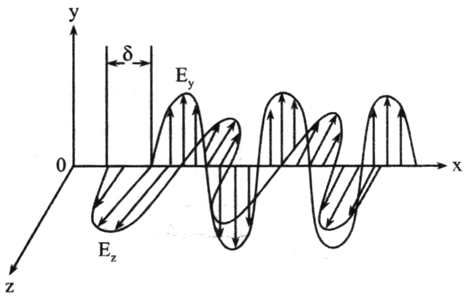


Fig. (9)

Figure (9) shows an elliptically polarized wave, which is formed by combining horizontally and vertically polarized waves with distinct magnitudes and a phase difference by an arbitrary angle 'δ'. Here the electric field vector 'E' of Z-polarized wave will be at maximum at the same time the electric field vector 'E' of y-polarized wave will be minimum and vice versa,

4.2 Brewster's Law

Q4. State Brewster's Law.

(OR)

Write a short note on Brewster's Law on polarization.

Ans :

Brewster's Law

Brewster performed a number of experiments to study the polarization of light by reflection at the surfaces of different media. He found that ordinary light is completely plane polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the angle of polarization.

- He was able to prove that the tangent of the angle of polarization is numerically equal to the refractive index of the medium.
- Moreover, the reflected and the refracted rays are perpendicular to each other.

Suppose a beam of unpolarized light AB is incidental and angle equal to the polarizing angle on the glass surface at B. It is reflected along BC and refracted along BD.

From snell's law

$$\mu = \frac{\sin i}{\sin r}$$

From Brewster's law

$$\mu = \tan i = \frac{\sin i}{\cos i}$$

From both laws. We get,

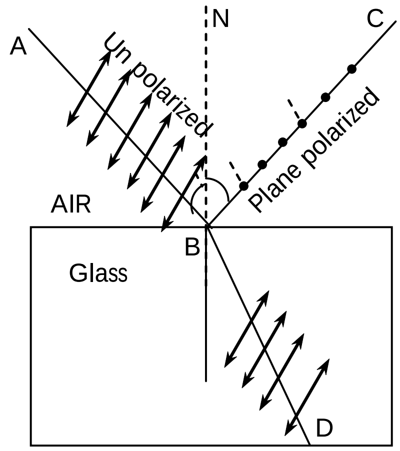
$$\cos i = \sin r = \cos (\pi / 2 - r)$$

$$\therefore \quad i = (\pi / 2 - r) \text{ or } i + r = \pi/2$$

As $i + r = \pi/2$, $<$CBD is also equal to $\pi/2$. Therefor, the reflected and the refracted rays are at right angles to each other.

From Brewster's law, it is clear that for crown glass of refractive index 1, 52, the value of 'i' is given by

$$i = \tan^{-1} (1.52) \text{ or } i = 56.7°$$



- Brewster proved that the tangent of the angle of polarization (p) is numerically equal to the refractive index (u) of the medium.

  i.e., $\boxed{\mu = \tan p}$

- This is known as Brewster's law. He also prove that the reflected and refracted rays are perpendicular to each other.
- One of the important applications of Brewster's law and Brewster's angle is in the design of a glass window that enables 100% transmission of light. Such a type of window is used in lasers is known as Brewster window.

4.3 Malus Law

Q5. State and explain Malus Law.

Ans : **(May-18)**

When a beam of light, polarized by reflection at one plane surface is allowed to fall on the second plane surface at the polarizing angle the intensity of the twice reflected beam varies with the angle between the planes of the two surfaces. The law of malus states that the "intensity of the polarized light transmitted through the analyzer varies as the square of the cosine of the angle between the plane of transmission of the analyzer and the plane of the polarizes'.

The proof of the law is based on the fact that any polarized vibration may be resolved into two rectangular components (i) parallel (ii) perpendicular to the plane of transmission of the analyzer.

Let OP = a be the amplitude of the vibrations transmitted or reflected by the polarizer and θ is the angle between the planes of the polarized and the analyzer.

Reserve OP into two components,

(i) a cos θ, along OA and

(ii) a sin θ, along OB

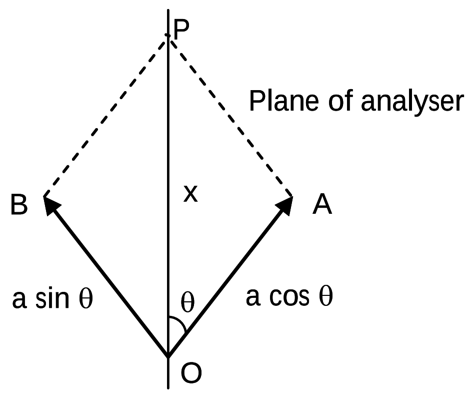
Only the a cos θ component is transmitted through the analyzer. Therefore, intensity of the transmitted light through the analyzer.

$$I_1 = (a \cos \theta)^2 = a^2 \cos^2 \theta$$

But $I = a^2$

Where I is the intensity of incident polarized light

Therefore, $I_1 = I \cos^2 \theta$, and $I_1 \alpha a^2 \cos^2 \theta$



When $\theta = 0$, i.e., the two planes are parallel

$I_1 = I$ because $\cos 0 = 1$

When $\theta = \pi/2.$, i.e., the two planes are at right angles to each other.

Therefore, $I_1 = I (\cos \pi/2)^2 = 0$.

This law fails when the incident light is unpolarized. Since in unpolarized light the electric field vector vibrates in all possible directions in a plane perpendicular to the direction of propagation. Hence we, have to take average value of $\cos^2 \theta$, over all possible values of θ. Thus the intensity of the beam transmitted is given by

$$I_1 = I < \cos^2 \theta > = 1/2$$

Hence, an ideal polarizer is one that transmits 50% of the incident unpolarized light as plane polarized one.

4.4 Nicol Prism Polarizer and Analyzer

Q6. Explain about nicol prism as a polarizer and analyzer.

(OR)

Describe the construction of Nicol Prism and show how it can be used as a polarizer and as an analyzer.

Ans : **(May-18, June-18)**

Nicol Prism

Nicol prism is a convenient optical device made from calcite crystal for producing and analyzing the plane polarized light. It was invented by william Nicol in 1826.

Principle

It is based upon the principle of double refraction. When an unpolarized light is passed through an uniaxial crystal it splits up into plane polarized ordinary and extraordinary rays. If by some optical means, one of the two rays is eliminated. The ray emerging through the crystal will be plane polarizes. In Nicol prism, ordinary ray is eliminated by the principle of total internal reflection so that the extraordinary ray is transmitted out as plane polarized light.

Construction

A calcite crystal whose length is three times in breadth and angels in the principal section should be 68° & 112°. The crystal is then cut into two pieces along the plane joining the two blunt corners and perpendicular to the principal section. The cut surfaces are ground and polished optically flat and then cemented together by Canada balsam. Canada balsam is a transparent liquid and its refractive index lies in between refractive indices of ordinary and extraordinary rays. Due to this property, it is optically denser for E-rays and rarer for o-rays. From sodium light (5893 A°)

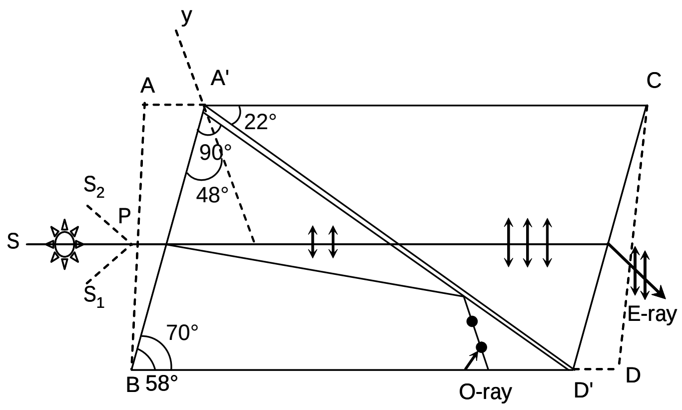
Refractive index of calcite for a O-ray, $\mu_0 = 1.66$

Refractive index of canada barsam, $\mu = 1.55$

Refractive index of calcite for E-ray, $\mu_e = 1.49$

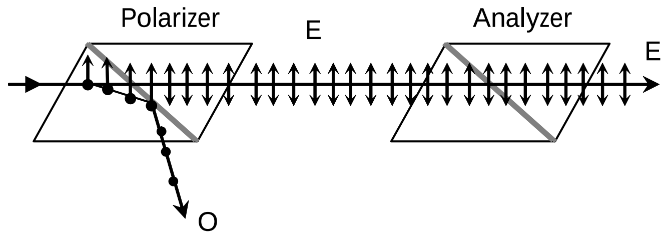
Thus canada balsam is optically denser than calcite of E ray and rarer for O-ray. Finally the crystal is enclosed in a tube painted black inside.

The Nicol prism in shown in diagram

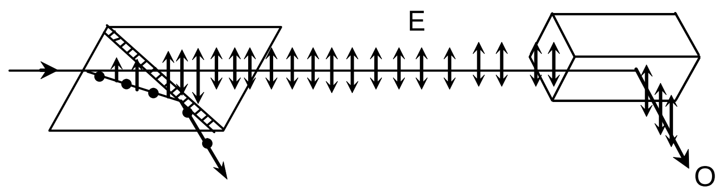


(a) Nicol Prism as a Polarizer

When an unpolarized light SP enters the face AB, it splits into ordinary and extraordinary rays. When the ordinary ray reacher the canada balsam layer, it suffers total internal reflection because the canada balsam acts as a rarer medium for O-rays coming from a denser medium while reaching at canada balsam layer and therefore E-ray is transmitted through the nicol prism. This E-ray is plane polarized and has vibrations in the principal section parallel to the shorter diagonal of the end face of the crystal. Thus, the Nicol prism can be used as a polarizer as known diagram.



(a)



(b)

(b) Nicol Prism as an Analyzer

When two Nicol prisms are arranged coaxially then the first nicol which produces plane polarized E-ray is called polarizer. The beam emerging from the polarizer falls on the second Nicol prism, called the analyzer. When the principal section of the second Nicol prism is parallel to that of the first Nicol Prism, then the E-ray transmitted by first prism is easily transmitted through the second prism. Now, when the second Nicol Prism is rotated gradually then the intensity of transmitted f-ray from second prism decreases and finally becomes zero when the principal section of the second Nicol prism becomes perpendicular to that of the first Nicol prism. In this position E-ray has its vibrations in the principal section of polarizer and hence normal to the principal section of analyzer. Therefor E-ray emerging from polarizer enters the analyzer it has no vibrations in the principal section of analyzer and therefore it acts as ordinary ray (o-ray) inside the analyzer and is totally reflected at the canada-balsam surface and therefore no light is transmitted through the analyzer. Thus, the first Nicol prism acts as a polarizer and the second Nicol prism acts as an analyzer in diagram (b).

Limitations of Nicol Prism as Polarizing Device

If the incident ray makes an angle much smatter than BPS with the surface A'B the ordinary ray will strike the balsam layer at an angle less than critical angle and hence will be transmitted.

If the incident ray makes an angle grater than BPS the E-ray will become more and more parallel to the optic axis A'y and hence its refractive index will become nearly equal to that of calcite for the o-ray. This will then also suffer total internal reflection like the o-ray. Hence no light will emerge out of the Nicol's prism. A Nicol's prism, therefore cannot be used for highly convergent or divergent beams. The angle between extreme rays of the incoming beam is limited to about 28°.

4.5 Refraction of Plane Wave Incident on Negative and Positive Crystals

Q7. Explain refraction of plane incident on negative and positive crystals.

(OR)

Explain Huygen's explanation of double refraction.

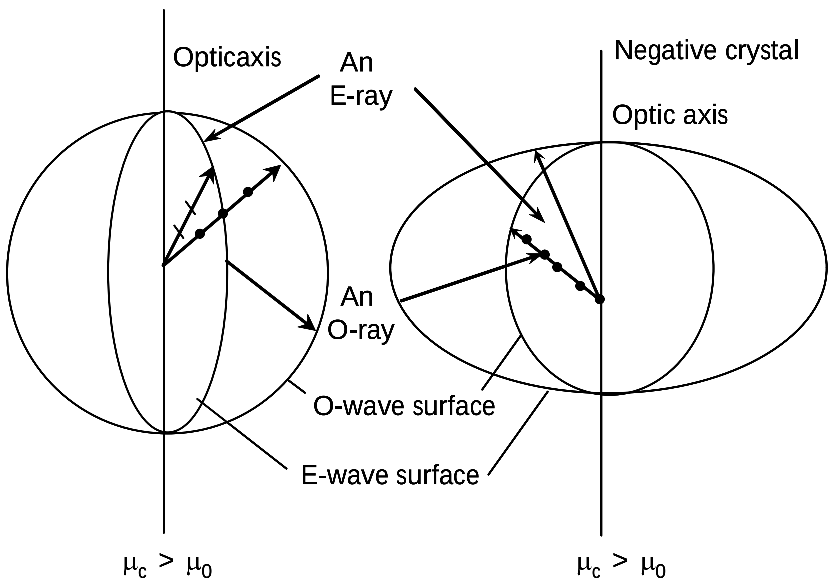
Ans :

Huygens explained the phenomena of double refraction in a uniaxial crystal with the help of the principle of secondary wavelets. In a doubly refracting crystal, a point source of light is the origin of two wave fronts. The wave front is spherical for O-ray because the velocity of light is the same in all directions for it. For the extraordinary ray, the velocity varies with the direction and hence the wave front in an ellipsoid of revolution. The velocities of O-ray end t-ray are the same along the optic axis. The wavefronts are shown in diagram (a) & (b).

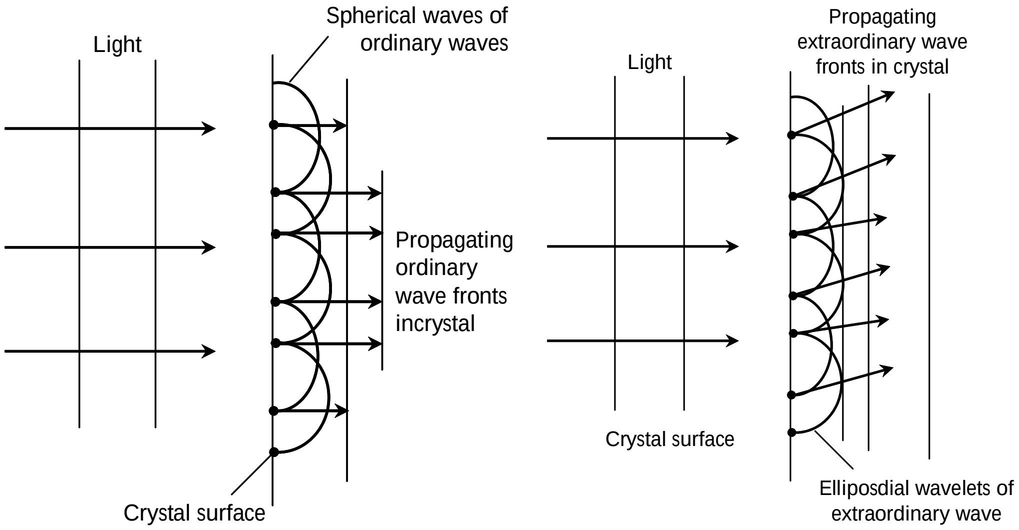
For negative crystals like calcite, the special wavefront due to ordinary ray lies within the ellipsoidal wavefront formed due to extraordinary ray. For a positive crystal like quartz, the extraordinary wavefront lies within the ordinary wave front.

1. For the positive uniaxial crystal (i.e., quartz), $\mu_e > \mu_0$. The velocity of the E-ray is least in the direction at right angles to the optic axis. It is maximum and equal to the velocity of the ordinary ray along the optic axis shown in (a).

2. For the negative crystal (i.e., calcite) $\mu_0 > \mu_e$. The velocity of the E-ray is equal to the velocity of the o-ray along the optic axis but it is maximum at right angles to the directions of the optic axis shown in (b).



(a) **(b)**



(c)

Huygen's principle explained the propagation of two waves in the following way. The plane wavefront arriving normally at the crystal surface generate spherical wavelets, travelling with equal speed V_0 in all direction. The tangent to these waves lies straight a head and by successive application of principle, the plane wave propagates straight a head with speed V_o. The only difference between this and normal refraction is that the travelling wave is completely polarized.

The extraordinary wavelets are more subtle. The wavelets spread out in ellipsoidal shape. The common tangent to these ellipsoids after a little time is the new wavefront. The line from the point of generation of each ellipsoid is off at an angle, represented by the dotted arrows in the diagram. These arrows define the direction of travel of the extraordinary wavefronts. What is odd is that the wavefronts are NOT perpendicular to their direction of travel. Inside the birefringent crystal, the extraordinary wave fronts are not transverse. The diagram shows the propagation of the waves.

4.6 Quarter Wave Plate and Half Wave Plate

Q8. What is a wave plate? Explain quarter wave plate and half wave plate.

Ans : **(June-19)**

An optical device which is capable of altering the polarization state of a light wave travelling through it is known as waveplate.

There are two types of waveplates

1. Quarter Wave Plate
2. Half wave plate

1. Quarter Wave Plate

Wave plates, also called retarders or converter of polarization forms with the help of appropriate retarder a given form of polarization can be converted into any other form.

It is made up of a doubly refracting crystal like a calcite or quartz. The refreshing faces of the crystal are cut parallel to the direction of optic axis and its thickness is such that it introduces a phase difference of $\pi/2$ or a path difference of $\lambda/4$ between the emerging ordinary and extraordinary rays.

If μ_0 & μ_e are refractive indices for the O & E-rays respectively, then for normal incidence, the path difference between O & E-rays is

For negative crystals $= (\mu_0 - \mu_e)t$ $\quad (\because \mu_0 > \mu_e)$

For positive crystals $= (\mu_e - \mu_0)t$ $\quad (\because \mu_e > \mu_0)$

Where 't' is the thickness of the quarter wave plate. But the quarter wave plate introduces a path difference of $\lambda/4$.

Hence, for negative crystals;

$$(\mu_0 - \mu_e)\ t = \lambda/4$$

$$\text{(or)}\ \ t = \frac{\lambda}{4(\mu_0 - \mu_e)}$$

And for positive crystals,

$$(\mu_e - \mu_0)t = \lambda/4$$

$$\text{or} \quad t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

This plate is used to produce circularly and elliptically polarized light when introduced in the path of a plane polarized light. As the thickness depends upon the wavelength, it is useful only for the wavelength for which it is constructed. In combination with a Nicol Prism, it is used for analyzing all kinds of polarised lights.

2. Half Wave Plate

It is made up of doubly refracting crystals. Its refracting faces are cut parallel to optic axis is and its thinkness (t) is such that it introduces a phase difference of π or a path difference $\lambda/2$ between emerging O & E-rays.

For negative crystal, the path difference

$$= (\mu_0 - \mu_e)\ t$$

And for positive crystal, the path difference

$$= (\mu_0 - \mu_e)\ t$$

But the half - wave plate introduce a path difference of $\lambda/2$ when a plane polarized light is passed through it.

Hence for negative crystals

$$(\mu_0 - \mu_e)\ t = \lambda/2$$

$$\text{or} \quad t = \frac{\lambda}{2(\mu_0 - \mu_e)}$$

For positive crystals

$(\mu_e - \mu_0)\ t = \lambda/2$

or $t = \dfrac{\lambda}{2(\mu_e - \mu_0)}$

The quarter and half wave plates are called retardation plates are called retardation plates because they retard motion of the rays.

Q9. Compare and contrast quarter wave plate and Half wave plate.

(OR)

How do you distinguish between a quarter wave plate and Half wave plate?

Ans : **(Jan.-21)**

Quarter Wave Plate	Half Wave Plate
1. When the thickness of a thin plate of birefringent crystal is such that it introduces a phase difference of 90° or path difference of $\lambda/4$ between e-ray and o-ray, then it is known as quarter wave plate.	1. When the thickness of a thin plate of birefringent crystal is such that it introduces a phase difference of 180° or path difference of $\lambda/4$ between e-ray and o-ray, then it is known as half wave plate.
2. Depending on the orientations of the quarter wave plate with respect to the plane of vibrations, the light emerging from the plate may be elliptical, circular or plane polarized light.	2. Depending on the orientations of the half wave plate with respect to the plane of vibrations, the light emerging from the plate is only plane polarized light,
3. The thickness (t) is expressed as, $t = \dfrac{\lambda}{4(\mu_o - \mu_e)}$ (For negative crystal) $t = \dfrac{\lambda}{4(\mu_e - \mu_o)}$ (For positive crystal)	3. The thickness (t) is expressed as, $t = \dfrac{\lambda}{2(\mu_o - \mu_e)}$ (For negative crystal) $t = \dfrac{\lambda}{2(\mu_e - \mu_o)}$ (For positive crystal)

4.7 Babinet's Compensator

Q10. Explain the construction and working of Babinet's compensator.

Ans : **(Jan.-21)**

We have studied that a quarter wave plate produces a fixed path difference between O & E-rays. This can be used for light for a particular wavelength for which it is constructed.

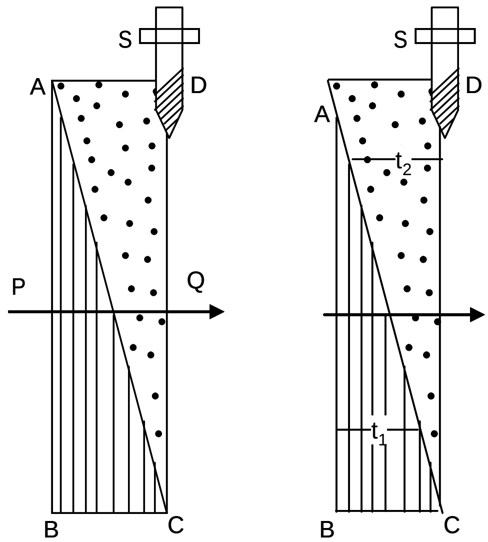
1. To overcome the limitation of a quarter wave plate, a Babinet's compensator is used where the thickness can be varied.
2. Babinet's compensator is an improvement over the quarter wave plate and is used to produce and analyze the elliptically polarized light of different wavelengths.

Construction

The babinet compensator consists of two small-angled wedge-shaped pieces of quarts placed with their hypotenuses in contact so as to form a small rectangular block ABCD. The optic axis of the left wedge is parallel to its refracting edge AB while that of right wedge is perpendicular to optic axis of the left wedge. Thus the optic axes of the two wedges are perpendicular to one another and also both are perpendicular to the incident beam. One of the edges is fixed while the other can be moved relative to fixed edge in its own plane by a micrometer screw's PQ is light entering the compensator.

Theory

Let a plane polarized light fall normally on face AB of the first wedge with the plane of vibration inclined at certain angle θ with the optic axis.



The light beam is broken upto into ordinary component and extraordinary component. As quartz is positive crystal, the ordinary component travels faster than extra ordinary components.

On reaching the second wedge, the ordinary component becomes extraordinary and extraordinary components becomes ordinary component because the optic axis is perpendicular to first wedge.

Thus the two components exchange their velocities in passing from one edge to other edge. Hence, the two wedges tend to cancel each other's effect.

Let μ_e & μ_0 be the refractive indices of quartz for extraordinary and ordinary rays and t_1 & t_2 be the thickness of the two wedges, respectively, traversed by a particular ray.

The path difference introduced between the two components by the first wedge is $(\mu_e - \mu_0)t_1$ by or the phase difference.

$$\Delta_1 = \frac{2\pi}{\lambda}(\mu_e - \mu_0)\, t_1$$

Further, the difference introduced between the two components by the second wedge is

$(\mu_0 - \mu_e)\, t_2$. i.e., $-(\mu_e - \mu_0)\, t_2$ or phase difference

$$\Delta_2 = \frac{2\pi}{\lambda}(\mu_0 - \mu_e)\, t_2$$

Thus the resultant phase difference & introduced by the compensator is given by,

$$\delta = \Delta_1 + \Delta_2$$

$$\delta = \frac{2\pi}{\lambda}(\mu_e - \mu_0)\, t_1 + \frac{2\pi}{\lambda}(\mu_0 - \mu_e)\, t_2$$

$$= \frac{2\pi}{\lambda}(\mu_e - \mu_0)\, t_1 - \frac{2\pi}{\lambda}(\mu_e - \mu_0)\, t_2$$

$$= \frac{2\pi}{\lambda}(\mu_e - \mu_0)\, (t_1 - t_2)$$

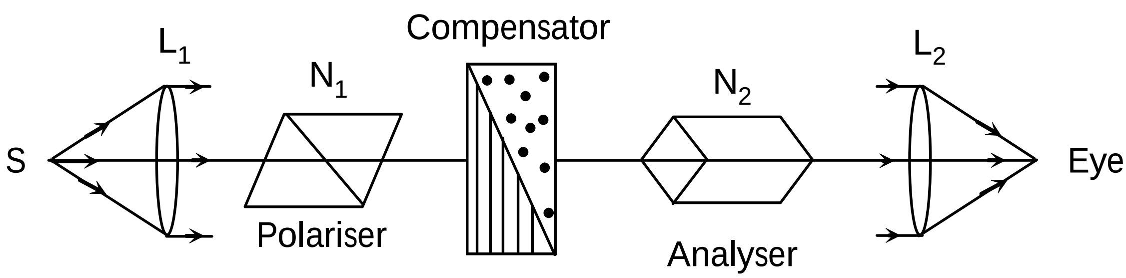
At the centre of the compensator, where $t_1 = t_2$, the resultant path difference and hence the phase difference is zero. In this case, the emergent light from compensator is plane polarized light in original plane.

Analysis of Elliptically Polarized Light

Before we use the Babinet's compensator to study the elliptically polarized light, the micrometer eye - piece of the compensator is first calibrated for the wavelength for which it is to be used. After calibration, it may be used to determine the characteristics of elliptically polarized light viz., (i) phase difference between its components, (ii) position of the axes and (iii) the ratio of the axes.

(i) Calibration

The experimental arrangement for the calibration of Babinet's compensator is shown in diagram.



The compensator 'C' is arranged between two crossed Nicol prisms N_1 & N_2. N_1 works as polarizer while N_2 works as an analyzer where's is the source and L_1 & L_2 are converging lenses. The polarizer is so oriented that the plane polarized light emerging from it and falling normally on the compensator. Makes an angle or with the optic axis of the compensator. Due to variation of the effective thickness along the compensator from the mid point it behaves as a wave plate. For those points of the compensator for which the path difference is 0, λ, 2λ, .. $n\lambda$, the emergent light from it is again plane polarized with vibrations in the same plane as that of transmitted by N_1. As Nicol prism N_2 is in a crossed position, these vibrations are stopped. Thus we get a set of dark bands in the field of view. At the intermediate positions where the path difference between the component vibrations is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, ..., the emergent light from the compensator is plane polarized with vibrations inclined at an angle of 2θ with incident plane of vibration. The Nicol prism N_2 will not extinguish this light. When $\theta = 45°$.

To calibrate the instrument, the movable wedge is displaced with the help of micrometer screw. Now the dark bands move laterally across the field of view. The movable wedge is adjusted in such a way that a dark band appears on the cross wire.

(ii) Phase Difference

Consider that elliptically polarized light is allowed to incident on Babinet's compensator. This light may be resolved into two components, one parallel to the optic axis and the other perpendicular to it. Let these components be.

$X = A \sin(\omega t + \alpha)$ & $y = B \sin(\omega t + \beta)$

The phase difference between two components is $(\alpha - \beta)$. Let the compensator introduce a phase change of δ. Then on passing through the compensator, the phase difference becomes $\alpha - \beta + \delta$, where $\delta = \frac{2\pi}{\lambda}(\mu_e - \mu_0)(t_1 - t_2)$, the total phase difference changes from point to point.

At those points where the total phase difference is an even multiple of π, we get a dark bond. On the other hand, at those points where the total phase difference is add multiple of π, we get a bright band.

The given elliptically polarized light is produced by introducing a $\lambda/4$ plate between polarizer and compensator using a monochromatic source of light is passed through the compensator and analyses.

Nicol prism N_2. Now the central dark band will be displaced in the field of view. The screw is adjusted until the central band is again comes under the cross wire, if the displacement of screw is 'x', then the phase difference between the two component of elliptically polarized light is given by

$$\phi = \frac{2\pi x}{\alpha}$$

where $\phi = \alpha - \beta$

(iii) Position of Axes

The position of the major and minor axes of the given elliptical vibration can be found as follows.

Using a white light source, plane polarized light is obtained by Nicol prisms N_1. The compensator is illuminated by this light and micrometers screw is adjusted to bring the central dark band under the cross wire.

The micrometer screw is turned through $\alpha/4$ so that the compensator introduces a path difference of $\pi/4$ or a space difference at cross wire. Now the plane polarized light is replaced by a given elliptically polarized light. The cross wire will not be on the dark band. Now the compensator is rotated in its own plane until the central dark is under the cross wire. Here the analyzer N_2 may also be rotated so that the central band is maximum black. The axes of the incident elliptically polarized light are now parallel to the optic ax of the wedges of the compensator. This determines the orientation of the axes of the elliptically polarised light.

(iv) Ratio of the Axes

In the above experiment if ϕ be the angle through which the compensator has to be rotated, then ratio of the axes is given by $\tan \phi = b/a$.

4.8 Optical Activity

Q11. What is Optical Activity?

Ans :

This property of rotating the plane of vibration of plane polarised light about its direction of travel by some crystal is known as optical activity. This phenomenon is known as optical rotation and the angle through which the plane of polarization is rotated is known as angle of rotation.

Some Observed Facts about Optical Rotation

Biot observed the following facts about the optical rotation.

1. There are two types of optically active substances. The substance which rotate the plane of polarization in clockwise (looking against the direction of light) are called levorotatory or left handed.

2. The amount of rotation (θ) produced by an optically active substance is proportional to its thickness (l) traversed i.e., $\theta \alpha l$.

3. In case of solution and vapours, the amount of rotation for a given path length is proportional to the concentration (C) of the solution or vapour.

   i.e., $\theta \alpha c$

4. The rotation varies inversely as the square of wavelength (λ) of light employed, i.e., $\theta \alpha l/\lambda^2$. Thus, it is least for red & greatest for violet.

5. The total rotation (θ) produced by a number of optically active substance is the algebraic sum of the rotations (θ_1, θ_2, θ_3, etc) produced by individual specimens, i.e.,

   $$\theta = \theta_1 + \theta_2 + \theta_3 + ...$$

   the rotation in the anti-clockwise direction being taken as positive and that in the clock wise direction as negative.

4.9 Analysis of Light by Laurent s Half Shade Polarimeter

Q12. Define polarimeter. Explain Laurent's half-shade polarimeter.

(OR)

Describe the construction and working mechanism of Laurent's half-shade polarimeter.

Ans : **(Jan.-21, June-18, May-18)**

Polarimeter is an instrument, which is used to measure the optical rotation produced by an optically active substance. It consists of two Nicol prisms capable of rotating about a common axis. It is also provided with a hollow tube for filling the solution of optically active substance.

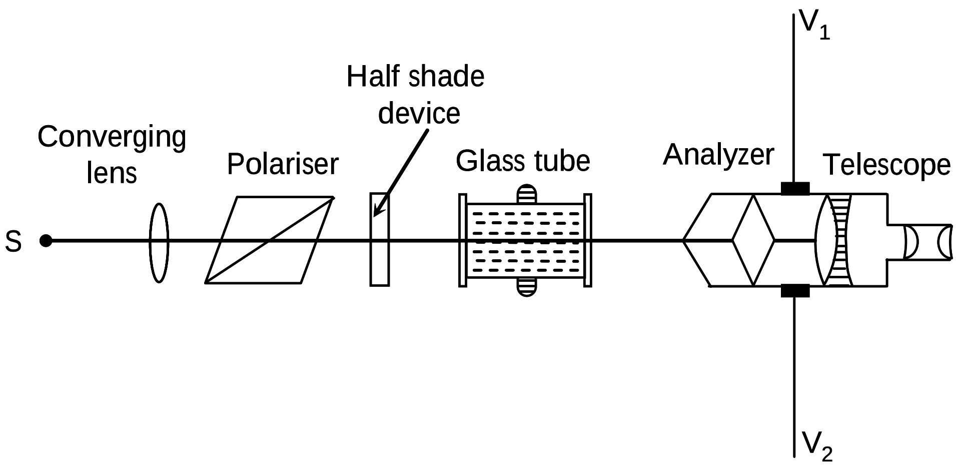
There are two types of polarimeters in use

1. Laurent's half shade polarimeter
2. Bi-quartz polarimeter.

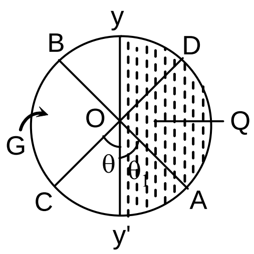
When these polarimeters are used to determine the concentration of sugar in a sugar solution, they are called saccharimeter.

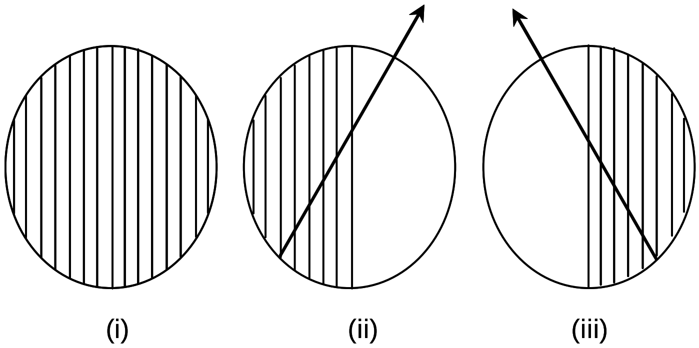
Laurent's half-shade polarimeter consists of two Nicol prisms N_1 & N_2. Prism N_1 is used as polarizer & prism N_2 is used as an analyzer. Behind prism N_1 a half the field of view, the other half is a simple glass plate. The amount of light absorbed by the other half is a simple glass plate. The amount of light absorbed by the glass plate is the same as that of the quartz plate. After the half - shade device a hollow glass tube which has a larger diameter it its middle is placed. This tube is filled by optically active solution and it is closed at its ends by over slips. There must be no air bubbles. The arrangement is shown in diagram (a).

When a beam of light through converging lens passes through the polarized nicol prism, the emerging light is plane polarized light. One half of the plane polarized light passes through the quartz plate and the other half passes through the glass plate.

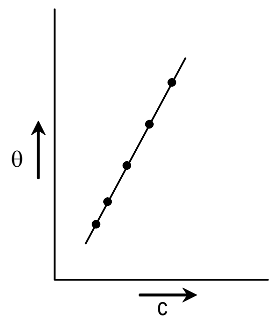


(a)





(b) (c)



(d)

Let the plane of vibration of plane - polarized light on the half shade plate be along AB.AB makes an angle θ with yy'. When a beam passes through the quartz plate, it splits up into two rays, ordinary and extraordinary rays, which travel with different velocities in the same direction. After emerging, they have a phase difference of π or a path difference of $\lambda/2$.

The vibrations of the beam emerging out of the glass are along AB and that of quartz plates are along CD.

If yy' is the direction of the principal section of analyzer, and it bisects the angle AOC, then the amplitude of light incident on both halves of the analyzer is equal. Hence, the field of view will be equally bright (i).

To find the specific rotation of an optically active substance, the analyzer is set in the position of equal brightness without any optically active solution filled in the tube. The readings of verniers are to be noted. When an optically active solution of known concentration is filled in the tube, the vibrations from the quartz half & the glass half are rotated. If the solution filled is sugar solution, then AB & DC are rotated in the clock wise direction. Therefore, the field of view is not equally bright.

Then the analyzer is rotated in clockwise direction and brought to the position of equal brightness. The readings of verniers are noted for this position.

If the analyzer is rotated to the right of yy', the right half is brighter than the left half as in (ii). If the analyzer is rotated to the left of yy', then the left half is brighter than the right half an in (iii). The angle through which the analyzer is rotated gives the angles through which the analyzer is rotated gives the angles through which the plane of vibrations of incident beam has been rotated by the optically active sugar solution.

In the actual experiment, for different concentrations of sugar solution the angles of rotation (θ, in degree) (c, gm/cc) are determined for each concentration on being plotted, a graph between c and the θ is found to be a straight line as shown in diagram.

For a particular value of θ & c the specific rotation (α) can be calculated as $a = \frac{10\theta}{l c}$.

Where 'l' is the length of the tube in cm, c is the concentration of the solution in gm/cc and θ is the rotation in degrees.

Problems

1. A ray of light is incident on the surface of a glass plate of refractive index 1.732 at the polarizing angle. Calculate the angle of refraction of the ray.

Sol :

According to Brewster's Law

$\mu = \tan i$

or $1.732 = \tan i$

or $i_p = \tan^{-1}(1.732) = 60°$

Since, the angle between i_p & r is 90°, when the ray is incident at polarization angle, then

$i_p + r - 90°$

or $r = 90 - i_p = 90° - 60° = 30°$

2. Calculate the thickness of a half wave plate of quartz for a wavelength of 5000 A°. Here μ_e = 1.553 & μ_0 = 1.544.

Sol :

For a half plate of positive crystal,

$$t = \frac{\lambda}{2(\mu_e - \mu_0)}$$

Given, $\gamma = 5000$ A° $= 5 \times 10^{-5}$ cm;

$\mu_e = 1.553$ & $\mu_0 = 1.544$

$$t = \frac{5\times10^{-5}}{2(1.553-1.544)} = \frac{5\times10^{-5}}{2\times0.009} = \frac{5\times10^{-5}}{1.8\times10^{-2}} = 2.78 \times 10^{-3} \text{ cm}$$

3. Find the thickness of a quarter wave plate when the wavelength of light is 5890 A°, μ_e = 1.553 & μ_0 = 1.544.

Sol :

The thickness of the quarter wave plate of positive crystal is given by,

$$t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

given $\lambda = 5890$ A° $= 5.890 \times 10^{-5}$ cm

$\mu_c = 1.553 \; ; \; \mu_0 = 1.544$

$$t = \frac{5.890 \times 10^{-5}}{4(1.533 - 15.44)}$$

$$= \frac{5.890 \times 10^{-5}}{4 \times 0.009} = \frac{5.890 \times 10^{-5}}{3.6 \times 10^{-2}}$$

$= 1.636 \times 10^{-3}$ cm

4. Calculate the thickness of a doubly refracting plate capable of producing a path difference $\lambda/4$ between extraordinary and ordinary waves (λ = 5890A°, μ_0 = 1.53; μ_e = 1.54).

Sol :

For producing a path difference $\lambda/4$, the conditions is

$$(\mu_e - \mu_0)\, t = \lambda/4$$

or $$t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

$$= \frac{5.890 \times 10^{-5}}{4(1.54 - 1.53)} = \frac{5.890 \times 10^{-5}}{4 \times 0.01}$$

$$= \frac{5.890 \times 10^{-5}}{4 \times 10^{-2}} = 1.47 \times 10^{-3} \text{ cm}$$

5. A half wave plate is constructed for a wavelength of 600A°. For what wavelength does it work as a quarter wave plate.

Sol :

For quarter wave plate, we know

$$t = \frac{\lambda_1}{4(\mu_e - \mu_0)} = t(\mu_e - \mu_0) = \lambda/4$$

For half-wave plate (t) = $\frac{\lambda_2}{2(\mu_e - \mu_0)}$; $(\mu_e - \mu_0)t = \frac{\lambda_2}{2}$

$$\therefore \quad \frac{\lambda_1}{4} = \frac{\lambda_2}{2} \text{ or } \lambda_1 = 2\lambda_2$$

The above wave plate (i.e., 2 × 600A° = 1200°A) acts as quarter wave plate for 1200A°.

6. A tube 20 cm long containing sugar solution rotates the plane of polarization through an angle of 13.5°. If the specific rotation is 66°. Find the amount of sugar present in a litre of the solution.

Sol :

Given that $\lambda = 20$ cm, $\theta = 13.5°$; $\alpha = 66°$

$\therefore$ Concentration $C = \dfrac{10\theta}{\alpha l}$

$$C = \frac{10 \times 13.5°}{20 \times 66} = 0.1 \text{ gm/cc}$$

Sugar present in 1 litre solution, m = 0.1 × 1000 = 100 gm

---

7. Two polarizing sheets have polarizing direction parallel to each other, so that the intensity of the transmitted light is a maximum. Through what angle must either sheet be turned so that the intensity becomes one half the initial value.

Sol :

From malus law, we have $I = I_0/2$

$\therefore$ $I = I_0 \cos^2\theta$ or $\cos^2\theta = 1/2$ (or) $\cos\theta = \pm(1/2)^{1/2}$

or $q = \cos^{-1}\left(\pm\dfrac{1}{\sqrt{2}}\right) = \pm 45°$

---

8. The refractive indices of glass and water are 1.54 & 1.33 respectively. Calculate the polarizing angle for a beam incident from (i) glass to water (ii) glass to air (iii) air to glass.

Sol :

i) $\mu = \dfrac{1.33}{1.54} = 0.866$

$\therefore$ $i_p = \tan^{-1}(0.666) = 40°52'$

ii) $\mu = \dfrac{1}{1.54} = 0.65$

$\therefore$ $i_p = \tan^{-1}(0.65) = 33°$

iii) $\mu = \dfrac{1.54}{1} = 1.54$

$\therefore$ $i_p = \tan^{-1}(1.54) = 57°$

9. The polarizing angle for air glass interference is 56.30°. What is the angle of refraction if light in incident from air on a glass slab at polarizing angle?

Sol :

We know $r + i_p = \pi/2$

Hence $r = \pi/2 - i_p = 90° - 56.3° = 33.7°$

$\therefore \quad r = 33.7°$

10. A half-wave plate is fabricated for a wavelength of 3000 A°. For what wavelength does it work as a quarter-wave plate?

Sol :

The thickness of a half wave plate is $d = \frac{\lambda l}{2(\mu_e - \mu_0)}$

The same plate is required to act as a quarter waveplate.

Therefore we can write $d = \frac{\lambda_2}{4(\mu_e - \mu_0)}$

$$d = \frac{\lambda_1}{2(\mu_e - \mu_0)} = \frac{\lambda_2}{4(\mu_e - \mu_0)}$$

$\lambda_2 = 2\lambda_1 = 2 \times 3800$ A° $= 7600$ A°

11. A tube 20 cm long containing sugar solution rotates the plane of polarization through an angle 13.5°. If the specific rotation is 66°. Find the amount of sugar present in litre of solution.

Sol : **(June-18)**

Given that

Specific rotation, $S = 66°$

Length of the tube, $l = 20$ cm $= 20 \times 10^{-1}$ dm $= 2$ dm

Rotation of plane of polarization, $\theta = 13.5°$

The expression for specific rotation is given by,

$$\delta = \frac{\theta}{l \times C}$$

$$\Rightarrow \quad C = \frac{\theta}{l \times \delta}$$

Substituting the corresponding values in above equation,

$$C = \frac{13.5}{66 \times 2} = 0.1 \text{ gm/c.c}$$

Unit concentration 1% = $\frac{1}{100}$

= 0.01 gm/cc

= 100 gm/litre

$\Rightarrow$ C = 1 × 10^2 gm/lit

Therefore, the amount of sugar present in a litre of solution is 1 × 10^2 gm/lit.

12. Find the thickness of a birefringent crystal, which introduction a phase difference of 60° between e and orays (μ_e = 1.553, μ_o = 1.544 and λ = 5400 Å).

Sol : **(June-19)**

Given that,

For a birefringent crystal,

Phase difference, $\delta = 60° = \frac{\pi}{3}$

Refractive index of *e*-ray, μ_e = 1.553

Refractive index of o-ray, μ_e = 1.544

Wavelength, λ = 5400 Å = 5400 x 10 ^{10} m

The expression for thickness of a crystal is given as,

$$\Delta = (\mu_o - \mu_e)\, d$$

$$\Rightarrow \quad d = \frac{\Delta}{(\mu_e - \mu_o)}$$

Here,

Δ – Path difference between E and O-rays

For crystal,

$$\Delta = \frac{\lambda}{2\pi}\,\delta$$

$$\Rightarrow \quad \Delta = \frac{\lambda}{2\pi}, \frac{\pi}{3} = \frac{\lambda}{6}$$

Substituting the corresponding values in equation (1),

$$d = \frac{5400 \times 10^{-10}}{6(1.553 - 1.544)} = 1 \times 10^{-5}$$

$\therefore$ d = 10 μm.

13. A ray of light is incident on the surface of a glass plate of refractive index 1.55 at the polarizing angle. Calculate the angle of refraction.

Sol : **(Jan.-21)**

Given that,

A ray of light is incident on the surface of a glass plate.

Refractive index of the glass plate, $\mu = 1.55$

The expression to obtain angle of refraction is given by,

$$r = \frac{\pi}{2} = i_P \quad . \quad .. (1)$$

But, $\mu = \tan i_P$

$\Rightarrow \quad i_p = \tan^1(\mu) \quad ... (2)$

Substituting equation (2) in (1).

$$r = \frac{\pi}{2} - \tan^{-1}(\mu) = \frac{\pi}{2} - \tan^{-1}(1.55) = \frac{\pi}{2} + 57.171^o$$

$$= 90° - 57.171^o$$

$$= 32.829^o$$

$\therefore$ Angle of refraction of the ray = 32.829^o.

14. Calculate the thickness of a quarter wave plate. Given μ_e = 1.533, μ_o = 1.544 and λ = 5000Å.

Sol : **(Jan.-21)**

Given

$\mu = 1.533$

$\mu^o = 1.544$

$\lambda = 5000°A$

$= 5000 \times 190^{-8}$ cm

The expression for thickness of quarter wave plate is given as,

$$\text{Thickness} = \frac{\lambda}{4(\mu_e - \mu_o)}$$

$$= \left| \frac{5000 \times 10^{-8}}{4(1.533 - 1.544)} \right|$$

$$= \frac{5000 \times 10^{-8}}{4(11 \times 10^{-3})}$$

$= 0.001136$ cm

$\therefore$ Thickness of quarter wave plate is 0.001136 cm

15. Calculate the specific rotatory power if the plane of polarization is turned through 26.4° transversing 20 cm length of 20% sugar solution.

Sol : **(Jan.-21)**

For a sugar solution

Angle of notation, $\theta = 26.4^o$

Length of plane, $l = 20$ cm

$= 20 \times 10^{-1}$ dm $= 2$ dm

Sugar solution, $\delta = 20\%$

$$= \frac{20}{100} = 0.2 \text{ gm/cc}$$

The expression for specific rotation is given by,

$$\delta = \frac{\theta}{l \times \delta}$$

$$\Rightarrow \quad S = \frac{26.4^o}{2 \times 0.2}$$

$$= 66^o$$

$\therefore \quad S = 66^o\ (dm^{-1})\ (f/cc)^{-1}$

Short Question and Answers

1. How do you distinguish between a quarter wave plate and Half wave plate?

Ans :

Quarter Wave Plate	Half Wave Plate
1. When the thickness of a thin plate of birefringent crystal is such that it introduces a phase difference of 90° or path difference of $\lambda/4$ between e-ray and o-ray, then it is known as quarter wave plate.	1. When the thickness of a thin plate of birefringent crystal is such that it introduces a phase difference of 180° or path difference of $\lambda/4$ between e-ray and o-ray, then it is known as half wave plate.
2. Depending on the orientations of the quarter wave plate with respect to the plane of vibrations, the light emerging from the plate may be elliptical, circular or plane polarized light.	2. Depending on the orientations of the half wave plate with respect to the plane of vibrations, the light emerging from the plate is only plane polarized light,
3. The thickness (t) is expressed as, $t = \frac{\lambda}{4(\mu_o - \mu_e)}$ (For negative crystal) $t = \frac{\lambda}{4(\mu_e - \mu_o)}$ (For positive crystal)	3. The thickness (t) is expressed as, $t = \frac{\lambda}{2(\mu_o - \mu_e)}$ (For negative crystal) $t = \frac{\lambda}{2(\mu_e - \mu_o)}$ (For positive crystal)

2. Define plane, circular and elliptical polarized light.

Ans :

Generally, a polarized light specifies the shape and locus of the tip of the electric field (E) vector at a particular point in space as a function of time. Based on the locus of the tip of the electric field (E) vector, light exhibit three types of polarization. They are,

1. Plane or linear polarization
2. Circular polarization
3. Elliptical polarization

1. Plane or linear polarization

The polarization in which the occurrence of oscillations will be restricted to a single plane perpendicular to the direction of propagation is referred to as plane polarization. The direction of field vector are along a plane perpendicular to the direction of propagation at each point in space and time. Hence, this type of polarization is also referred as linear polarization.

2. Circular Polarization

The polarization in which the amplitude of electric field vector *'E'* remains fixed and revolves at a constant rate around the direction of propagation and also forms a circular helix in space is referred to as circular polarization, which is shown in figure (5)

3. Elliptical Polarization

The polarization in which the amplitude of electric field vector 'E' varies time and revolves around the direction of propagation and also forms a flattened helix in space is referred to as elliptical polarization.

3. What are uniaxial and Biaxial crystals?

Ans :

(i) Uniaxial Crystals

The crystals in which the two refracted rays will propagate along a single direction (optic axis) with equal velocity are known as uniaxial crystals.

Example: Calcite, tourmaline and quartz.

(ii) Biaxial Crystals

The crystals in which the two refracted rays will propagate along two directions with equal velocities are known as biaxial crystals.

Example: Topaz, aragonite etc.

4. State and explain Malus Law.

Ans :

When a beam of light, polarized by reflection at one plane surface is allowed to fall on the second plane surface at the polarizing angle the intensity of the twice reflected beam varies with the angle between the planes of the two surfaces. The law of malus states that the "intensity of the polarized light transmitted through the analyzer varies as the square of the cosine of the angle between the plane of transmission of the analyzer and the plane of the polarizes'.

The proof of the law is based on the fact that any polarized vibration may be resolved into two rectangular components (i) parallel (ii) perpendicular to the plane of transmission of the analyzer.

Let OP = a be the amplitude of the vibrations transmitted or reflected by the polarizer and θ is the angle between the planes of the polarized and the analyzer.

Reserve OP into two components,

(i) a cos θ, along OA and

(ii) a sin θ, along OB

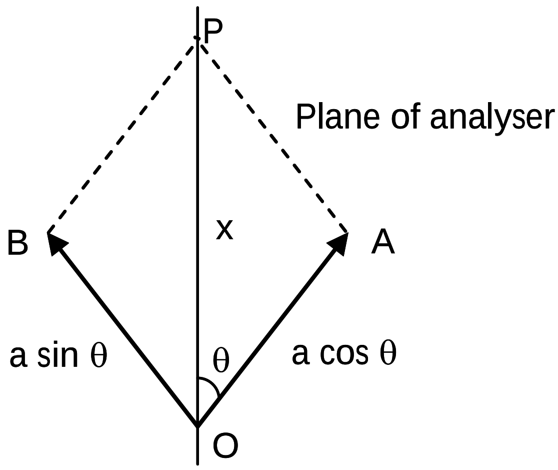
Only the a cos θ component is transmitted through the analyzer. Therefore, intensity of the transmitted light through the analyzer.

$I_1 = (a \cos\theta)^2 = a^2 \cos^2\theta$

But $I = a^2$

Where I is the intensity of incident polarized light

Therefore, $I_1 = I \cos^2\theta$, and $I_1 \propto a^2 \cos^2\theta$



When $\theta = 0$, i.e., the two planes are parallel

$I_1 = I$ because $\cos 0 = 1$

When $\theta = \pi/2$., i.e., the two planes are at right angles to each other.

Therefore, $I_1 = I (\cos \pi/2)^2 = 0$.

This law fails when the incident light is unpolarized. Since in unpolarized light the electric field vector vibrates in all possible directions in a plane perpendicular to the direction of propagation. Hence we, have to take average value of $\cos^2\theta$, over all possible values of θ. Thus the intensity of the beam transmitted is given by

$$I_1 = I < \cos^2\theta > = 1/2$$

Hence, an ideal polarizer is one that transmits 50% of the incident unpolarized light as plane polarized one.

5. Define polarized light and polarization.

Ans :

- A light wave that is vibrating in more than one plane is referred to as unpolarized light.
- The waves in which the vibrations occur in a single plane called polarized light waves.
- The process of transforming unpolarized light into polarized light is known as polarization.

6. State Brewster's Law.

Ans :

Brewster's Law

Brewster performed a number of experiments to study the polarization of light by reflection at the surfaces of different media. He found that ordinary light is completely plane polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the angle of polarization.

- He was able to prove that the tangent of the angle of polarization is numerically equal to the refractive index of the medium.
- Moreover, the reflected and the refracted rays are perpendicular to each other.

Suppose a beam of unpolarized light AB is incidental and angle equal to the polarizing angle on the glass surface at B. It is reflected along BC and refracted along BD.

From snell's law

$$\mu = \frac{\sin i}{\sin r}$$

From Brewster's law

$$\mu = \tan i = \frac{\sin i}{\cos i}$$

From both laws. We get,

$$\cos i = \sin r = \cos(\pi/2 - r)$$

$$\therefore \quad i = (\pi/2 - r) \text{ or } i + r = \pi/2$$

As $i + r = \pi/2$, $<$CBD is also equal to $\pi/2$. Therefor, the reflected and the refracted rays are at right angles to each other.

7. Quarter Wave Plate.

Ans :

Wave plates, also called retarders or converter of polarization forms with the help of appropriate retarder a given form of polarization can be converted into any other form.

It is made up of a doubly refracting crystal like a calcite or quartz. The refreshing faces of the crystal are cut parallel to the direction of optic axis and its thickness is such that it introduces a phase difference of $\pi/2$ or a path difference of $\lambda/4$ between the emerging ordinary and extraordinary rays.

If μ_0 & μ_e are refractive indices for the O & E-rays respectively, then for normal incidence, the path difference between O & E-rays is

For negative crystals $= (\mu_0 - \mu_e)t$ $\quad (\because \mu_0 > \mu_e)$

For positive crystals $= (\mu_e - \mu_0)t$ $\quad (\because \mu_e > \mu_0)$

Where 't' is the thickness of the quarter wave plate. But the quarter wave plate introduces a path difference of $\lambda/4$.

Hence, for negative crystals;

$$(\mu_0 - \mu_e)\ t = \lambda/4$$

$$\text{(or)}\ \ t = \frac{\lambda}{4(\mu_0 - \mu_e)}$$

And for positive crystals,

$$(\mu_e - \mu_0)t = \lambda/4$$

$$\text{or} \qquad t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

This plate is used to produce circularly and elliptically polarized light when introduced in the path of a plane polarized light. As the thickness depends upon the wavelength, it is useful only for the wavelength for which it is constructed. In combination with a Nicol Prism, it is used for analyzing all kinds of polarised lights.

8. Half Wave Plate

Ans :

It is made up of doubly refracting crystals. Its refracting faces are cut parallel to optic axis is and its thinkness (t) is such that it introduces a phase difference of π or a path difference $\lambda/2$ between emerging O & E-rays.

For negative crystal, the path difference

$$= (\mu_0 - \mu_e)\ t$$

And for positive crystal, the path difference

$$= (\mu_0 - \mu_e)\ t$$

But the half - wave plate introduce a path difference of $\lambda/2$ when a plane polarized light is passed through it.

Hence for negative crystals

$$(\mu_0 - \mu_e)\ t = \lambda/2$$

$$\text{or} \ \ t = \frac{\lambda}{2(\mu_0 - \mu_e)}$$

For positive crystals

$$(\mu_e - \mu_0)\ t = \lambda/2$$

$$\text{or} \qquad t = \frac{\lambda}{2(\mu_e - \mu_0)}$$

The quarter and half wave plates are called retardation plates are called retardation plates because they retard motion of the rays.

Choose the Correct Answers

1. Which of the following phenomenon proves the transverse characteristics of light [c]

   (a) Interference (b) Dispersion

   (c) Polarization (d) Diffraction

2. At polarization angle the reflected ray from a glass is [b]

   (a) Partially polarized light (b) Completely polarized light

   (c) Scattered ray (c) Diffracted ray

3. At polarization angle the angle between reflected ray & refracted ray [a]

   (a) 90° (b) 45°

   (c) 180° (d) 0°

4. Which of the following phenomenon is responsible for polarization of light? [c]

   (a) Interference (b) Double reflection

   (c) Double refraction (d) Diffraction

5. The velocity of O-ray & E-ray in a crystal is same along the direction of [a]

   (a) Optic axis (b) Geometrical axis

   (c) Perpendicular to the optic axis (d)

6. If polarization angle of a medium is 60° the angle of refracting is [c]

   (a) 40° (b) 150°

   (c) 30° (d) 60°

7. The action of Nicol prism is based on [b]

   (a) Scattering (b) Double refracting

   (c) Reflection (d) Diffraction

8. Light produced by a Nicol prism is [a]

   (a) Plane polarized (b) Elliptically polarized

   (c) Circularly polarized (d) Unpolarized

9. In quarter wave plate the phase difference between E-ray and O-ray is [b]

   (a) π (b) $\pi/2$

   (c) $\pi/4$ (d) Zero

10. Light produced by polaroid is [a]

   (a) Plane polarized (b) Elliptically polarized

   (c) Circularly polarized (d) Ordinary light.

Fill in the Blanks

1. Linear and circular polarization schemes are special cases of ________ .
2. For circularly polarized light, the electric field has constant ________ .
3. In elliptial polarization the electric field rotates, and the ________ is not constant along the rotation.
4. Light from the sun is ________ light.
5. Light from an LCD source, such as a lap top screen is ________ light.
6. Polaroid is a sheet of film which only allows ________ light.
7. ________ can convert linearly polarized light to circularly polarized light & vice versa.
8. A light wave that is vibrating in more than one plane is referred to as ________.
9. Crystals which have only one optic axis are called ________.
10. In Laurent's polarimeter ________ is used.

ANSWERS

1. Elliptial polarization
2. Amptitude
3. Amplitude
4. Unpolarized
5. Linearly polarized
6. Linearly polarized
7. Wave retarder
8. Unpolarized light
9. Uniaxial crystals
10. Half shade plate.

One Mark Answers

Q1. Define polarization.

Ans :

The process of transferring unpolarized light into polarized light is known as polarization.

Q2. Define Refraction.

Ans :

Refraction is the change in direction of wave propagation due to change in its transmission medium. The phenomenon is explained by conservation of energy.

Q3. Define Brewster's law.

Ans :

It states that when light is incident at polarizing angle at the intereface of a refracting medium, the refractive index of the medium is equal to the tangent of the polarizing angle.

Q4. Define Malus's law.

Ans :

The intensity of the light transmitted by the analyzer is directly proportional to the square of the cosine of angle between the transmission axes of the analyser and the polarizer.

Q5. What is optical activity?

Ans :

The ability of a substance to rotate the plane of polarization of a beam of light that is passed through it.

FACULTIES OF SCIENCE

B.Sc. IV - Semester(CBCS) Examination

January - 2021

Subject : Physics

PAPER-IV : OPTICS

Time : 2 Hours] **[Max. Marks : 80**

SECTION - A (4 × 5 = 20 M)

[Short Answer Types]

Note : Answer any **FOUR** questions.

ANSWERS

1. State and explain spatial and temporal coherence **(Unit-I, SQA-1)**
2. Explain colours of thin film. **(Unit-II, SQA-2)**
3. Explain Rayleigh - criterion of resolution **(Unit-III, SQA-1)**
4. Describe the construction of zone plate and its working. **(Unit-III, SQA-2)**
5. A ray of light is incident on the surface of a glass plate of refractive index 1.55 at the polarizing angle. Calculate the angle of refraction. **(Unit-IV, Prob.13)**
6. How do you distinguish between a quarter wave and half wave plate. **(Unit-IV, SQA-1)**
7. Explain the defect coma and its minimization. **(Out of Syllabus)**
8. State some applications of fibre optics. **(Out of Syllabus)**

SECTION - B (3 × 20 = 60 M)

[Essay Answer Types]

Note : Answer any **THREE** questions.

9. Explain the features of interference pattern produced with a biprism. Describe the experimental arrangement of Fresnel biprism to find the wave length of light. **(Unit-II, Q.No.4,6)**
10. Describe the theory of Newton's rings experiment to determine the wave length of monochromatic source of radiation. In a Newton's rings experiment the diameter of 3^{rd} and 23^{rd} dark rings are 0.2 cm and 0.56 cm respectively. If the radius of curvature of piano convex lens is 92 cm, find the wave length of light. **(Unit-II, Q.No.16, Prob.11)**
11. What is diffraction? How the intensity distribution is studied in the case of double slit due to fraunhofer diffraction. **(Unit-III, Q.No.1,7)**

12. Write the difference between Zone plate and Convex lenses. Describe the diffraction due to a straight edge. **(Unit-III, Q.No. 14,16)**

13. Define specific rotation. How it is experimentally determined using Laurent's half, shade polarimeter. Calculate the specific rotatory power if the plane of polarization is turned through 26.4° transversing 20 cm length of 20% sugar solution. **(Unit-IV, Q.No.12, Prob.15)**

14. Explain the construction and working of Babinets compensator. Calculate the thickness of a quarter wave plate. Given $\mu_c = 1.533$, $\mu_o = 1.544$ and $\lambda = 5000°A$. **(Unit-IV, Prob.12)**

15. Derive the conditions of achromatism for two lenses of focal lengths f_1 and f_2,
    (i) When they are made of different materials but placed in contact and **(Out of Syllabus)**
    (ii) When they are made of same material but separated by a distance. **(Out of Syllabus)**

16. Explain the classification of optical fibres according to the type of
    (i) Material **(Out of Syllabus)**
    (ii) Modes and **(Out of Syllabus)**
    (iii) Refractive index. **(Out of Syllabus)**

FACULTIES OF SCIENCE

B.Sc. IV - Semester(CBCS) Examination

May / June - 2019

Subject : Physics

PAPER-IV : OPTICS

Time : 3 Hours] **[Max. Marks : 80**

SECTION - A (5 × 4 = 20 M)

[Short Answer Types]

Note : Answer any **FOUR** questions.

ANSWERS

1. Explain the conditions for sustainable interference of light. **(Unit-II, SQA-3)**
2. The diameter of a one of the dark ring in the Newton's rings experiment is 6 mm. Find the diameter of same ring when the experiment is conducted in the liquid of refractive index is 1.5. **(Unit-II, Prob.12)**
3. Define zone plate and explain the construction of a zone plate. **(Unit-III, SQA-2)**
4. Find the possible order of diffraction with a grating of element 0.12×10^{5} m and wavelength is 6000 Å. **(Unit-III, Prob-11)**
5. Define polarized light, plane, circularly and elliptically polarized light. **(Unit-IV, SQA-2)**
6. Find the thickness of a birefringent crystal, which introduces a phase difference of 60° between e and o-rays ($\mu_e = 1.553$, $\mu_o = 1.544$ and $\lambda = 5400$ Å). **(Unit-IV, prob.12)**
7. The combination of two thin lenses separated by a distance is used to satisfy the chromatic aberration and minimizing spherical aberration. This combination has focal length 50 cm. Then find the focal length of lenses. **(Out of Syllabus)**
8. What is an aberration? Mention different types of aberrations. **(Out of Syllabus)**

SECTION - B (4 × 15 = 60 M)

[Essay Answer Types]

Note : Answer any **THREE** questions.

9. (a) Explain the action of a biprism and describe how wavelength of a given light is determined by using biprism experiment. **(Unit-II, Q.No.5)**

OR

(b) Describe the working of a Michelson interferometer and obtain an expression to determine the difference of two neighbouring wavelengths. **(Unit-II, Q.No.18,20)**

10. (a) Explain the Fraunhofer diffraction at single slit and derive an equation for the intensity diffraction pattern. **(Unit-III, Q.No.4)**

OR

(b) Discuss Fresnel diffraction at straight edge and discuss the condition for maxima and minima intensity. **(Unit-III, Q.No.16)**

11. (a) Explain various methods to produce the plane polarized light. **(Unit-IV, Q.No.2)**

OR

(b) What is a waveplate? Mention the types of waveplates and explain the working of waveplates. **(Unit-IV, Q.No.8)**

12. (a) Define chromatic aberration and obtain the expression for chromatic aberration for an object at infinity distance. **(Out of Syllabus)**

OR

(b) Classify the optical fibers based on the refractive indices of core and cladding. Explain the advantages of optical fibers in communications. **(Out of Syllabus)**

FACULTIES OF SCIENCE

B.Sc. IV - Semester(CBCS) Examination

June - 2018

Subject : Physics

PAPER-IV : OPTICS

Time : 3 Hours] **[Max. Marks : 80**

SECTION - A (5 × 4 = 20 M)

[Short Answer Types]

Note : Answer any **FOUR** questions.

ANSWERS

1. What are the conditions necessary for observing interference fringes? **(Unit-II, SQA-3)**
2. What is meant by phase change on reflection? **(Unit-II, SQA-4)**
3. If white light is used in young's double slit experiment, what will happen to the interference bands? **(Unit-III, SQA-9)**
4. Explain the Rayleigh criterion for resolution. **(Unit-III, SQA-1)**
5. What are different types of polarizations? **(Unit-IV, SQA-2)**
6. What are Uniaxial and Biaxial Crystals? **(Unit-IV, SQA-3)**
7. Explain the methods for elimination of astigmatism. **(Out of Syllabus)**
8. Explain the advantages of fiber optic communications. **(Out of Syllabus)**

SECTION - B (4 × 15 = 60 M)

[Essay Answer Types]

Note : Answer any **THREE** questions.

9. (a) What are Newton's Rings? Derive an expression for the diameter of bright rings. **(Unit-II, Q.No.16)**

OR

(b) Describe Fresnel's Biprism method for the determination of the Wavelength of light. **(Unit-II, Q.No.4)**

10. (a) Discuss the Fraunhofer diffraction due to single slit. In a grating spectrum which spectral line in fourth order will overlap with third order line of 5461 °A. **(Unit-III, Q.No.4, Prob.2)**

OR

(b) Describe and explain the phenomenon of diffraction due to straight edge. Explain why the bands are neither equidistant not equality illuminated. **(Unit-III, Q.No.17)**

11. (a) Describe the construction and working of Nicol prism. **(Unit-IV, Q.No.6)**

OR

(b) Describe the construction and working of Laurent's half shade Polarimeter. **(Unit-IV, Q.No.12)**

12. (a) What is meant by spherical aberration? Explain how it is minimized by the co-axial lenses separated by distance. **(Out of Syllabus)**

OR

(b) Define numerical aperture and acceptance angle. Derive a relation between them. **(Out of Syllabus)**

FACULTIES OF SCIENCE

B.Sc. IV - Semester(CBCS) Examination

May / June - 2018

Subject : Physics

PAPER-IV : OPTICS

Time : 3 Hours] **[Max. Marks : 80**

SECTION - A (5 × 4 = 20 M)

[Short Answer Types]

Note : Answer any **FOUR** questions.

ANSWERS

1. Define the principles of Superposition and explain it. **(Unit-II, SQA-5)**
2. Mention the difference between Biprism and Lloyd's Mirror Fringes. **(Unit-II, SQA-9)**
3. What are Fresnel's assumption used to explain diffraction? **(Unit-III, SQA-4)**
4. Explain Rayleigh criterion for resolution. **(Unit-III, SQA-1)**
5. What are uniaxial and Biaxial crystals? **(Unit-IV, SQA-3)**
6. State and explain Malus Law. **(Unit-IV, SQA-4)**
7. What is Coma? **(Out of Syllabus)**
8. What are various types of losses in the optical fibers? **(Out of Syllabus)**

SECTION - B (4 × 15 = 60 M)

[Essay Answer Types]

Note : Answer any **THREE** questions.

9. (a) What are Newton's Rings? Show that the diameter of Newton's dark rings are proportional to the square root of natural numbers. **(Unit-II, Q.No.16)**

OR

(b) Describe Michelson's interferometer. How it is used to determine the thickness of thin sheet? **(Unit-II, Q.No.18,20)**

10. (a) Derive an expression for radius of a Fresnel half period zone.

Find the angular width of a central bright maximum in the Fraunhofer diffraction pattern of slit of width 12×10^{-3} cm when the slit is illuminated by monochromatic light of wave length 6000Å. **(Unit-III, Q.No.12, Prob.6)**

OR

(b) Explain the phenomenon of Fresnel diffraction at straight edge. Indicate the intensity distribution of diffraction pattern by a diagram. **(Unit-III, Q.No.17)**

11. (a) Describe the construction and working of Nicol prism. **(Unit-IV, Q.No.6)**

OR

(b) Describe the polarimeter and explain how it is used to measured the strength of sugar solution.

A tube 20 cm long containing sugar solution rotates the plane of polarization through an angle 13.5°. If the specific rotation is 66°. Find the amount of sugar present in litre of solution. **(Unit-IV, Q.No.12, Prob.11)**

12. (a) Explain the defects of images due to astigmatism and curvature.

Discuss the methods for eliminating them. **(Out of Syllabus)**

OR

(b) Explain the differences between step index and graded index fibers.

Obtain an expression for numerical aperture. **(Out of Syllabus)**

FACULTIES OF SCIENCE

B.Sc. IV - Semester(CBCS) Examination

Model Paper - I

Subject : Physics

PAPER-IV : WAVES & OPTICS

Time : 3 Hours] **[Max. Marks : 80**

SECTION - A (8 × 4 = 32 M)

[Short Answer Types]

Note : Answer any **EIGHT** of the following questions.

ANSWERS

1. Define the laws of vibrations in strings fixed at both ends ? **(Unit-I, SQA-1)**
2. Explain transverse wave propagation along a stretched string. **(Unit-I, SQA-4)**
3. The fundametnal frequency of vibration of a stretched string of length 1m is 256 Hz. Find the frequency of the same string of half the original length under identical conditions. **(Unit-I, Prob.3)**
4. What is meant by phase change on reflection? **(Unit-II, SQA-4)**
5. Explain different types of fringes. **(Unit-II, SQA-8)**
6. Using sodium light (l = 5893 A°) interference fringes are formed from a think our wedge, when viewed normally 10 fringes are observed in a distance of 1cm. Calculate the angle of the wedge. **(Unit-II, Prob.1)**
7. Rayleigh's Criterion for resoltion. **(Unit-III, SQA-1)**
8. Explain phase reversal zone plate. **(Unit-III, SQA-7)**
9. Calculate the angles at which the first dark bnand and then next bright band are farmed in the fraunhofer diffraction pattern of slit 0.3 mm wide ? **(Unit-III, Prob.1)**
10. How do you distinguish between a quarter wave plate and Half wave plate? **(Unit-IV, SQA-1)**
11. Half Wave Plate **(Unit-IV, SQA-8)**
12. Calculate the thickness of a doubly refracting plate capable of producing a path difference $\lambda/4$ between extraordinary and ordinary waves (λ = 5890A°, μ_0 = 1.53; μ_e = 1.54) **(Unit-IV, Prob.4)**

SECTION - B (4 × 12 = 48 M)

[Essay Answer Types]

Note : Answer all the following questions

13. (a) Derive the differential equation for a Transverse Wave Propagation along a Stretched String **(Unit-I, Q.No.2)**

OR

(b) Write a brief note on tuning fork ? **(Unit-I, Q.No.22)**

14. (a) What is meant by phase change on reflection? **(Unit-II, Q.No.7)**

OR

(b) Explain Newton Rings in reflected light with contact between lens and Glass plate ? **(Unit-II, Q.No.16)**

15. (a) Discuss Fraunhofer diffraction due to a single slit. Explain the distribution of intensity of light in the diffraction pattern. **(Unit-III, Q.No.4)**

OR

(b) Determine the wavelength of light in normal and oblique incidence methods using diffraction grating. **(Unit-III, Q.No.11)**

16. (a) Explain various methods to produce the plane polarized light. **(Unit-IV, Q.No.2)**

OR

(b) Define polarimeter. Explain Laurent's half-shade polarimeter? **(Unit-IV, Q.No.12)**

FACULTIES OF SCIENCE

B.Sc. IV - Semester(CBCS) Examination

Model Paper - II

Subject : Physics

PAPER-IV : WAVES & OPTICS

Time : 3 Hours] **[Max. Marks : 80**

SECTION - A (8 × 4 = 32 M)

[Short Answer Types]

Note : Answer any **EIGHT** of the following questions.

ANSWERS

1. Define the terms Transverse Waves, Overtones, Transverse impedances? **(Unit-I, SQA-2)**

2. Write the differences betwee longitudinal and Transverse vibrations? **(Unit-I, SQA-7)**

3. A flexible string of length 1m and mass 1gm is stretched by a tension T. The string is found to vibrate in three segments at a frequency of 512 Hz. Calculate the tension. **(Unit-I, Prob.5)**

4. Write short a note on Principle of Superposition. **(Unit-II, SQA-5)**

5. Differentiate Lloyd's and Biprism mirror fringes **(Unit-II, SQA-9)**

6. Two optically plane glass strips of length 10cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light used has wavelength 5900 °A, find the fringe width. **(Unit-II, Prob.8)**

7. Describe the construction of Zone plates. **(Unit-III, SQA-2)**

8. What are the difference between interference and diffraction? **(Unit-III, SQA-8)**

9. A plane transmission grating has 80000 lines in all. Find in the wavelength region of 6000 Å, in the second order

   a) The resolving power of the grating

   b) The smallest wavelength difference that can be resolved ? **(Unit-III, Prob.3)**

10. Define plane, circular and elliptical polarized light. **(Unit-IV, SQA-2)**

11. What are uniaxial and Biaxial crystals? **(Unit-IV, SQA-3)**

12. A half wave plate is constructed for a wavelength of 600Å. For what wavelength does it work as a quarter wave plate? **(Unit-IV, Prob.5)**

SECTION - B (4 × 12 = 48 M)

[Essay Answer Types]

Note : Answer all the following questions

13. (a) Obtain a wave equation for longitudinal vibrations in bars and find its general solution ? **(Unit-I, Q.No.10)**

OR

(b) Derive the wave equations for transverse vibrations along a stretched string. **(Unit-I, Q.No.15)**

14. (a) Determine the thickness of a Transparent material using Biprism. **(Unit-II, Q.No.5)**

OR

(b) Determine the difference in wavelength of sodium D_1, D_2 lines ? and Find the thickness of thin transparent plate. **(Unit-II, Q.No.20)**

15. (a) Define limit of resolution and obtain an expression for Rayleigh's criterion? **(Unit-III, Q.No.6)**

OR

(b) Describe and explain the phenomenon of diffraction due to a straight edge. Explain why the bands are neither equidistant nor equally illuminated? **(Unit-III, Q.No.3)**

16. (a) Explain different types of polarization. **(Unit-IV, Q.No.3)**

OR

(b) Explain the construction and working of Babinet's compensator? **(Unit-IV, Q.No.10)**

FACULTIES OF SCIENCE

B.Sc. IV - Semester(CBCS) Examination

Model Paper - III

Subject : Physics

PAPER-IV : WAVES & OPTICS

Time : 3 Hours] **[Max. Marks : 80**

SECTION - A (8 × 4 = 32 M)

[Short Answer Types]

Note : Answer any **EIGHT** of the following questions.

ANSWERS

1. Write the differences betwee longitudinal and Transverse vibrations. **(Unit-I, SQA-7)**
2. Write the uses of tuning fork. **(Unit-I, SQA-9)**
3. The speed of a transverse wave on a stretched string is 500 m/s, when it is stretched under a tension of 19.6N. If the tension is altered to a value of 78.4N, what will be the spee fo the wave ? **(Unit-I, Prob.7)**
4. Explain the terms temporal and spatial coherence. **(Unit-II, SQA-1)**
5. Determination of Wavelength **(Unit-II, SQA-10)**
6. Light of wavelength 6000Å falls normally on a thing wedge film of refractive index 1.4, forming fringes taht are 2mm apart. Find the angle of the wedge. **(Unit-II, Prob.2)**
7. If white light is used in young's double slit experiment, what will happen to the interference bands. **(Unit-III, SQA-9)**
8. Explain fresnel's half period zones. **(Unit-III, SQA-5)**
9. Find the angular width of the central bright maximum in the fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000Å. **(Unit-III, Prob-6)**
10. State and explain Malus Law. **(Unit-IV, SQA-4)**
11. State Brewster's Law. **(Unit-I, SQA-1)**
12. A tube 20 cm long containing sugar solution rotates the plane of polarization through an angle of 13.5°. If the specific rotation is 66°. Find the amount of sugar present in a litre of the solution. **(Unit-IV, Prob.6)**

SECTION - B (4 × 12 = 48 M)

[Essay Answer Types]

Note : Answer all the following questions

13. (a) Explain the modes of vibrations of strings clamped at the ends. **(Unit-I, Q.No.5)**

OR

(b) Explain the concept of transverse impedance. **(Unit-I, Q.No.18)**

14. (a) Write about coherent sources and its types. Explain briefly. **(Unit-II, Q.No.3)**

OR

(b) Explain Newton's Rings in reflected light without contact between lens and glass plate, and explain Haidinger fringes ? **(Unit-II, Q.No.17)**

15. (a) Explain diffraction of grating by fraunhofer diffraction pattern with N slits? **(Unit-III, Q.No.9)**

OR

(b) Explain the concept of Fresnel half period zone plates. **(Unit-III, Q.No.13)**

16. (a) State Brewster's Law. **(Unit-IV, Q.No.4)**

OR

(b) Explain about nicol prism as a polarizer and analyzer? **(Unit-IV, Q.No.6)**