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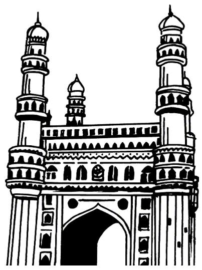

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B.Sc.

II Year III Sem

ELECTROMAGNETIC THEORY

PHYSICS PAPER - III

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- **Objective Type**
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SYLLABUS

UNIT - I

Electric Field: Concept of electric field lines and electric flux, Gauss's law (Integral and differential forms), application to linear, plane and spherical charge distributions. Conservative nature of electric field 'E', Irrotational field. Electric potential:- Concept of electric potential, relation between electric potential and electric field, potential) energy of a system of charges. Energy density in an electric field. Calculation of potential from electric field for a spherical charge distribution.

UNIT - II

Magnetostatics

Concept of magnetic field 'B' and magnetic flux, Biot-Savart's law, B due to a straight current carrying conductor. Force on a point charge in a magnetic field. Properties of B, curl and divergence of B, solenoidal field. Integral form of Ampere's law, Applications of Ampere's law: field due to straight, circular and solenoidal currents. Energy stored in magnetic field. Magnetic energy in terms of current and inductance. Magnetic force between two current carrying conductors. Magnetic field intensity. Ballistic Galvanometer:- Torque on a current loop in a uniform magnetic field, working principle of B.G., current and charge sensitivity, electromagnetic damping, critical damping resistance.

UNIT - III

Electromagnetic Induction and Electromagnetic waves: Faraday's laws of induction (differential and integral form), Lenz's law, self and mutual Induction. Continuity equation, modification of Ampere's law, displacement current, Maxwell equations. Maxwell s equations in vacuum and dielectric medium, boundary conditions, plane wave equation: transverse nature of EM waves, velocity of light in vacuum and in medium. Poynting's theorem.

UNIT - IV

Varying and alternating currents: Growth and decay of currents in LR. CR and LCR circuits - Critical damping. Alternating current, relation between current and voltage in pure R, C and L-vector diagrams - Power in ac circuits'. LCR series and parallel resonant circuit - Q-factor. AC & DC motors-single phase, three phase (basics only).

Network Theorems: Passive elements, Power sources, Active elements, Network models: T and n Transformations, Superposition theorem, Thevenin's theorem, Norton's theorem. Reciprocity theorem and Maximum power transfer theorem (Simple problems).

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Important Questions

UNIT - I

1. Define coulomb's law ?

Ans :

Refer Unit-I, Q.No. 2.

---

2. What do you mean by electric flux.

Ans :

Refer Unit-I, Q.No. 5.

---

3. State and prove Gauss theorem in electrostatistics.

Ans :

Refer Unit-I, Q.No. 6.

---

4. Using Gauss law, obtain an Expression for Electric field intensity at a point due to a line of charge of infinite length

Ans :

Refer Unit-I, Q.No. 7.

---

5. Give mathematical description on of electric field at points inside, outside and on the surface of uniformly charged cylinder?

Ans :

Refer Unit-I, Q.No. 10.

---

6. Show that electric field is conservative in nature.

Ans :

Refer Unit-I, Q.No. 11.

---

7. What is meant by electric potential.

Ans :

Refer Unit-I, Q.No. 13.

8. Explain about energy density in electrostatic field?

Ans :

Refer Unit-I, Q.No. 17.

9. Derive Expression for potential & Electric field due to Uniformly Circular disc.

Ans :

Refer Unit-I, Q.No. 19.

UNIT - II

1. State and explain Biot - Savart law.

Ans :

Refer Unit-II, Q.No. 2.

2. Derive an expression for the magnetic induction due to long straight conductor carrying current.

Ans :

Refer Unit-II, Q.No. 3.

3. Explain the various properties of magnetic field.

Ans :

Refer Unit-II, Q.No. 5.

4. Obtain the relationship between magnetic flux density 'B' magnetising force 'H' intensity of magnetisation?

Ans :

Refer Unit-II, Q.No. 6.

5. State and Explain integral form of Amperes law?

Ans :

Refer Unit-II, Q.No. 9.

6. Calculate the intensity of magnetic field at a point on the axis of circular coil carrying current?

Ans :

Refer Unit-II, Q.No. 10.

7. Derive the Expression for the force between two parallel current carrying conductors.

Ans :

Refer Unit-II, Q.No. 13.

8. Explain the principle and working of a moving coil Ballastic galvanometer.

Ans :

Refer Unit-II, Q.No. 15.

UNIT - III

1. State and Explain Faraday's laws of Electromagnetic Induction. Derive the differential and Integral forms of Faraday's Law.

Ans :

Refer Unit-III, Q.No. 1.

2. State and Explain Lenz's Law obtain an Expression for induced E.M.F.

Ans :

Refer Unit-III, Q.No. 2.

3. What is self induction? Define coefficient of self induction and obtain an expression for self induction of a long solenoid.

Ans :

Refer Unit-III, Q.No. 3.

4. Obtain the Expression for self inductance of long solenoid.

Ans :

Refer Unit-III, Q.No. 4.

5. Define Mutual Induction. Derive an expression for the coefficient of mutual induction between a pair of coils.

Ans :

Refer Unit-III, Q.No. 6.

6. Mention the differences between self induction and mutual induction.

Ans :

Refer Unit-III, Q.No. 7.

7. Derive the Maxwell correction to Ampere's law.

Ans :

Refer Unit-III, Q.No. 9.

8. Write Maxwell's equations in differential and integral forms.

Ans :

Refer Unit-III, Q.No. 10.

9. Derive the Maxwell's Electromagnetic wave equation for E & B in dielectronic medium and vacuum (or) free space.

Ans :

Refer Unit-III, Q.No. 11.

10. Derive Boundary conditions for D, B, E, and H.

Ans :

Refer Unit-III, Q.No. 12.

11. Show that Electromagnetic waves are transverse in nature.

Ans :

Refer Unit-III, Q.No. 14.

12. Explain energy conservation law in electromagnetism?

Ans :

Refer Unit-III, Q.No. 17.

UNIT - IV

1. Write a brief note on Growth of current in LR circuit?

Ans :

Refer Unit-IV, Q.No. 1.

2. Explain about growth and decay of charge in RC-circuit?

Ans :

Refer Unit-IV, Q.No. 3.

3. Explain the phenomenon of critical damping.

Ans :

Refer Unit-IV, Q.No. 5.

4. Explain about power in AC circuit.

Ans :

Refer Unit-IV, Q.No. 9.

5. Explain the LCR circuit in series and parallel resonant condition?

Ans :

Refer Unit-IV, Q.No. 11.

6. Explain about 3-phase AC-motors?

Ans :

Refer Unit-IV, Q.No. 13.

7. Express the elements of π-network interms of Y and ABCD parameter.

Ans :

Refer Unit-IV, Q.No. 21.

8. State and prove superposition theorem.

Ans :

Refer Unit-IV, Q.No. 24.

9. State and prove Thevenin's theorem.

Ans :

Refer Unit-IV, Q.No. 25.

UNIT I	**Electric Field:** Concept of electric field lines and electric flux, Gauss's law (Integral and differential forms), application to linear, plane and spherical charge distributions. Conservative nature of electric field 'E', Irrotational field. Electric potential:- Concept of electric potential, relation between electric potential and electric field, potential) energy of a system of charges. Energy density in an electric field. Calculation of potential from electric field for a spherical charge distribution.

1.1 Electric Charge

Q1. Define the term electric charge.

Ans :

Greek Philosopher Thales discovered that when amber is rubbed with wool, it acquires the property of attracting light bodies bodies such as piece of paper. Later on, Gilbert, an English Philosopher discovered that several other materials such as glass ebonite, sulphur etc., also after being rubbed behave in same manner. The bodies exhibiting the above property are said to be electrified (or) charged.

The cause due to which the body aquires this property is called electricity.

Further it has been observed that bodies electrified with different kind of charges attract each other. Thus like charge repel and unlike charges attract each other.

Thus charge is having following properties

(i) Charge is scalar quantity

(ii) Two types of charges positive charge and negative charge

(iii) Similar charges repel each other while opposite charges attract each other.

(iv) Charges is conserved.

1.2 Coulomb's Law

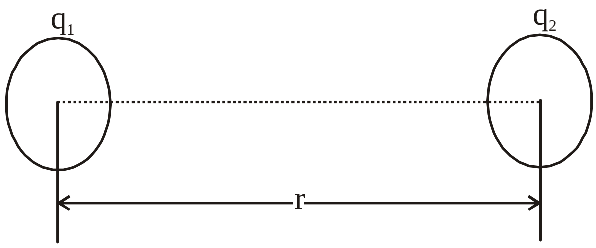
Q2. Define coulomb's law ?

Ans : **(Imp.)**

In 1785, Coulomb gave two laws for the force of attraction (or) repulsion between to electrically charged bodies seperated from each other by a definite distance.

Coulomb's law states that the force of attraction (or) repulsion between two stationary point charges is directly propotional to the product of the two charges and inversly propotional to the square of the distance between them.

If q_1 and q_2 be the two point charges separated from each other by a distance r



then force 'F' acting between them is given by

$$F \propto q_1 q_2 \text{ and } F \propto \frac{1}{r^2}$$

$$\therefore \qquad F \propto \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \qquad .....(1)$$

where $\frac{1}{4\pi\varepsilon_0}$ propotionality constant.

If q_1, q_2 are in coulomb, r in metre and force in newton then.

$$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 \text{ newton - metre}^2 / \text{coulomb}^2$$

Hence, the force between two point charges placed in vaccum (or air) is

$$\boxed{F = 9.0 \times 10^9 \cdot \frac{q_1 q_2}{r^2} \text{ newton}}$$

The constant ε_0 is called permitivity of free space given by

$$\varepsilon_0 = 8.9 \times 10^{-12} \text{ coulomb}^2 / \text{newton/metre}^2$$

Coulomb's law in Medium :

If instead of vaccum some insulting material is placed between the charges then force acting between them is given by

$$F = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{q_1 q_2}{r^2}$$

K = dielectric constant of the medium

If $\varepsilon_0 K =$ where ε is called permitivity of the dielectric

Hence $\boxed{F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}}$

1.3 Concept of Electric Field

Q3. Define Electric field and Intensity of electric field ?

Ans :

i) Electric Filed

The region surrounding an electric charge or a group of charges in which another charge experiences a force is called electric field.

ii) Intensity of electric field

The intensity of electricfield at a point in the field is defined as the force experienced by a unit positive charge placed at that point. Let 'F' be the force experienced by a test charge q_0 placed at a point in the electric field then intensity of electric field 'E' at that point is given by

$$E = \frac{F}{q_0} = \frac{\text{Newton}}{\text{Coulomb}}$$ is a vector quantity

But from Coulomb's law

The force on unit positive charge is given by

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{Q \times 1}{r^2}$$

$$E = \frac{F}{1} = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$$

1.3.1 Electric filed lines

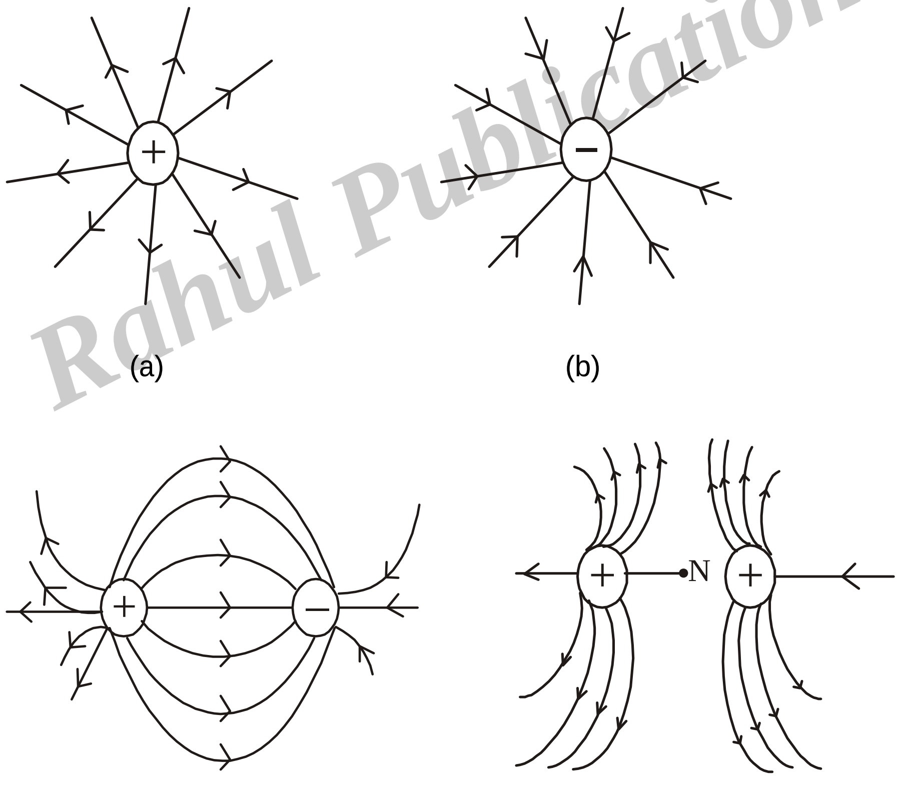
Q4. Write about Electric lines force?

Ans :

An electric line of force is that imaginary smooth curve drawn in an electric field along which a free isolated positive charge will move. The tangent at any point on it gives the direction of the field at that point. We can represent an electric field by lines of force as shown in fig (1).

In fig. (a) we have drawn lines of force in an electric field produced by a positively charged sphere. For a positive charge the line of force move towards infinity due to repulsive force.

Similarly for a negative charge the line of force moves towards charge from infinity as shown in fig (b).



(a) (b)

(c) **Fig (1)** (d)

Fig (c). shows the lines of force due to two equal and opposite charges. The lines start from positive charge and end at the negative charge Fig (d) shows lines of force due to two similar positive charges. It clear that at the midpoint 'N' field produced by one charge is equal opposite to the field produced by other point charge. Thus at this point field becomes zero.

Thus intensity of electric field at point can also defined as number of lines of force passing through unit area round that point normally.

Properties

- The tangent drawn at any point on the lines of force gives the direction of electric field at that point.
- The electric line of force start from positive charge and end on a negative charge.
- No two lines of force intersect each other.
- The electric lines of force are perpendicular to equipotential surfaces.
- The number of lines of forces crossing unit area in normal direction is propotional to electric field intensity.

1.3.2 Electric Flux

Q5. Explain about Electric flux ?

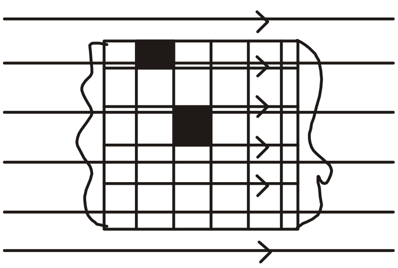
(OR)

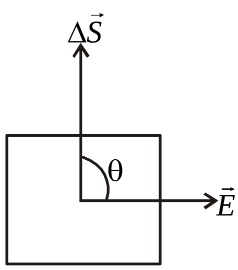
What do you mean by electric flux.

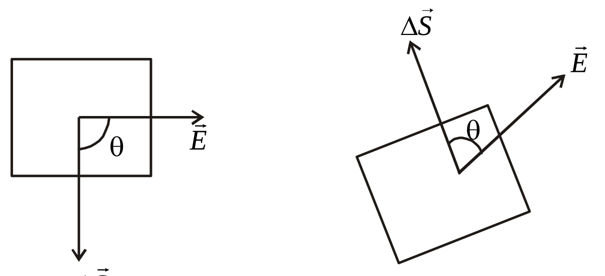
Ans : **(Imp.)**

The electric flux through a surface placed inside electric field represents the total number of electric lines of force crossing the surface in a direction normal to the surface.

In order to explain electric flux, Let us consider an electric field as shown in figure. Now imagine an arbitary closed surface immersed in this field. The surface is divided into number of elementary squares.







Each square on the surface may be represented by a vector ΔS whose magnitude is equal to its area and the direction taken as the outward normal drawn on this surface. E is the electric field vector acting on the surface.

The scalar product E. ΔS is defined as the electric flux for the surface. The total flux ϕ_E through the entire surface is given by

$$\boxed{\phi_E = \oint E \cdot \Delta S = E \cdot S}$$

If 'θ' is the angle between E and ΔS then the scalar product is given by

$$E \cdot \Delta S = E \cdot ds \cos\theta$$

Electric flux $\phi_E = \oint E \cdot ds \cos\theta$

$$\phi_E = E\cos\theta \oint ds$$

$$\phi_E = E\cos\theta A \;(\because \oint ds = A = \text{area of surface})$$

$$\boxed{\phi_E = EA\cos\theta}$$

Thus the flux ϕ_E of electric field is measured by number of lines of force that cut the surface.

1.4 Gauss Law (Integral and Differential Forms)

Q6. State Gauss law in electrostatics and derive its Integral and differential form ?

(OR)

State and prove Gauss theorem in electrostatistics.

Ans : **(Imp.)**

Gauss law states total normal electric flux ϕ_E over a closed surface is $(1/\varepsilon_0)$ times the total charge 'Q' enclosed within the surface. i.e.,

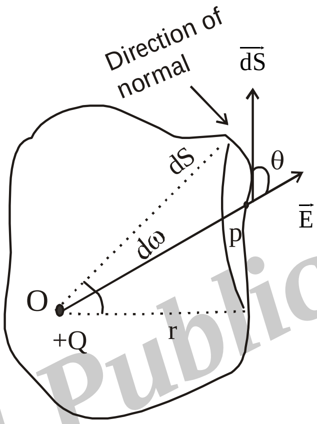
$$\phi_E = \oint E \cdot ds = \oint E ds \cos\theta = \left(\frac{1}{\varepsilon_0}\right) Q$$

Where ε_0 is the permitivity of the free space

Proof

(i) When the charge is with in the surface

Let a charge $+Q$ is placed at a point 'O' with in closed surface of irregular shape as shown figure.



Consider a point 'P' on the surface at a distance 'r' from 'O'. Consider a small area 'ds' around 'P'. The normal to the surface ds is represented by a vector $\vec{ds}$ which makes an angle 'θ' with the direction of electric field 'E' along OP.

The electric flux $d\phi_E$ outwards through the area 'ds' is given by

$$d\phi_E = E \cdot ds = E ds \cos\theta \qquad .....(1)$$

Where 'θ' is angle between E and ds

From coulomb's law the electric intensity 'E' at point P distance 'r' from a point charge 'Q' is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2}\right) \qquad .....(2)$$

From e.q. (1) and (2) we get

$$d\phi_E = \frac{Q}{4\pi\varepsilon_0 r^2} \cdot ds\cos\theta$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{ds\cos\theta}{r^2}\right)$$

Where $\frac{ds\cos\theta}{r^2}$ is solid angle 'dw' subtended by ds at a point 'O'.

Hence

$$d\phi_E = \frac{Q}{4\pi\varepsilon_0}.d\omega$$

The total flux ϕ_E over the entire whole surface is given by

$$\phi_E = \frac{Q}{4\pi\varepsilon_0}\oint d\omega$$

where $\oint d\omega$ is the solid angle subtended by the whole surface at a point '0' which is equal to 4π

Hence $\phi_E = \frac{Q}{4\pi\varepsilon_0} \times 4\pi$

$$\boxed{\phi_E = \frac{Q}{\varepsilon_0}}$$

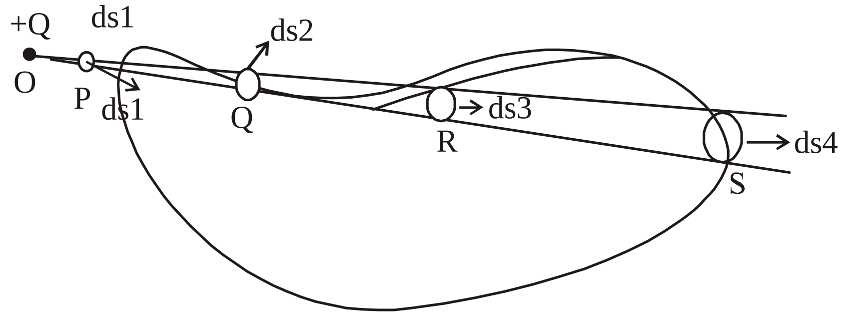
If the closed surface encloses several charges say Q_1, Q_2 ,Q_3 ..... Then the total flux is given by

$$\phi_E = \frac{1}{\varepsilon_0}[Q_1 + Q_2 + ...]$$

$$\phi_E = \frac{1}{\varepsilon_0}\Sigma Q$$

(ii) When the charge is out side the surface

Consider a point charge $+Q$ situated at a point '0' outside the closed surface as shown in figure.



Here the cone of solid angle $d\omega$ from '0' cuts the surface areas ds_1, ds_2, ds_3, ds_4 at points P, Q, R and S respectively. The Electric flux for an outward normal is positive while for inward drawn normal is negative. Thus flux through areas ds_2 and ds_4 are positive while for areas ds_1 and ds_3 are negative. Therefore

Electric flux at 'P' through area $ds_1 = \left(\frac{-Q}{4\pi\varepsilon_0}\right)d\omega$

Electric flux at 'Q' through area $ds_2 = \left(\frac{+Q}{4\pi\varepsilon_0}\right)d\omega$

Electric flux at 'R' through area $ds_3 = \left(\frac{-Q}{4\pi\varepsilon_0}\right)d\omega$

Electric flux at 'S' through area $ds_4 = \left(\frac{+Q}{4\pi\varepsilon_0}\right)d\omega$

$$\therefore \text{ Total flux } = \frac{-Q}{4\pi\varepsilon_0}d\omega + \frac{Q}{4\pi\varepsilon_0}d\omega - \frac{Q}{4\pi\varepsilon_0}d\omega + \frac{Q}{4\pi\varepsilon_0}d\omega = 0$$

Thus total Electric flux over the whole surface due to an external charge is zero.

Differential form of Gauss Law

According to Gauss law

$$\oint E.ds = \frac{Q}{\varepsilon_0}$$

$$\varepsilon_0 \oint E.ds = Q \quad .....(1)$$

If 'Q' is the charge distributed over a volume 'V' and ρ be the density of charge. Then

$$Q = \iiint \rho dv \quad ......(2)$$

From eq (1)

$$\varepsilon_0 \oint E.ds = \iiint \rho dv \quad .....(3)$$

According to Gauss divergence theorem.

$$\oint E.ds = \iiint \text{div } E dv \quad .....(4)$$

Sub (4) in (3), we get

$$\varepsilon_0 \iiint \text{div } E \ dv = \iiint \rho dv \quad .....(5)$$

Eq (5) is true for any arbitary volume, Hence integrands must be equal

$$\varepsilon_0 \text{div } E = \rho \ \text{(or)} \ \boxed{\text{div } E = \frac{\rho}{\varepsilon_0}}$$

We know that $D = \varepsilon_0 E$ (or) $E = \dfrac{D}{\varepsilon_0}$

$$\therefore \text{div } E = \nabla.E = \frac{1}{\varepsilon_0}(\nabla.D) = \frac{\rho}{\varepsilon_0}$$

$$\boxed{\nabla.D = \rho}$$

which represents differential form of Gauss law.

1.4.1 Application of Gauss Law to Linear Charge Distribution

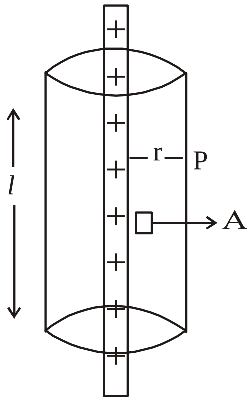
Q7. Using Gauss law, obtain an Expression for Electric field intensity at a point due to a line of charge of infinite length

Ans : **(Imp.)**

Consider a line charge of infinite length as shown in figure.

Let the charge per unit length be λ.

Suppose 'P' is point at the distance 'r' from the charge distribution where Electric field intensity 'E' is to be calculated.



To make the use of Gauss theorem here we draw the cylindrical Gaussian surface of length 'ℓ' and radius 'r' whose curved surface passes the point 'p'. The lines of force pass through the curved surface of Gaussian surface perpendicularly and no lines of force pass through the cross section of the Cylinder.

If ϕ represents the Electric flux passing through the Gaussian surface, by definition of flux.

ϕ = E.A where A = $2\pi rl$. (Surface Area of curved part)

$\therefore \quad \phi = E.2\pi rl.$ ....(1)

Now, charge enclosed by Gaussian surface 'q'

= charge present in length 'ℓ'

$q = \lambda \ell$

But According to Gauss law

$$\phi = \frac{\text{Charge enclosed by Gaussian surface}}{\epsilon_0}$$

$$\phi = \frac{q}{\epsilon_0} = \frac{\lambda\ell}{\varepsilon_0} \qquad ....(2)$$

Equating equations (1) & (2)

$$E.2\pi rl = \frac{\lambda\ell}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

$$\therefore \quad E \, \alpha \, \frac{1}{r}$$

Hence, the Electric field intensity produced by a linear charge distribution is non uniform rather it depends upon the linear charge density, and the distance of the concerned point.

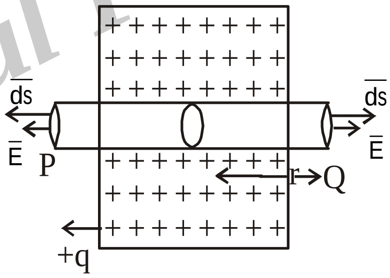
1.4.2 Application of Gauss Law to Plane Charge Distribution

Q8. Obtain the Expression for Electric field due to infinite sheet of charge.

Ans :

Consider a thin conducting, infinite sheet charge either sides with $+q$ charge. Let σ be the surface charge density.

Let 'p' be a point at a distance' 'r' from the sheet at a distance 'r' from the sheet at which Electric field 'E' is to calculated. In order to calculate field, consider another point 'Q' Symmetrical to 'P' on the other side of sheet and imagine a cylindrical Gaussian surface as shown in figure.



Let 'A' be the cross sectional area of the Gaussian Surface. By symmetry the Electric field 'E' is every where perpendicular to the plane. For curve surface electric flux is zero because $\bar{E}$ and $\overline{ds}$ are perpendicular to each other. For end planes of Gaussian surface $\bar{E}$ and $\overline{ds}$ are parallel to each other hence they contribute to electric flux.

If ϕp and ϕ_Q represent electric flux through $\overline{ds}$ at points P and Q then the total Electric flux is

$\phi_E = \phi_P + \phi_Q$

$= E\, ds \cos 0^o + E\, ds \cos 0^o$

($\therefore$ $\theta = 0$ between E and ds due to plane sheet)

$\phi_E = 2Eds$ ....(1)

But from Gauss law $\phi_E = \frac{q}{\varepsilon_0} = \frac{\sigma ds}{\varepsilon_0}$ ....(2)

From equations (1) & (2)

$$2Eds = \frac{\sigma ds}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

This equation would becomes $E = \frac{\sigma}{\varepsilon_0}$ if $+q$ charge is present only on one side of plane sheet.

1.4.3 Application of Gauss law to spherical charge distribution

Q9. In case of uniformly charged sphere evaluate Electric field at points inside, outside and on the surface of sphere.

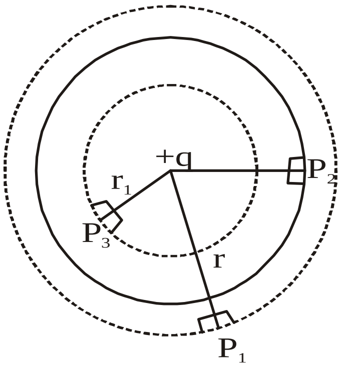
Ans :

Consider a sphere of radius 'R' with its centre at '0' and is uniformly charged with $+q$ as shown in fig.(8) volume charge density ρ can be written as

$$\rho = \frac{q}{\frac{4}{3}\pi R3}$$

$\therefore$ Charge dq in volume dv becomes $dq = \rho\, dv$

From the symmetry of charge distribution, the Electric field at all points of the surface is same and will be normal to the surface.



As Shown in the fig imagine a Gaussian surface outside the sphere at a distance 'r' from the centre '0' and let the Electric flux at point P_1, on this surface through an element area ds be '$d\phi_1$'.

Similarly imagine another Gaussian surface in the sphere at a distance r_1 from the centre and electric flux through similar elemental area 'ds' at a point P_3 on this surface be '$d\phi_3$'. Also electric flux through small area ds around the point P_2 on sphere be '$d\phi_2$'. Then by Gauss law we can find intensities of Electric field at these points.

(i) Outside the sphere

Electric flux $d\phi$, through element area ds on Gaussian surface at point p_1 at a distance 'r' from the centre is

$d\phi_1 = Eds \cos 0$

$= Eds$

Total flux through entire gaussian surface

$$\phi_1 = \int d\phi_1 = \int Eds = E\int ds = E\left(4\pi r^2\right) \qquad \text{........(1)}$$

But According to Gauss law

$$\phi_1 = \frac{q}{\varepsilon_0} \qquad \text{......(2)}$$

From equation (1) & (2)

$$E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{\phi}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

$$\therefore \; E \alpha \frac{1}{r^2}$$

(ii) On the surface of the Sphere

When point 'P_2' is on the surface of sphere then Electric flux 'ϕ_2' through the surface of sphere of radius R is

$$\phi_2 = E\int ds = \frac{q}{\varepsilon_0}$$

$$E\left(4\pi R^2\right) = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}}$$

(iii) Inside the sphere

When the point p_3 is inside the sphere on the Gaussian surface at a distance 'r_1' from the centre the Electric charge on the Gaussian surface

$$q_1 = \rho\left(\frac{4}{3}\pi r_1^3\right)$$

$\therefore$ Electric flux ϕ_3 through the entire Gaussian surface is

$$\phi_3 = E\int ds = \frac{q_1}{\varepsilon_0}$$

$$E\left(4\pi r_1^2\right) = \rho\frac{\left(\frac{4}{3}\pi r_1^3\right)}{\varepsilon_0}$$

$$\therefore \boxed{E = \frac{\rho}{3\varepsilon_0}(r_1)} \text{ (or) } E \propto r_1 \qquad ......(3)$$

If charge q is uniformly distributed over a sphere of radius R. Then

$$\rho = \frac{\text{total charge}}{\text{volume}} = \frac{q}{4/3\,\pi R^3} = \frac{3q}{4\pi R^3}$$

Equation (3) becomes

$$E = \frac{3q\,r_1}{4\pi R^3 . 3\varepsilon_0} = \frac{q\,r_1}{4\pi\varepsilon_0 R^3}$$

The variation of Electric field with distance 'r' from the centre is shown in fig (9)

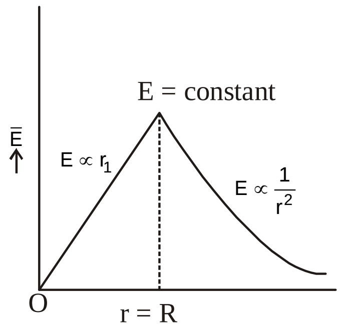


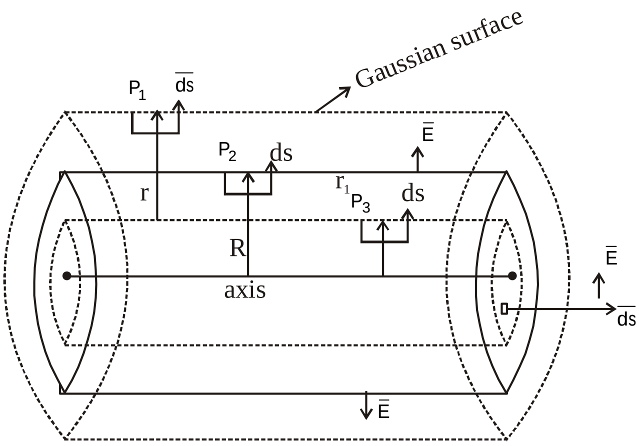
Fig (9)

Starting from the centre of the uniformly charged sphere Electric field strength increase propotionally with distance $(E \alpha r_1)$ and reaches to maximum value on the surface of the sphere and then decreases inversly as square of the distance.

Q10. Give mathematical description on of electric field at points inside, outside and on the surface of uniformly charged cylinder?

Ans : **(Imp.)**

Let the cylinder is uniform changed with 'q' throughout the volume and ρ is the charge per unit volume. Let us calculate the electric field strength at points (i) outside the surface (P_1) (ii) on the surface (P_2) (iii) inside the surface (P_3).



To caculate intencities of electric field at points P_1, P_2 and P_3 imagine cylindrical Gaussian surfaces of length 'L' and radii r, R and r_1 respectively with same axis as that of charged cylinder as shown in figure.

Due to symmetry the intensity of electric field 'E' is same at every point on the curved surface of the cylinder and is directed radially outward along the normal drawn to the surface, hence for the curved $\bar{E}$ and $\overline{ds}$ are parallel with each other, so there will be flux through curved surface of the cylinder. Flux due to circular ends is zero because $\bar{E}$ and $\overline{ds}$ are perpendicular to each other. Therefore, the total electric flux 'ϕ_E' through the entire cylindrical Gaussian surface is given by

ϕ_E = Electric flux through circular ends + Electric flux through curved surface

$$\phi_E = \int \bar{E} \cdot d\bar{s} + \int \bar{E} \cdot d\bar{s}$$

$$= \int Eds \ \cos 0^o + \int Eds \cos 90^o$$

$$= \int Eds + 0 \qquad = \int Eds$$

Assuming length of cylindrical gaussian surface is 'L' and radius R the volume charge density is given by

$$\rho = \frac{q}{\pi R^2 L}$$

(i) Outside the surface of cylinder

The electric flux through the Gaussian surface of radius 'r' at P_1 is

$$\phi_E = \int Eds = \frac{q}{\varepsilon_0}$$

$$\therefore \qquad E\int ds = \frac{q}{\epsilon_0} \quad \text{(or)} \quad E\left(2\pi r \cdot L\right) = \frac{q}{\epsilon_0}$$

$$\therefore \qquad \boxed{E = \frac{1}{2\pi \epsilon_0 \ L} \frac{q}{r}} \quad \text{(or)} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \left[\because \lambda = \frac{q}{L}\right]$$

$$\therefore \qquad E \propto \frac{1}{r}$$

As the distance from the axis of the cylinder increases the electric field intensity decreases and is inversely propotional to the distance 'r'.

(ii) On the surface of the cylinder

On the surface of the cylinder the distance 'r' becomes 'R' and hence electric field intensity at point P_2 is given by

$$\int E ds = \frac{q}{\epsilon_0}$$

$$E\int ds = \frac{q}{\epsilon_0}$$

$$E(2\pi RL) = \frac{q}{\epsilon_0}$$

$$\boxed{E = \frac{q}{2\pi \epsilon_0 LR}} \text{ constant (or) } E = \frac{\lambda}{2\pi\varepsilon_0 R}$$

This expression shows that electric field intensity is constant (or) same at all points on the surface of the cylinder.

(iii) Inside the surface of the cylinder

As shown in fig.(10) if we imagine a cylindrical Gaussian surface of length 'L' and radius r_1 at the point P_3 then electrical charge on Gaussian surface becomes

$$q_1 = \rho\left(\pi r_1^2 L\right)$$

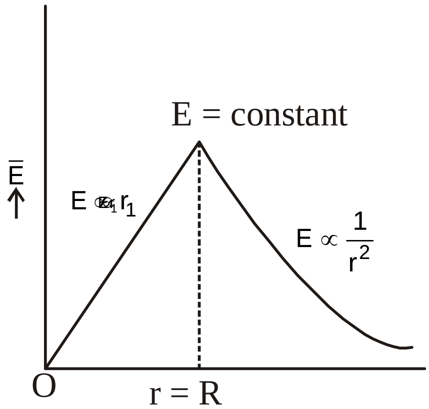
$$\phi_E = \int E ds = \frac{q_1}{\epsilon_0}$$

$$E\int ds = \frac{\rho\left(\pi r_1^2 L\right)}{\epsilon_0}$$

$$E(2\pi r_1 L) = \frac{\rho \pi r_1^2 L}{\epsilon_0}$$

$$\therefore \quad E = \frac{\rho}{(2\epsilon_0)} r_1$$

The variation on of electric field it length with distance from the axis of the cylinder is shown in figure.



From figure we can notice that inside cylinder $(E \propto r_1)$ electric field is directly propotional to r_1, on the surface 'E' is constant. Outside the surface 'E' decreases inversely with 'r'.

1.5 Conservative Nature of Electric Field 'E'

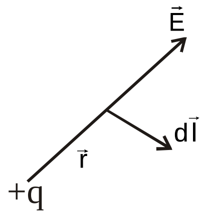
Q11. Show that electric field is conservative in nature.

Ans : **(Imp.)**

The conservative nature of electric field means that the work done to move a charge from one point to another point in electric field is independent of path, but it depends only on the initial and final positions of the charge

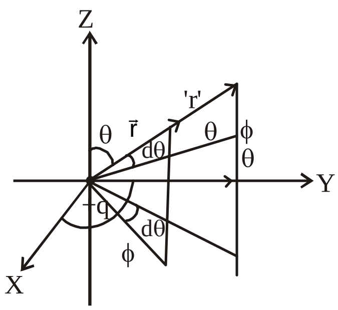
i.e., $\oint \bar{E} \cdot dl = 0$ (or) $\bar{\nabla} \times \bar{E} = 0$

consider a positive charge $+q$ along a curv and dl is small elements then.



According to coulmb's law electric field is given by

$$\vec{E} = \frac{1}{4\pi \in_0} \cdot \frac{q}{r^2} \hat{r}$$

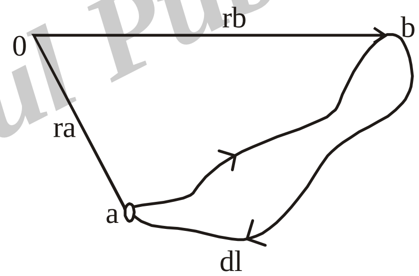


In spherical coordinates

$$d\vec{l} = dr\ \hat{r} + r\ d\theta\ \hat{\theta} + r\sin\theta\ d\phi\ \hat{\phi}$$

$$\vec{E} \cdot d\vec{l} = \frac{1}{4\pi \in_0}\frac{q}{r^2} \cdot dr\left(\hat{r} \cdot \hat{r}\right) + \frac{1}{4\pi \in_0} \cdot \frac{q}{r^2}\hat{r} \cdot rd\theta\ \hat{\theta} + \frac{1}{4\pi \in_0} \cdot \frac{qr}{r^2}\sin\theta\ d\phi\ \hat{\phi} \cdot \hat{r}$$

$$\vec{E} \cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} dr \qquad [\because \hat{r} \cdot \hat{\theta} = 0 \ \ r \perp {'}\theta \text{ and } \hat{\phi} \cdot \hat{r} = 0 \ r \perp {'}r\phi]$$



Consider charge move from point a to b through distance r_a and r_b as shown in fig(13)

$$\therefore \qquad \int_a^b E \cdot d\bar{l} = \frac{q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}dr$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_{r_a}^{r_b}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

where r_a is the distance from origin to point a, r_b is the distance to b.

closed line integral of electric field is zero.

i.e. $$\oint \vec{E}\cdot d\bar{l} = \int_a^b \bar{E}\cdot d\bar{l} + \int_b^a \bar{E}\cdot dl$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right] + \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_a} + \frac{1}{r_b} - \frac{1}{r_a}\right]$$

$$\oint \vec{E}\cdot dl = 0$$

According to stokes theorem

$$\oint E\cdot dl = \oint \nabla\times E\cdot ds = 0$$

$\therefore$ $\boxed{\nabla\times E = 0}$ Thus electric field is conservative in nature.

1.6 Irrotational Field

Q12. What is meant by irrotational field.

Ans :

The electric field is vector field whose curl is zero every where. Thus Electric field is known as irrotational field. i.e., $\nabla\times E = 0$

1.7 Electric Potential

1.7.1 Concept of Electric Potential

Q13. Explain the Concept of Electric Potential.

(OR)

What is meant by electric potential.

Ans : **(Imp.)**

Electric potential (or) potential difference between two points is a scalar quantity. Electric field and Electric potential are intimately related to each other.

Let us consider two points A and B in an Electric field, and imagine a test charge q_0 in between two points. If the charge is moved from A to B against electricfield direction, work has to be done (W_{AB})

$\therefore$ The workdone per unit charge (W_{AB}/q_0) can be considered as potential difference.

i.e. $$V_B - V_A = \frac{W_{AB}}{q_0}$$

If a point 'A' is at infinite distance then $V_A = 0$

$$\therefore \quad V_B = \frac{W_{AB}}{q_0}$$

There fore, the workdone to move a unit electric charge an infinite distance to a point against electric field is called Electric potential at that point.

Potential difference is measured in volts

1 volt = 1Joule/1 coulomb (in SI unit)

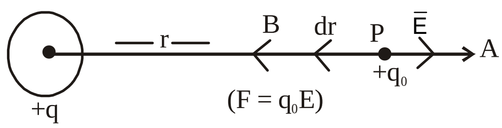
Definition :

"If the workdone to move an electric charge of one coulomb between two points against the direction of electric field is 1 Joule then potential difference between two points is equal to one volt.

Q14. Obtain the expression for potential due to point charge ?

Ans :

Consider a point charge +q as shown in figure.



The aim of this article is to calculate potential at point B situated at a distance r from the charge +q. For this purpose we select two points A and B along radial line. Let a test charge q_0 be moved from A to B.

The force exerted by charge q on test charge q_0 is q_0 E.

Now to move the test charge q_0 towards 'B' a force $-q_0E$ must be applied.

The work done by external agent to move charge q_0 through small distance dr is given by

$$dw = q_0 E.dr = q_0 E\, dr \cos 180^0$$

$$= -q_0 Edr$$

Further $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

$$\therefore \quad dw = \frac{-1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\, dr$$

The total work done in moving test charge from A to B

$$W_{AB} = \int_{r_A}^{r_B} -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}dr$$

$$= \frac{-qq_0}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)_{r_A}^{r_B} = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$\therefore$ Potential difference between two points will be

$$V_B - V_A = \frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

If the reference point 'A' is taken at infinity so that $V_A = 0$

$$\therefore V_B = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r_B}$$

on dropping suffix $\boxed{V = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}}$

1.7.2 Relation between Electric potential and Electric field

Q15. Express the relation between electricpotential and electricfield?

Ans :

Let us Consider conductor AB of length 'd' be placed in Electric field intensity as shown in fig.(a)

The work done (W_{AB}) in moving test charge (q_0) from A to B by applying a force $\left(\overline{F} = -q_0\overline{E}\right)$ opposite to the direction of Electric field $\vec{E}$.

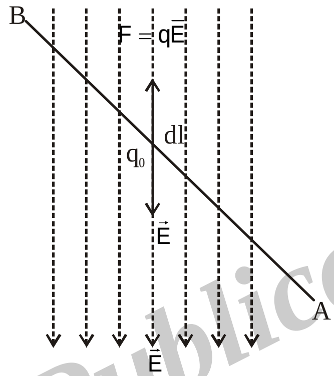


Figure (a)

Let dw be the workdone in moving test charge q_0 through small displacement dl

$$dw = \overline{F}.\overline{dl} = (-q_0E) . \overline{dl} = -q_0E\, dl \cos 180$$

$$= q_0\, E\, dl \text{ ....(1)}$$

The total work done W_{AB} to move the test charge q_0 from A to B is obtained by integrating the equation (1)

$$W_{AB} = \int_A^B q_0 E dl = q_0 E \int_A^B dl = q_0 E.d \text{ .... (2)}$$

But $\dfrac{W_{AB}}{q_0} = V$ hence $\dfrac{W_{AB}}{q_0} = V = E.d$

(or) $\boxed{E = \dfrac{V}{d}}$

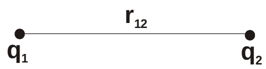
1.7.3 Potential Energy of System of Charges

Q16. Derive the expression for potential energy of system of charges?

Ans :

Consider two point charges q_1 & q_2 seperated by distance 'r' then potential energy of two charges seperated by distance is given by

$$U_2 = K\frac{q_1 q_2}{r_{12}}$$



The energy may be pasitive (or) negative depending upon the sign of charges.

Now consider 4 point charges q_1, q_2, q_3, q_4 as shown in fig.(1) seperated by different distances .

Then the potential energy of the system of 4 charges is given by

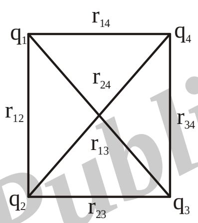


Figure (1)

$$U_4 = K\frac{q_1 q_2}{r_{12}} + K\frac{q_1 q_3}{r_{13}} + K\frac{q_1 q_4}{r_{14}} + K\frac{q_2 q_3}{r_{23}} + K\frac{q_2 q_4}{r_{24}} + K\frac{q_3 q_4}{r_{34}}$$

$\therefore$ The total potential energy of 'n' number of system of charges can be written as

$$\boxed{U = \sum_{i=1}^{n}\sum_{j=1}^{n} K\frac{q_i q_j}{r_{ij}}} \quad \text{(where } i < j\text{)}$$

1.7.4 Energy Density in an Electric Field

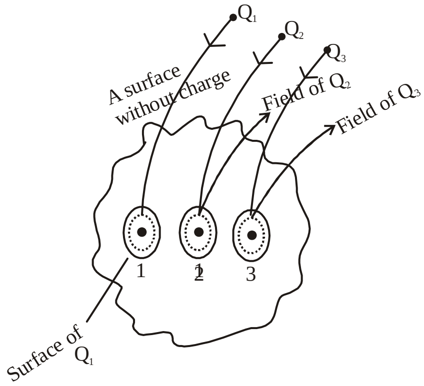
Q17. Explain about energy density in electrostatic field?

Ans : **(Imp.)**

Let us calculate the potential energy of system consisting 'n' number of charges.

First of all we shall find potential energy of system due to three charges q_1, q_2, q_3 and then generalize it for 'n' charges.

For this purpose, we have to calculate the workdone by external source in positioning these charges on a surface as shown as in fig.



The surface is assumed to be without charge let us bring the charge q_1 from infinity to position 1, no work is done because the charge does not exist on the system. So, $w_1 = 0$.

Now the charge q_2 taken from infinity to position 2.

The moment of the charge q_2 is taking place in field of the charge q_2 . Then workdone is given by W_2 = charge × potential

$= q_2 \times v_{21}$ where v_{21} is potential at q_2 due to q_1

Similarly in bring charge q_3 from infinity to position 3, the movement will be in the field of charges q_1 and q_2. Then workdone is given by

$W_3 = q_3 v_{31} + q_3 v_{32}$ where v_{31} and v_{32} are potential at position 3 due to charges q_1 and q_2 respectively.

Now, total workdone

$$W_e = W_1 + W_2 + W_3$$

$$W = 0 + q_2 v_{21} + q_3 v_{31} + q_3 v_{32} \qquad .....(1)$$

Now let us consider the reverse order i.e. charge q_3 is taken first and q_2 and finally charge q_1. Then the workdone is given by

$$W = 0 + q_2 v_{23} + (q_1 v_{13} + q_1 v_{12}) \qquad .....(2)$$

Adding equations (1) + (2)

$$2W = q_1 v_{12} + q_1 v_{13} + q_2 v_{21} + q_2 v_{23} + q_3 v_{31} + q_3 v_{32}$$

$$2W = q_1 (v_{12} + v_{13}) + q_2 (v_{21} + v_{23}) + q_3 (v_{31} + v_{32})$$

$$2W = q_1v_1 + q_2v_2 + q_3v_3$$

where v_1, v_2, v_3 are total potential at charges q_1, q_2, q_3

Ingeneral $W = \frac{1}{2}\left[q_1v_1 + q_2v_3 + q_3v_3\right]$

$$\boxed{W = \frac{1}{2}\sum_{i=1}^{n} q_i v_i} \quad .....(3)$$

We know that workdone for spherical charge distribution in integral form is given by

$$W = \frac{1}{2}\iiint_v v dv$$

According to Gauss law $\nabla \cdot D = \rho$

$$\therefore \quad W = \frac{1}{2}\iiint_v (\nabla \cdot D) v dv \quad .....(4)$$

where 'v' is scalar D is a vector field. To obtain value of volume integral we use the following vector indentity

$$\nabla \cdot (VD) = V(\nabla \cdot D) + D \cdot (\nabla V) \quad .....(5)$$

$$V(\nabla \cdot D) = \nabla \cdot (VD) - D \cdot \nabla V$$

Substituting this value in equation (4)

$$W = \frac{1}{2}\left[\iiint_v (\nabla \cdot VD) dv - \iiint_v D \cdot (\nabla V) dv\right]$$

Using Gauss divergence theorem the first volume integral of this equation can be changed in to closed surface integral.

i.e. $W = \frac{1}{2}\oint_v VDds - D \cdot (\nabla V) dv$

The surface integral over this closed surface is zero.

$\therefore \qquad W = \frac{-1}{2}\iiint_V D\cdot(\nabla V)\,dv$

$= \frac{-1}{2}\iiint (D\cdot -E)\,dv \qquad [\because -\nabla V = E]$

$= \frac{1}{2}\iiint_V (D.E)\,dv$

$= \frac{1}{2}\iiint_V (\varepsilon_0 E\cdot E)\,dv \qquad (\because D = \varepsilon_0 E)$

$\therefore \qquad W = \frac{1}{2}\iiint_V \varepsilon_0 E^2 dv$

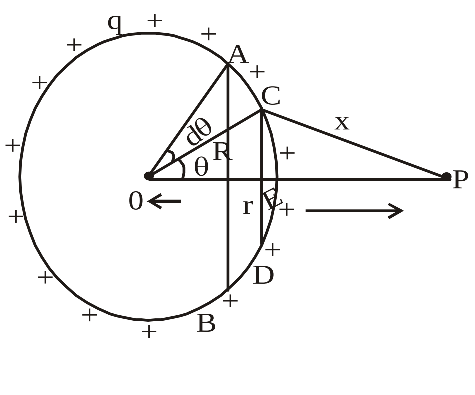
This is known as energy density for an electric field 'E'.

1.7.5 Calculation of Potential from Electric Field for a Spherical Charge Distribution

Q18. Derive the Expression for Electric field from potential due to charged Spherical conductor.

Ans :

The figure shows an electrically charged sphere of radius 'R' with centre '0' and +q is uniformly spread on its surface.



Let us find out the electric potential due to charged spherical conductor at different points.

1. When point 'P' lies outside the sphere
2. When point 'P' lies on the surface
3. When point 'P' lies insides the shell

1. when point 'p' lies outside the sphere

Let 'σ' be the surface charge density. Now we have to calculate the potential at an external point 'p' at distant 'r' from the centre of spherical conductor. For this purpose we divide the sphere into a number of rings with centres on OP. Further consider one such ring ABCD between two planes AB and CD. Let CP = x, $\angle COP = \theta$ and $\angle AOC = d\theta$

From right angled triangle OEC,

CE = OC $\sin\theta$ = R $\sin\theta$

From sector AOC, AC = R $d\theta$

The circumference of the ring $= 2\pi \times (R\sin\theta)$

Area of the ring $= 2\pi R\sin\theta \times Rd\theta$

$= 2\pi R^2 \sin\theta d\theta$

charge on the ring = Area of ring × surface density

$= 2\pi R^2 \sin\theta d\theta \times \sigma$

Where $\sigma = \dfrac{\text{total charge on shell}}{\text{total surface area}}$

$= \dfrac{q}{4\pi R^2}$

$\therefore$ charge on the ring dq = $2\pi R^2 \sin\theta d\theta \times \dfrac{q}{4\pi R^2}$

$$dq = \frac{q\sin\theta d\theta}{2} \qquad .....(1)$$

So potential at 'p' due to the charge on the ring $dv = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{x}$

$$dv = \frac{1}{4\pi\varepsilon_0 x} \cdot \frac{q \sin\theta d\theta}{2} \quad [\therefore \text{ From eq (1)}]$$

$$dv = \frac{q \sin\theta d\theta}{8\pi\varepsilon_0 x} \qquad \text{...... (2)}$$

From figure $x^2 = R^2 + r^2 - 2Rr\cos\theta$

Differentiating this equation, we get

$2x\,dx = 2Rr \sin\theta\, d\theta$

$$\sin\theta d\theta = \frac{xdx}{Rr} \qquad \text{.....(3)}$$

Substituting the value of $\sin\theta d\theta$ from eq (3) in eq (2) we have

$$dv = \frac{qxdx}{8\pi\varepsilon_0 Rrx} = \frac{qdx}{8\pi\varepsilon_0 Rr}$$

In order to obtain the potential due to whole spherical shell we integrate the above equation within the limits $x = r - R$ and $x = r + R$, Hence,

$$V = \int_{r-R}^{r+R} dv = \int_{r-R}^{r+R} \frac{qdx}{8\pi\varepsilon_0 Rr}$$

$$= \frac{q}{8\pi\varepsilon_0 Rr}[x]_{r-R}^{r+R} = \frac{q}{8\pi\varepsilon_0 Rr}[r + R - r + R]$$

$$= \frac{q}{8\pi\varepsilon_0 Rr} \times 2R$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}} \qquad \text{.....(4)}$$

It is clear that the potential decreases with distance in inverse proportion.

2. When point 'p' lies on the surface

In this case r = R

Potential at the surface = $\frac{1}{4\pi\varepsilon_0}.\frac{q}{R}$ ......(5)

3. When point 'p' lies inside the shell

In this case, the limits of integration becomes x = R–r and x = R + r, Hence,

$$V = \int_{R-r}^{R+r} \frac{q\,dx}{8\pi\varepsilon_0 Rr} = \frac{q}{8\pi\varepsilon_0 Rr}\int_{R-r}^{R+r} dx$$

$$= \frac{q}{8\pi\varepsilon_0 Rr}[R + r - R + r] = \frac{q}{8\pi\varepsilon_0 Rr}\times 2r$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R}} \quad .... (6)$$

Thus, the potential at an internal point is same as that on the surface.

The fig (19) shows variation of potential due to charged sphere with distance from centre

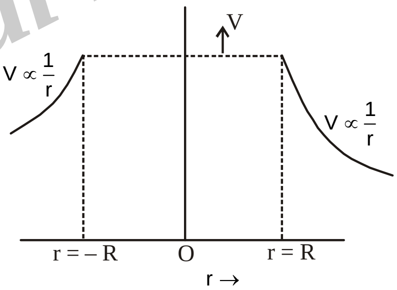


Fig (19)

From graph it is clear that between r = –R and r = + R the potential is maximum and constant then it varies inversly with distance.

Intensity of Electric field :

1. outside the charged sphere

From eq (4) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$$E = -\frac{\partial v}{\partial r} = \frac{-\partial}{\partial r}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right) = \frac{-q}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r}\right)$$

$$= \frac{-q}{4\pi\varepsilon_0}.\left(\frac{-1}{r^2}\right)$$

$$\therefore \quad \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

2. On the surface of charged sphere

From eq (5) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ = constant

$$E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R^2}$$

3. Inside the charged sphere

From eq (6)

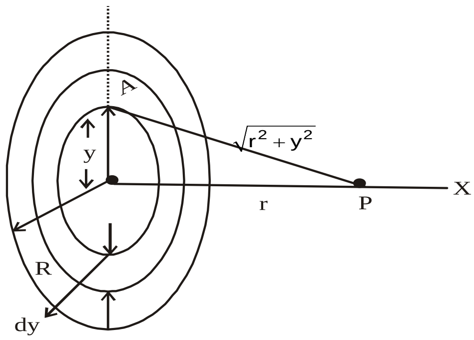
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad \therefore \quad E = \frac{-\partial v}{\partial r} = 0$$

Thus Electric field intensity is zero from inside upto the centre, maximum on the surface and as the distance increases from the surface of the sphere the intensity 'E' decreases inversly with square of distance.

Q19. Derive Expression for potential & Electric field due to Uniformly Circular disc.

Ans : **(Imp.)**

Consider a uniformly charged circular disc with charge 'q' as shown in fig (20) Let 'σ' be the surface charge density. Here the aim is to calculate the potential 'V' at any point 'P' on the axis of disc at a distance 'r' from the centre O.



For this purpose we divide the disc into large number of flat circular strips. Consider one such strip of radius y and width dy. As the width of the strip is very small, each point of this trip can be assumed to be at equal distance AP = $\sqrt{r^2+y^2}$ from point 'P'.

The charge contained by the strip

$$dq = \sigma \times \text{Area of strip}$$

$$= \sigma . (2\pi y dy)$$

Potential at 'P' due to charge Element will be

$$dv = \frac{1}{4\pi\varepsilon_0}.\frac{dq}{AP} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi y dy}{\sqrt{r^2+y^2}} \quad ....(1)$$

The potential due to whole disc can be obtained by integrating eq (1) within the limits O to R

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{ydy}{\sqrt{r^2+y^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\int_0^R (r^2+y^2)^{-1/2}.ydy$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+r^2}-r\right] \quad ....(2)$$

At the centre of disc r = 0 hence eq (2) becomes

$$V_0 = \frac{\sigma R}{2\varepsilon_0} \quad ....(3)$$

In case when $r \geq R$ the quantity $\sqrt{r^2+R^2}$ can be approximated by Binomial theorem

$$\sqrt{r^2 + R^2} = r\left(1 + \frac{R^2}{r^2}\right)^{1/2}$$

$$= r\left(1 + \frac{R^2}{2r^2} + ...\right)$$

$$= r + \frac{R^2}{2r}$$

$$\therefore \qquad V = \frac{\sigma}{2\varepsilon_0}\left[r + \frac{R^2}{2r} - r\right]$$

$$= \frac{\sigma R^2}{4\varepsilon_0 r} = \frac{\pi R^2 \sigma}{4\pi\varepsilon_0 r}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}} \quad \left[\because q = \sigma \times \text{area} = \sigma \times \pi R^2\right]$$

Intensity of Electric field :

The Electric field intensity 'E' at any point 'p' at distance 'r' from the axis of disc is equal to negative gradient of potential.

$$E = \frac{-\partial v}{\partial r}$$

$$\text{From eq (2) } E = \frac{-d}{dr}\left[\frac{\sigma}{2\varepsilon_0}\left(R^2 + r^2\right)^{1/2} - r\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\frac{1}{2}\left(R^2 + r^2\right)^{1/2}.(2r) - 1\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{r}{\left(R^2 + r^2\right)^{1/2}}\right]$$

At the centre of disc $r = 0$

$$\boxed{E_0 = \frac{\sigma}{2\varepsilon_0}}$$

Problems

1. If a point charge 'q' is placed at the centre of cube what is the flux linked (a) with the cube (b) with each face of the cube ?

Sol:

a) According to Gauss law, the flux linked with a close body is $\left(1/\varepsilon_0 \right)$ times the charge enclosed. Here the charge 'q' is enclosed within the cube, hence flux

$$\phi = 1/\varepsilon_0 \times q = \frac{q}{\varepsilon_0}$$

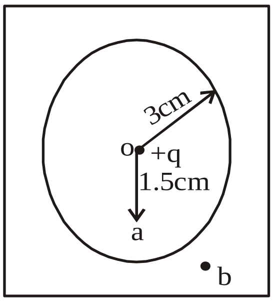
b) The cube is symmetrical body with six faces, so flux linked with each face

$$\phi = \frac{1}{6} \times \frac{1}{\varepsilon_0} \times q = \frac{q}{6\varepsilon_0}$$

2. A point charge q = 2 × 10^{-7} coulomb is placed at centre of spherical cavity of radius 3.0 cm in metal piece . Find Electric intensities at a + b.

Sol :

(i) Electric intensity at a point 'a' can be obtained by considering Gaussian surface of radius 1.5 cm



This surface encloses charge q.

Hence $E \times 4\pi(0.015)^2 = \frac{q}{\varepsilon_0}$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(0.015)^2}$$

$$= 9 \times 10^9 \times \frac{2.0 \times 10^{-7}}{(0.15)^2}$$

$= 8 \times 10^5$ newton/coulomb

(ii) WKT the charge resides on the outer surface of the conductor. Hence 'E' at point 'b' will be zero

3. The charge on the spherical conductor is 3×10^{-9} C. Radius of conductor is 0.1 m. Find the potential of spherical conductor.

Sol :

Given $q = 3 \times 10^{-9}$ C

$r = 0.1$ m

The potential of spherical conductor

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r}$$

$$= 9 \times 10^9 \times \frac{3 \times 10^{-9}}{0.1}$$

= 270 volt.

4. What is Electripotential at the surface of nucleus of gold? The radius of nucleus is 6.6×10^{-15} m. The atomic number of gold is 79.

Sol :

Given charge in the nucleus = 79 e

$7.9 \times 1.6 \times 10^{-19}$ C

Radius of nucleus $= 6.6 \times 10^{-15}$ m

Considering nucleus to be spherically symmetrical the potential on the surface

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r}$$

$$V = 9 \times 10^{9} \times \frac{79 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-15}}$$

$$= 17 \times 10^{6} \text{ volt}$$

5. At a distance of 5 cm and 10 cm from the surface of sphere, the potentials are 600V and 420V. Find the potential of its surface.

Sol :

Given $V_1 = 600V$

$V_2 = 420\ V$

Let 'r' be the radius of sphere. Then

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r} \qquad .....(1)$$

Given that $V_1 = 600 = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r+0.05)}$ ..... (2)

$$V_2 = 420 = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r+0.1)} \qquad ..... (3)$$

Dividing eq (2) by (3) , we get

$$\frac{r+0.1}{r+0.05} = \frac{600}{420}$$

(or) $r = \frac{0.2}{3}$ m

Dividing eq (1) by eq (2) we get

$$\frac{V}{600} = \frac{r+0.05}{r} = \frac{0.2/3 + 0.05}{0.2/3}$$

$\therefore$ $V = 1050$ volt

6. Two spheres of radius 6 cm have charges 10^{-8} coulomb and -3×10^{-8} coulomb. Find the potential of two spheres and potential at the midpoint 0 centres if the distance between the centres is 2 metre.

Sol :

Given radius of sphere = 6 cm = 6 × 10^{-2} metre

Potential of sphere with charge 10^{-8}C

$$=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}=9\times10^9\times\frac{10^{-8}}{6\times10^{-2}}$$

$$=1.5\times10^3\,\text{volt}$$

Potential of sphere with charge 3×10^{-8} Coulomb

$$=\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q}{r}\right)=9\times109\times\frac{3\times10^{-8}}{6\times10^{-2}}$$

$$=-4.5\times10^3\,\text{volt}$$

Potential at the midpoint of line of centre

$$=\frac{1}{4\pi\varepsilon_0}\left[\frac{10^{-8}}{1}-\frac{3\times10^{-8}}{1}\right]$$

$$=9\times10^9\left(-2\times10^{-8}\right)$$

$$=-180\,\text{volt}$$

7. The potential difference between two parallel metal sheets sepereted by a distance of 0.2 m is 800V. If a positive charge of 5×10^{-9}C is kept in between sheets. What is force acting on it.

Sol :

Given

dv = 8000 v, dx = 0.2 m, q = 5×10^{-9}c

$$E=-\frac{dv}{dx}=\frac{8000}{0.2}=40,000\,\text{vm}^{-1}$$

WKT $F = qE$

$$F = 5 \times 10^{-9} \times 40,000 = 2 \times 10^{-4}\,N$$

$\therefore$ $F = 2 \times 10^{-4}\,N$

8. Two large metal plates of area 1.0 m^2 face each other are seperated by 5 cm and carry equal and opposite charges on their inner surface. If 'E' between the plates 55 N/C, find the charge on plates.

S0l :

Electric filed intensity of two metal plates separated by a distance is given by

$$E = E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$$

We know that $\sigma = \frac{q}{A}$

$$E = \frac{q}{A\varepsilon_0}$$

Substituting the given values

$$55 = \frac{q}{1 \times 8.85 \times 10^{-12}}$$

$q = 4.88 \times 10^{-10}$ coulomb.

Short Question and Answers

1. Derive the differential form of Gauss law.

Ans :

Differential form of Gauss Law

According to Gauss law

$$\oint E.ds = \frac{Q}{\varepsilon_0}$$

$$\varepsilon_0 \oint E.ds = Q \quad .....(1)$$

If 'Q' is the charge distributed over a volume 'V' and ρ be the density of charge. Then

$$Q = \iiint \rho dv \quad ......(2)$$

From eq (1)

$$\varepsilon_0 \oint E.ds = \iiint \rho dv \quad .....(3)$$

According to Gauss divergence theorem.

$$\oint E.ds = \iiint \text{div } E dv \quad .....(4)$$

Sub (4) in (3), we get

$$\varepsilon_0 \iiint \text{div } E \ dv = \iiint \rho dv \quad .....(5)$$

Eq (5) is true for any arbitary volume, Hence integrands must be equal

$$\varepsilon_0 \text{div } E = \rho \ \text{(or)} \ \boxed{\text{div } E = \frac{\rho}{\varepsilon_0}}$$

We know that $D = \varepsilon_0 E$ (or) $E = \dfrac{D}{\varepsilon_0}$

$$\therefore \text{div } E = \nabla.E = \frac{1}{\varepsilon_0} (\nabla.D) = \frac{\rho}{\varepsilon_0}$$

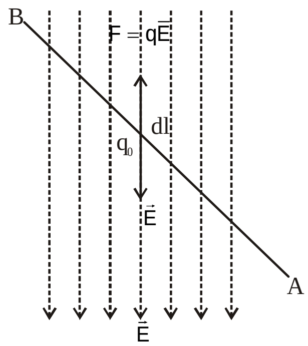
$$\boxed{\nabla.D = \rho}$$

which represents differential form of Gauss law.

2. Derive the relationship between electric field and electric potential.

Ans :

The work done (W_{AB}) in moving test charge (q_0) from A to B by applying a force $\left(\overline{F} = -q_0\overline{E}\right)$ opposite to the direction of Electric field $\vec{E}$.



Let dw be the workdone in moving test charge q_0 through small displacement dl

$$dw = \overline{F}.\overline{dl} = (-q_0E)\ .\ \overline{dl} = -\ q_0E\ dl\cos 180$$

$$= q_0\ E\ dl \ ....(1)$$

The total work done W_{AB} to move the test charge q_0 from A to B is obtained by integrating the equation (1)

$$W_{AB} = \int_A^B q_0 Edl = q_0 E\int_A^B dl = q_0 E.d \ .... (2)$$

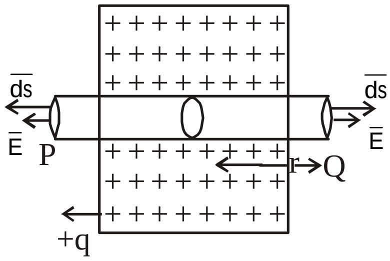
But $\dfrac{W_{AB}}{q_0} = V$ hence $\dfrac{W_{AB}}{q_0} = V = E.d$

(or) $\boxed{E = \dfrac{V}{d}}$

3. Obtain the expression for electric field due to infinite sheet of charge.

Ans :

Consider a thin conducting, infinite sheet charge either sides with $+q$ charge. Let σ be the surface charge density.



Let 'p' be a point at a distance' 'r' from the sheet at a distance 'r' from the sheet at which Electric field 'E' is to calculated. In order to calculate field, consider another point 'Q' Symmetrical to 'P' on the other side of sheet and imagine a cylindrical Gaussian surface as shown in figure.

Let 'A' be the cross sectional area of the Gaussian Surface. By symmetry the Electric field 'E' is every where perpendicular to the plane. For curve surface electric flux is zero because $\bar{E}$ and $\overline{ds}$ are perpendicular to each other. for end planes of Gaussian surface $\bar{E}$ and $\overline{ds}$ are parallel to each other hence they contribute to electric flux.

If ϕp and ϕ_Q represent electric flux through $\overline{ds}$ at points P and Q then the total Electric flux is

$$\phi_E = \phi_P + \phi_Q$$

$$= E\, ds \cos 0^o + E\, ds \cos 0^o$$

$(\because \theta = 0$ between E and ds due to plane sheet)

$$\phi_E = 2Eds \text{ ....(1)}$$

But from Gauss law $\phi_E = \dfrac{q}{\varepsilon_0} = \dfrac{\sigma ds}{\varepsilon_0}$ ....(2)

From equations (1) & (2)

$$2Eds = \frac{\sigma ds}{\varepsilon_0}$$

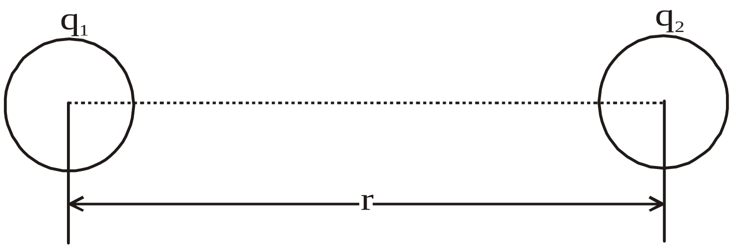
$$E = \frac{\sigma}{2\varepsilon_0}$$

4. Define coulomb's law ?

Ans :

Coulomb's law states that the force of attraction (or) repulsion between two stationary point charges is directly propotional to the product of the two charges and inversly propotional to the square of the distance between them.

If q_1 and q_2 be the two point charges separated from each other by a distance r



then force 'F' acting between them is given by

$$F \propto q_1 q_2 \text{ and } F \propto \frac{1}{r^2}$$

$$\therefore \quad F \propto \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

where $\frac{1}{4\pi\varepsilon_0}$ propotionality constant.

5. Define Electric field and Intensity of electric field.

Ans :

Electric Filed

The region surrounding an electric charge or a group of charges in which another charge experiences a force is called electric field.

Intensity of electric field

The intensity of electricfield at a point in the field is defined as the force experienced by a unit positive charge placed at that point. Let 'F' be the force experienced by a test charge q_0 placed at a point in the electric field then intensity of electric field 'E' at that point is given by

$E = \frac{F}{q_0} = \frac{\text{Newton}}{\text{Coulomb}}$ is a vector quantity

But from Coulomb's law

The force on unit positive charge is given by

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{Q \times 1}{r^2}$$

$$E = \frac{F}{1} = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$$

6. Write about Electric lines force?

Ans :

An electric line of force is that imaginary smooth curve drawn in an electric field along which a free isolated positive charge will move. The tangent at any point on it gives the direction of the field at that point.

Thus intensity of electric field at point can also defined as number of lines of force passing through unit area round that point normally.

1. The tangent drawn at any point on the lines of force gives the direction of electric field at that point.
2. The electric line of force start from positive charge and end on a negative charge.
3. No two lines of force intersect each other.
4. The electric lines of force are perpendicular to equipotential surfaces.
5. The number of lines of forces crossing unit area in normal direction is propotional to electric field intensity.

7. Explain about Electric flux.

Ans :

The electric flux through a surface placed inside electric field represents the total number of electric lines of force crossing the surface in a direction normal to the surface.

The scalar product $E.\Delta S$ is defined as the electric flux for the surface. The total flux ϕ_E through the entire surface is given by

$$\boxed{\phi_E = \oint E \cdot \Delta S = E \cdot S}$$

If 'θ' is the angle between E and ΔS then the scalar product is given by

$$E \cdot \Delta S = E \cdot ds \cos\theta$$

Electric flux $\phi_E = \oint E \cdot ds \cos\theta$

$$\phi_E = E\cos\theta \oint ds$$

$$\phi_E = E\cos\theta A \ (\because \oint ds = A = \text{area of surface})$$

$$\boxed{\phi_E = EA\cos\theta}$$

8. What is meant by irrotational field.

Ans :

The electric field is vector field whose curl is zero every where. Thus Electric field is known as irrotational field. i.e., $\nabla \times E = 0$

9. Explain the Concept of Electric Potential.

Ans :

Electric potential (or) potential difference between two points is a scalar quantity. Electric field and Electric potential are intimately related to each other.

Let us consider two points A and B in an Electric field, and imagine a test charge q_0 in between two points. If the charge is moved from A to B against electricfield direction, work has to be done (W_{AB})

$\therefore$ The workdone per unit charge (W_{AB}/q_0) can be considered as potential difference.

i.e. $$V_B - V_A = \frac{W_{AB}}{q_0}$$

If a point 'A' is at infinite distance then $V_A = 0$

$$\therefore \quad V_B = \frac{W_{AB}}{q_0}$$

There fore, the workdone to move a unit electric charge an infinite distance to a point against electric field is called Electric potential at that point.

Choose the Correct Answer

1. Expression for intensity of electric field inversly propational to [b]

   (a) r (b) r^2

   (c) r^3 (d) r^4

2. Expression for electric flux [a]

   (a) $\phi_E = \int E \cdot \Delta S$ (b) $\phi_E = \int B \cdot \Delta S$

   (c) $\phi_B = \int E \cdot \Delta S$ (d) None

3. Differential form of Gauss law [c]

   (a) $\nabla \cdot E = \frac{1}{\varepsilon_0}$ (b) $\nabla \cdot D = \frac{\rho}{\varepsilon_0}$

   (c) $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ (d) None

4. Intensity of electric field for infinite sheet of charge directly propational to [d]

   (a) λ (b) ρ

   (c) a & b (c) σ

5. Values of electric field intensity for charged cylinder on the surface [b]

   (a) $E = \frac{qL}{2\pi\varepsilon_0 R}$ (b) $E = \frac{q}{2\pi\varepsilon_0 LR}$

   (c) $E = \frac{qR}{2\pi\varepsilon_0 L}$ (d) None

6. The value of curl of electric field [c]

   (a) $\nabla \times E = \rho$ (b) $\nabla \times E = \frac{\rho}{\varepsilon_0}$

   (c) $\nabla \times E = 0$ (d) None

7. Potential energy of system of charges is given by [b]

(a) $U = \sum_{\substack{i=1 \\ (i>j)}}^{n} \sum_{j=1}^{n} \frac{kq_1 q_j}{r_{ij}}$ (b) $U = \sum_{\substack{i=1 \\ (i<j)}}^{n} \sum_{j=1}^{n} \frac{kq_1 q_j}{r_{ij}}$

(c) a and b (d) None

8. Intensity of electric field of circular disc At centre [a]

(a) 0 (b) $\frac{\sigma}{\varepsilon_0}$

(c) $\frac{\sigma}{2\varepsilon_0}$ (d) σ

9. The value of $\frac{1}{4\pi\varepsilon_0}$ [d]

(a) 9×10^7 (b) 9×10^5

(c) 9×10^4 (d) 9×10^9

10. Charge per length 'λ' is known as [a]

(a) Linear density (b) Surface charge density

(c) Volume density (d) None

Fill in the blanks

1. The total number of electric lines of force crossing the surface a direction normal to surface is known as _______ .
2. According to coulomb's law the force acting between two charges directly propational to _______ charges.
3. For positive charge the lines of force move towards _______ .
4. In Gauss law of charge is outside the surface the flux becomes _______ .
5. The Electric field for infinite sheet of charge if the charge present only on one side is E _______ .
6. Expression for energy density in Electric field = _______ .
7. If curl of Electric field is zero then 'Electric field E is _______ in nature.
8. Relation between Electric field and potential E = _______ .
9. Expression for potential of spherical conductor on the surface V = _______ .
10. Potential difference is measured in _______ .

Answers

1. Electric flux
2. Product
3. Infinity
4. Zero
5. $\frac{\sigma}{\varepsilon_0}$
6. $\frac{1}{2}\varepsilon_0 E^2$
7. Conservative
8. V/d
9. $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$
10. Volts

One Mark Answers

1. Define intensity Electric field .

Ans :

The intensity of Electric field at a point in the field is defined as the force experienced by a unit positive charge placed at that point.

2. Define Electric potential.

Ans :

Electric potential at a point in Electric field is defined as the workdone by an external agent in carrying a unit positive test charge from infinity to that point against the electric force of the field.

3. Define irrotational field.

Ans :

The Electric field is a vector field whose curl is zero every where then such a field is known as irrational field.

4. Define Gauss law.

Ans :

Gauss law stales that total normal Electric flux ϕ_E over a closed surface is $\left(1/\varepsilon_0\right)$ times the total charge Q Enclosed with in the surface.

Mathematically $\phi_E = \int E.ds = \left(1/\varepsilon_0\right)Q$

5. What is meant by energy density.

Ans :

Amount of energy stored in a given system (or) region of space per unit volume.

UNIT II	**Magnetostatics** Concept of magnetic field 'B' and magnetic flux, Biot-Savart's law, B due to a straight current carrying conductor. Force on a point charge in a magnetic field. Properties of B, curl and divergence of B, solenoidal field. Integral form of Ampere's law, Applications of Ampere's law: field due to straight, circular and solenoidal currents. Energy stored in magnetic field. Magnetic energy in terms of current and inductance. Magnetic force between two current carrying conductors. Magnetic field intensity. Ballistic Galvanometer:- Torque on a current loop in a uniform magnetic field, working principle of B.G., current and charge sensitivity, electromagnetic damping, critical damping resistance.

2.1 Concept of Magnetic Field 'B' and Magnetic Flux

Q1. Define magnetic field (B), magnetic flux?

Ans :

i) Magnetic Field

The intensity of magnetic field induction 'B' at a given point is equal to force acting on a test charge moving with velocity perpendicular to the direction of magneticfield induction.

It is given by $B = \frac{F_B}{q_0 V}$

where F_B is the magnetic force

q_0 is test charge

V is the velocity

Unit *I*: $B = \frac{1 \text{ Newton}}{1 \text{ Coulmb} \times 1\text{m / sec}} = \frac{\text{Newton}}{\text{Ampere} \cdot \text{metre}}$

$\therefore$ 1 webre / mt^2 = 1 newton / amp. metre

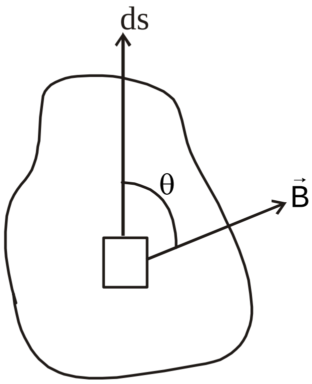
Line of magnetic induction

The path followed by a unit north pole when moved freely in a magnetic field created by magnet.

- The tangent to the line of induction gives the direction of 'B' at that point.
- The magnetic lines of force are always pointed from north pole to south pole.

ii) Magnetic flux

The magnetic flux denoted by ϕ_B can be defined as the total number of lines of induction cutting through a surface is called magnetic flux through that surface.



Consider an element of surface area represented by a vector ds in suppose it makes an 'θ' angle with magnetic induction 'B' on that area. Then scalar product B.ds will represent i.e. magnetic flux fϕ_B over ds. Thus,

$$d\phi_B = B.ds$$

Now, the magntic flux over the entire surface will be $\phi_B = \oint B \cdot ds$

If B is uniform over the entire area, then

$$\phi_B = B\oint ds = B.S$$

The unit of magnetic flux = webre / metre^2 × metre^2 = weber

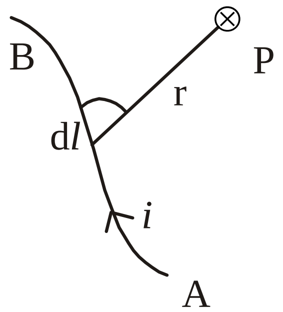
2.1.1 Biot-Savart's Law

Q2. State and explain Biot - Savart law.

Ans : **(Imp.)**

According to oersted's experiment, current carrying conductor produces a magnetic field around it. In 1820 Biot and savart performed a series of experiments to study the magnetic field produced by various current carrying conductors. They obtained a relation by means of which 'B' can be calculated at any point of space in which a current is passing. The relation is called as Biot and Savart law.

As shown fig let AB be a conductor of any arbitrary shape in which current 'i' is flowing. let P be a point at which filed is to be determined.



To evaluate 'B' at a point 'P' at a distance 'r' from point '0' divide the conductor of length '*l*' into large number of small segments of lengths $\overrightarrow{dl}$.

The field at point 'P' due to flow of current 'i' through 'dl' be $\overrightarrow{dB}$.

'θ' is the angle between the vectors $\overrightarrow{dl}$ and $\vec{r}$

According to to Biot-Savart law the magnitude of $\overrightarrow{dB}$ is

(i) directly propotional to strength of current 'i' and length of the element segment dl i.e. $\overline{dB} \propto i$ and $dB \propto dl$

(ii) inversely propotional to square of the distance between P and dl i.e.

$$d\bar{B} \propto \frac{1}{r^2}$$

(iii) directly propotional to angle 'θ' between the vector $\overline{dl}$ and $\bar{r}$

i.e. $$\overline{dB} \propto \sin\theta$$

From the above points

$$dB \propto \frac{idl\sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2} \qquad .....(1)$$

where $\frac{\mu_0}{4\pi}$ is propotionality constant and μ_0 is permeability of free space.

In vector form $\overline{dB} = \frac{\mu_0}{4\pi}\frac{i(d\overline{l} \times \overline{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{i(\overline{dl} \times \hat{r})}{r^2}$

$\frac{\mu_0}{4\pi} = 10^{-7}$ web / amp. metre .....(2)

where $\hat{r}$ is unit vector along position vector at 'P'

The resultant field at 'P' can be obtained by integrating equation .....(3)

$$\therefore \quad \overline{B} = \int d\overline{B} = \frac{\mu_0 i}{4\pi}\int \frac{\overline{dl} \times \hat{r}}{r^2}$$

2.2 Magnetic Field Due to Straight Current Carrying Conductor

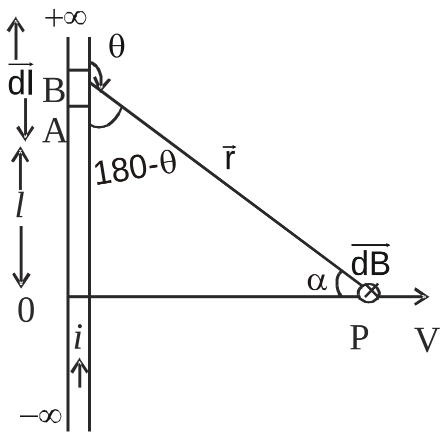
Q3. Calculate the intensity of magnetic field due to long straight conductor carrying current.

(OR)

Derive an expression for the magnetic induction due to long straight conductor carrying current.

Ans : **(Imp.)**

Consider an infinitely long conductor placed in vaccum and carrying a current 'i' as shown fig. Let us find the magnetic field induction at a point 'P located at a distance 'R' from the midpoint 'O' using Biot-Savart law.



Consider a small elemental length dl of conductor at a distance '*l*' from 0. Let 'r' be the distance of the element from the point p. Then the field at a point 'P' due to small length 'dl' is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{dl\sin\theta}{r^2}$$

where 'θ' is the angle between the current carrying conductor at dl and line joining the element to point 'P'.

The field due to whole conductor is given by

$$B = \int dB = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{+\infty}\frac{\sin\theta dl}{r^2} \qquad .....(1)$$

From fig $r = \left(l^2 + R^2\right)^{\frac{1}{2}}$ and $\sin\left(180 - \theta\right) = \sin\theta = \frac{R}{r}$

$$\therefore \qquad \sin\theta = \frac{R}{\left(l^2 + R^2\right)^{\frac{1}{2}}}$$

Substituting the value of sin q in eq (1)

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{+\infty}\frac{Rdl}{\left(l^2 + R^2\right)^{\frac{1}{2}}}\cdot\frac{1}{\left(l^2 + R^2\right)}$$

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{+\infty}\frac{Rdl}{\left(l^2 + R^2\right)^{\frac{3}{2}}}$$

From fig (3) $\text{Tan} \propto = \frac{\ell}{R}$

$$\ell = R \text{ Tan} \propto$$

$$dl = R\sec^2 \propto d \propto$$

The limits of integrations under this substitution become $\frac{-\pi}{2}$ to $\frac{\pi}{2}$

$$\therefore \quad B = \frac{\mu_0 i}{4\pi} \int_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} \frac{R \cdot R\sec^2 \propto d\propto}{\left(R^2 \tan^2 \propto + R^2\right)^{\frac{3}{2}}}$$

$$B = \frac{\mu_0 i}{4\pi} \int_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} \frac{R^2 \sec^2 \propto d\propto}{R^3\left(1 + \tan^2 \propto\right)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 i}{4\pi} \int_{\frac{-\pi}{2}}^{\frac{+}{2}} \frac{\sec^2 \propto d\propto}{R\left(\sec^2 \propto\right)^{\frac{3}{2}}} d\propto$$

$$= \frac{\mu_0 i}{4\pi} \int_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} \frac{d\propto}{R\sec\propto} = \frac{\mu_0 i}{4\pi R} \int_{\frac{-\pi}{2}}^{\frac{+\pi}{2}} \cos\propto d\propto$$

$$= \frac{\mu_0 i}{4\pi R}\left[\sin\propto\right]_{\frac{-\pi}{2}}^{\frac{+\pi}{2}}$$

$$= \frac{\mu_0 i}{4\pi R}\left[1+1\right] = \frac{\mu_0 i}{2\pi R}$$

$$\therefore \quad \boxed{B = \frac{\mu_0 i}{2\pi R}} \text{ web/m}^2$$

This the expression for the magnetic field inducting near a long straight conductor.

2.2.1 Force on a Point Charge in a Magnetic Field

Q4. Derive the expression for force on a conductor carrying current in magneticfield.

Ans :

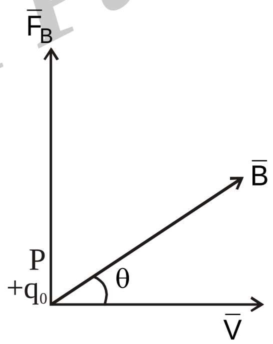
The intensity of magnetic field induction at any point in the space is defined as the force acting on test charge q_0 moving from that point. The force on the test charge at a given point is measured by projecting particle towards the point in different directions and with different velocities.

In this, when the test charge is projected with velocity 'v' the force F_B on it given by the product two vector $\bar{V}$ and intensity of magnetic induction($\bar{B}$).

$$\bar{F}_B = q_0\left(\bar{V} \times \bar{B}\right)$$

Let us consider positive test charge whose $\bar{F}_B$ is acting in upward direction as shown in fig.

The angle between $\bar{B}$ and $\bar{V}$ is θ



The magnitude of the force $F_B = q_0 VB \sin\theta$

(i) If $\theta = 90^0$ then the force is maximum given by $F_B = q_0 VB$

(ii) If $\theta = 0^0$ (or) 180^0 the force is maximum and equals to zero i.e., $F_B = 0$.

This shows that the force due to magnetic field to the direction of field is zero.

2.2.2 Properties of magnetic field

Q5. Explain the various properties of magnetic field.

Ans : **(Imp.)**

Let us know some basic properties of magnetic material such as (i) magnetic induction (B), (ii) magnetising force (H), (iii) intensity of magnetization (I), (iv) Permeability (u), (v) magnitude susceptibility (x).

(i) Magnetic induction (B)

We know that matter is formed by a group of atoms. The electrons with their spin orbiting around the nucleus constitute a closed loop carrying current. The rotation of electrons about an axis forms a magnetic dipole of moment. The net dipolemoment in unmagnetized substance is zero as the dipoles are randomly oriented. When substance is subjected to magnetic field the dipoles align parallel to the field. As the substance produces its own magnetic field due to alignment of atomic dipoles we say that substance is magnelized. The substance has two lines of force (i) due to magnetizing field (ii) due magnetization of substance it self.

Thus. the magnetic induction 'B' is defined as total number of lines of force per unit area due to both to the magnetizing field and to induced magnetism in the substance. units of B is weber / metre^2

(ii) Magnetic intensity (or) Magnetizing force (H)

The degree to which a magnetic field can magnetise a material is expressed by a physical quantity called magnetizing force and it is a vector quantity.

Magnetising force (H) is directly propotional to the intensity of magnetic induction caused by it in a medium.

Its direction is same as that of B in the medium

$$H \propto B \quad \text{(or)} \quad H = \frac{1}{\mu}B$$

$$\text{(or)} \quad B = \mu H \text{ (in medium)}$$

$$B = \mu_0 H \text{ (in vaccum)}$$

Units of H - ampere / metre

(iii) Intensity of magnetization (I)

When a magnetization is placed in a magnetic field the material acquires a magnetic moment M.

The magnetic moment per unit volume of substance is called intensity of magnetization.

Let M be the total magnetic moment of volume 'V' then

$$I = \frac{M}{V}$$

Suppose the material is a rectangular parallelopiped of uniform cross section on A and length 2*l* then

M = m × (2*l*) where m is pole strength

V = A × (2*l*)

$I = \frac{m \times (2l)}{A \times 2l} = \frac{m}{A} amp/m$ xxxxxxrce through it is called magnetic permeability. (or) Permeability is also defined as ratio between magnetic flux intensity (B) in medium to magnetising force applied on the medium.

$$H \propto B, \quad H = \frac{1}{\mu}B$$

$$\Rightarrow \qquad \mu = \frac{B}{H}$$

units : $\frac{\text{Web}}{\text{amp.meter}} = \frac{\text{Henry}}{\text{meter}}$

The value of 'μ' in free space is $4\pi \times 10^{-7}$ Henry / meter

(iv) Relative permeability

The relative permeability is defined as permeability of medium to the permeability of free space.

$$\mu_r = \frac{\mu}{\mu_0} \text{ (or) } \mu = \mu_0 \times \mu_r$$

we know that in vaccum $B_0 = \mu_0 H$

in medium $B = \mu H$

$$\therefore \quad \frac{\mu \not{H}}{\mu_0 \not{H}} = \frac{B}{BO} \Rightarrow \frac{\mu}{\mu_0} = \frac{B}{BO} = \mu_r$$

(v) Magnetic susceptibility ('χ') :

Magnetic susceptibility is defined as ratio of intensity of magnetization I to the magnetic intensity 'H'. Thus $\chi = \frac{I}{H}$

(or)

The intensity of magnetization in medium is directly propational to magnetising force H.

$$\therefore \quad I \propto H \quad \text{(or) } 1 = \chi H$$

where χ is called magnetic susceptibility.

'χ' is a ratio has no units.

Q6. Obtain the relationship between magnetic flux density 'B' magnetising force 'H' intensity of magnetisation?

Ans : **(Imp.)**

The magnetic flux density B due to magnetizing force H is sum of magnetic flux density created in vaccum (B_0) and magnetic flux density created in medium (B_m) due to magnetisation.

We have, $B = B_0 + B_m$

But $B_0 = \mu_0 H =$ lines of force crossing unit area in normal direction due to external dield

$B_m = \mu_0 I =$ lines of force crossing unit area normally due to intensity magnetization 'I'.

$\therefore$ The two sets of line of force inside the substance are called as lines induction.

$$\therefore \quad B = \mu_0 H + \mu_0 I$$

$$B = \mu_0 (H + I)$$

Dividing by H, we get

$$\frac{B}{H} = \mu_0 + \mu_0 \frac{I}{H}$$

But $\frac{B}{H} = \mu =$ Permeability of the material

$\frac{I}{H} = \chi$, susceptibility of the material

$$\therefore \quad \mu = \mu_0 + \mu_0\chi = \mu_0(1+\chi)$$

$$\mu = \mu_0(1+\chi)$$

$$\frac{\mu}{\mu_0} = (1+\chi)$$

$$\boxed{\mu_r = 1+\chi}$$

where μ_r is relative permeability.

2.2.3 Curl of Magnetic Filed

Q7. Show that curl of magnetic field $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$?

Ans :

According Biot-Savart law expression for magnetic field for volume current is given by

$$\vec{B} = \frac{\mu_0}{4\pi} \int \vec{J} \times \frac{\vec{r}}{r^3} dv$$

$$\vec{\nabla}\left(\frac{1}{r}\right) = \nabla(r-1) = -1 - r^{-1-2} \cdot \vec{r} = -1r^{-3}\vec{r} = \frac{-\vec{r}}{r^3}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int -\vec{J} \times \nabla\left(\frac{1}{r}\right) dv \qquad ..... (1)$$

We can write

$$\nabla \times \left(\frac{J}{r}\right) = \nabla\left(\frac{1}{r}\right) \times \vec{J} + \frac{\left(\nabla \times \vec{J}\right)}{0}\frac{1}{r}$$

$$= \nabla\left(\frac{1}{r}\right) \times \vec{J}$$

$$= -\vec{J} \times \nabla\left(\frac{1}{r}\right) \qquad \text{..... (2)}$$

Replacing (2) in (1)

$$= \frac{\mu_0}{4\pi}\int \nabla \times \left(\frac{\vec{J}}{r}\right) dv$$

It can be written as

$$\vec{B} = \vec{\nabla} \times \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{r} dv$$

$$\vec{\nabla} \times \vec{B} = \vec{\nabla}\left(\vec{\nabla} \times \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{r} dv\right)$$

We know that $\sum \nabla \times \left(\nabla \times \vec{A}\right)\ \nabla \times \left(\nabla \times A\right) = \nabla\left(\nabla . A\right) - \nabla^2 A$ then

$$\vec{\nabla} \times \vec{B} = \nabla\left(\nabla \cdot \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{r} dv\right) - \nabla^2\left(\frac{\mu_0}{4\pi}\int \frac{J}{r} dv\right)$$

Assume $I_1 = \nabla\left(\nabla \cdot \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{r} dv\right)$

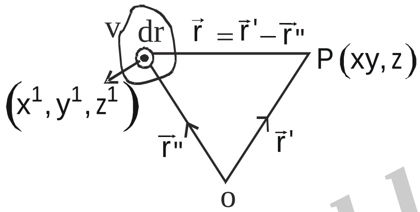
$$I_2 = \nabla^2\left(\frac{\mu_0}{4\pi}\int \frac{I}{r} dv\right)$$

$$I_1 = \vec{\nabla}\left(\vec{\nabla}\cdot\frac{\mu_0}{4\pi}\int\frac{\vec{J}}{r}dv\right)$$

$$= \vec{\nabla}\left(\frac{\mu_0}{4\pi}\int\vec{\nabla}\cdot\frac{\vec{J}}{r}dv\right)$$

$$= \vec{\nabla}\left(\frac{\mu_0}{4\pi}\int\vec{J}\vec{\nabla}\cdot\frac{1}{r}dv\right)$$

Consider surface volume with position vectors (x^1, y^1, z^1) and another point P(x,y,z). and join these points w.r. to origin ‘0’.



$$\vec{r}' = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r}'' = x'\hat{i} + y'\hat{j} + z'\hat{k}$$

$$\vec{r} = (xi + yi + zk) - (x'i + y'j + z'k)$$

$$= r' - r''$$

$$\vec{\nabla} \rightarrow \vec{r}',\ \vec{\nabla}' \rightarrow \vec{r}''$$

$$\vec{\nabla}\left(\frac{1}{r}\right) = -\vec{\nabla}'\left(\frac{1}{r}\right) \text{ where } \vec{r} = \vec{r}' - \vec{r}''$$

$$\therefore \quad I_1 = -\vec{\nabla}\left(\frac{\mu_0}{4\pi}\int\vec{J}\vec{\nabla}\cdot\left(\frac{J}{r}\right)dv\right)$$

$$= -\vec{\nabla}\left(\frac{\mu_0}{4\pi}\oint_S\frac{J}{R}ds\right) \quad \text{[Using Divergence theorem]}$$

As $r \to \infty$ $J \to$ becomes zero

$$= 0$$

$$I_2 = \nabla^2\left(\frac{\mu_0}{4\pi}\int\frac{\vec{J}}{r}dv\right)$$

$$= \frac{\mu_0}{4\pi}\int\nabla^2\left(\frac{\vec{J}}{r}\right)dv \qquad = \frac{\mu_0}{4\pi}\int\vec{J}\vec{\nabla}^2\left(\frac{1}{r}\right)dv$$

$$= \frac{\mu_0}{4\pi}\int J\left[-4\pi\partial\left(r-r'\right)dv\right.$$

Assume $r' = \vec{r}\,''$

$$= \frac{\mu_0}{4\pi}J\left(-4\pi\right)$$

$$I_2 = -\mu_0 J$$

$$\therefore \quad \nabla\times B = \nabla\left(\vec{\nabla}\cdot\left(\frac{\mu_0}{4\pi}\int\frac{J}{R}dv\right)\right) - \nabla^2\left(\frac{\mu_0}{4\pi}\int\frac{J}{R}dv\right)$$

$$= 0 - \left(-\mu_0 J\right) = \mu_0 J$$

$$\therefore \quad \boxed{\nabla\times B = \mu_0\vec{J}}$$

2.2.4 Divergence of Magnetic Field

Q8. Show that divergence of magnetic field is zero?

Ans :

According to Biot-Savart law, the expression of B for volume current is

$$\vec{B} = \frac{\mu_0}{4\pi}\int\vec{J}\times\frac{\vec{r}}{r^3}dv$$

$$\vec{\nabla}\cdot\vec{B}=\frac{\mu_0}{4\pi}\int\underset{A}{\vec{\nabla}}\cdot\left(\underset{B}{\vec{J}}\times\underset{C}{\frac{\vec{r}}{r^2}}\right)dv$$

We know that $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\vec{C}\cdot\left(\vec{A}\times\vec{B}\right)-\vec{B}\cdot\left(\vec{A}\times\vec{C}\right)$

$$\vec{\nabla}\cdot\vec{B}=\frac{\mu_0}{4\pi}\left[\frac{\vec{r}}{r^3}\cdot(\nabla\times J)-\vec{J}\cdot\left(\vec{\nabla}\times\frac{\vec{r}}{\vec{r}}\right)\right]dv$$

$$\vec{\nabla}\cdot\vec{B}=\frac{\mu_0}{4\pi}\int O\cdot\vec{J}\cdot\left(\vec{\nabla}\times\frac{\vec{r}}{r^3}\right)dv$$

$$=-\frac{\mu_0}{4\pi}\int\vec{J}\cdot\vec{\nabla}\times\left(\underset{\phi}{r^{-3}}\underset{A}{\vec{r}}\right)dv \qquad .....(1)$$

$$\left(\nabla\times\phi\vec{A}\right)=\nabla\phi\vec{A}+\phi\left(\vec{\nabla}\times\vec{A}\right)$$

where ϕ is scalar 'A' is vector

Applying the above formula to eq (1)

$$\vec{\nabla}\cdot\vec{B}=\frac{-\mu_0}{4\pi}\int\vec{J}\cdot\left[\nabla\left(r^{-3}\right)\times\vec{r}+r^{-3}(\nabla\times r)\right]dv$$

We know that $\vec{\nabla}\left(r^n\right)=nr^{n-2}\vec{r}$

$$\vec{\nabla}\left(r^{-3}\right)=-3r^{-5}\vec{r}$$

$$\therefore\quad\vec{\nabla}\cdot\vec{B}=\frac{-\mu_0}{4\pi}\int J\cdot\left[-3r^{-5}\frac{\vec{r}\times\vec{r}}{0}+r^{-3}\cdot 0\right]dv$$

$$=\frac{-\mu_0}{4\pi}\int\vec{J}\cdot 0\ dv=0$$

$$\boxed{\therefore\vec{\nabla}\cdot\vec{B}=0}$$

2.3 Integral Form of Ampere's Law

Q9. State and Explain integral form of Amperes law?

(OR)

Define Ampere's Law.

Ans : **(Imp.)**

According to Ampere's law, the line integral of magnetic field 'B' along closed curve is equal to μ_0 times the current 'i' through the area bounded by the curve.

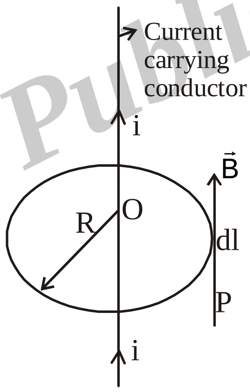
$$\oint B \cdot dl = \mu_0 i$$

where μ_0 is permeability of free space.

Amperes law can be used in finding magnetic field due to symmetrical current distributions.

Proof

Consider a long straight conductor carrying a current 'i' as shown in fig



Let the conductor be perpendicular to the page directed upward. Taking the point '0' as centre, if we consider a circular path of radius R, the magnetic induction 'B' at that point of circular path is

$$B = \frac{\mu_0 i}{2\pi R} \text{ (Biot-Savart law)}$$

Regarding B, low point should be remembered

(i) The magnitude of B is constant at all points on the circle.

(ii) This is parallel to the circuit element 'dl'

The line integral of magnetic field B along the circular path is given by

$$\oint B \cdot dl = \oint Bdl \cos 0^0$$

$$= \oint Bdl$$

$$= B\oint dl = B2\pi R \qquad ....(2)$$

where $2\pi R$ is circumference of circle.

Substituting the value of (B) from eq (1) into equation (2), we get

$$\oint B.dl = \frac{\mu_0 i}{2\pi R} \times 2\pi R$$

$$= \mu_0 i$$

$$\therefore \qquad \boxed{\oint B \cdot dl = \mu_0 i}$$

Thus, line integral of $\oint B \cdot dl$ is μ_0 times the current through the area bounded by the circle. This is Ampere's law.

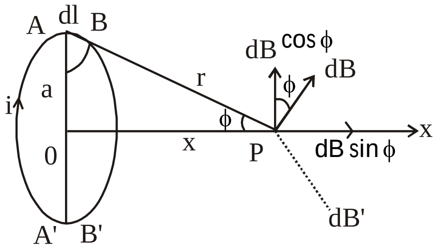
2.3.1 Applications of Ampere's Law

2.3.1.1 Magnetic field at a point on the axis of current carrying circular coil

Q10. Calculate the intensity of magnetic field at a point on the axis of circular coil carrying current?

Ans : **(Imp.)**

As shown in figure consider a circular coil of radius 'a' and carrying current 'i'. P is point on the axis of the coil at distance 'x' from point the centre. We are required to calculate the field at a point 'P'.



Consider a small element AB of length dl. Let ‘r’ be the distance of the element from the point P and θ be the angle which the direction of current makes with the line joining the element to the point 0.

The magnetic field dB at point ‘P’ due to current element AB of length dl is given by

$$dB = \frac{\mu_0}{4\pi} \times \frac{idl \times r}{r^3}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{idl \sin 90^0}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{idl}{r^2} \; (\because \text{angle between dl and } r = 90^0) \qquad .....(1)$$

The vector dB at a point ‘P’ resolved into two components one of magnitude dB $\cos\phi$ right angle to axis and other of magnitude dB $\sin\phi$ along the axis of the coil.

If we take another element A' B' diametrically opposite to AB of same length, it will also produce electric field dB at P. The direction of dB now will be opposite to the previous one. The components along the axis will add up while components perpendicular to the axis will cancel.

Similarly if we divide the whole circular coil into number of Elements, the vertical components will cancel while components along the axis will add up.

$\therefore$ Magnetic field along the axis $B = \int dB \sin\phi$

From eq (1)

$$\therefore \; B = \frac{\mu_0 i}{4\pi r^2} \int dl \sin\phi$$

$$B = \frac{\mu_0 i}{4\pi r^2} . \int dl . \frac{a}{r} \qquad \left(\because \sin\phi = \frac{a}{r} \right)$$

$$B = \frac{\mu_0 i_a}{4\pi r^3} \int dl$$

But $\int dl$ = circumference of the coil = $2\pi a$

From figure $r = (a^2 + x^2)^{1/2}$

$$\therefore \quad B = \frac{\mu_0 i_a}{4\pi\left(a^2 + x^2\right)^{3/2}} \times 2\pi a$$

$$= \frac{\mu_0 i a^2}{2\left(a^2 + x^2\right)^{3/2}}$$

If there are 'N' turns in the coil

$$B = \frac{\mu_0 N i a^2}{2\left(a^2 + x^2\right)^{3/2}} \text{ tesla} \qquad .....(2)$$

Thus the direction of B is along the axis of the coil.

(i) At the centre of the coil $x = 0$, Thus the value of 'B' becomes

$$B = \frac{\mu_0 N i a^2}{2a^3} = \frac{\mu_0 N i}{2a}$$

(ii) At very far off from the loop $x >> a$ and $(a^2 + x^2)^{3/2} = x^3$ then 'B' becomes

$$B = \frac{\mu_0 N i a^2}{2x^3}$$

(iii) Variation of field along the axis of coil

The variation of field B along the axis of coil is represented in fig (1). It is obvious from the figure that 'B' is greatest at the centre of the coil where $x = 0$ and decreases on bothsides as we move away from the centre

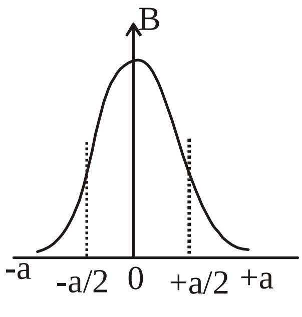


Figure (1)

The rate of variation of field can be calculated by using eq (2)

Differentiating eq (2) with respect to x, we get $\frac{dB}{dx} = \frac{\mu_0 Nia^2}{2}\frac{d}{dx}(a^2 + x^2)^{-3/2}$

$$= \frac{\mu_0 Nia^2}{2}\left(-3/2\right)\left(a^2+x^2\right)^{-5/2}(2x)$$

$$= \frac{-\mu_0 Nia^2}{2}(3x).\left(a^2+x^2\right)^{-5/2}$$

The rate of variation of field can be made uniform by $\frac{dB}{dx}$ constant (or) $\frac{d^2B}{dx^2} = 0$

Thus $\frac{d^2B}{dx^2} = \frac{-3}{2}\mu_0 Nia^2 \frac{d}{dx}\left\{x\left(a^2+x^2\right)^{-5/2}\right\}$

$$= \frac{-3}{2}\mu_0 Nia^2\left[x.\left(\frac{-5}{2}\right)\left(a^2+x^2\right)^{-7/2}(2x)+\left(a^2+x^2\right)^{-5/2}\right]$$

$$= \frac{-3}{2}\mu_0 Nia^2\left[\left(a^2+x^2\right)^{-5/2}-5x^2\left(a^2+x^2\right)^{-7/2}\right]$$

$$= \frac{-3}{2}\mu_0 Nia^2\left[a^2+x^2\right]^{7/2}\left(\left(a^2+x^2\right)-5x^2\right)$$

$$= \frac{-3}{2}\mu_0 Nia^2\left(a^2+x^2\right)^{-7/2}\left(a^2-4x^2\right)$$

If $\frac{d^2B}{dx^2} = 0$ then $a^2 - 4x^2 = 0 \Rightarrow 4x^2 = a^2$

(or) $x = \pm a/2$

This Expression shows that near points $\left(a/2\right)$ from the centre of coil on both sides the field decreases uniformly with increasing distance.

2.3.1.2 Magnetic field induction due to current carrying solenoid at a point on its axis

Q11. Derive the Expression for the magnetic field inside a long solenoid carrying current. Show that the field at the ends of such solenoid is half of that in the middle.

Ans :

A Long, tightly wound helical coil of wire is called as solenoid. The length of wire used to form solenoid is many times larger than the radius of the solenoid.

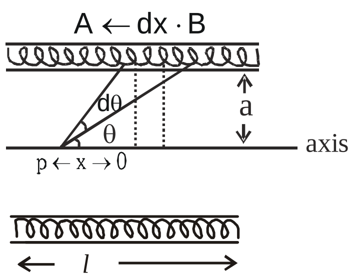


Fig (1)

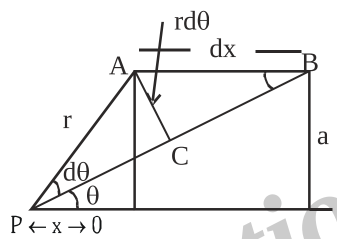


Fig (2)

Consider a long solenoid of length 'l' and radius a metre as shown fig (1) Let 'N' be total number of turns in the solenoid. Then the number of turns 'n' per metre will be N/ℓ. Suppose Solenoid carries a current i ampere. We shall calculate the field in the following cases

(i) Field at an inside point

(ii) Field at an axial end point

(iii) Field at the centre of solenoid of finite length

(i) Field at an inside point

Now, we have to calculate 'B' at a point 'p' inside the solenoid on axis. For this purpose divide solenoid into number of narrow equidistant coils. Consider one such coil of width dx. There will be ndx turns.

The field at 'p' due to elementary coil of width dx carrying current 'i' is given by

$$dB = \frac{\mu_0 (ndx) ia^2}{2(a^2 + x^2)^{3/2}} \text{ weber/metre}^2 \qquad .....(1)$$

(From eq (2) of last article)

From fig (2) consider triangle ABC,

$$\sin\theta = \frac{rd\theta}{dx} \quad dx = \frac{rd\theta}{\sin\theta}$$

From Δ APO, $a^2 + x^2 = r^2$ $\therefore (a^2 + x^2)^{3/2} = r^3$

substituting the value s in eq (1) we get

$$dB = \frac{\mu_0 n\left(\frac{rd\theta}{\sin\theta}\right)ia^2}{2r^3}$$

$$dB = \frac{\mu_0 nia^2 d\theta}{2r^2\sin\theta}$$

$$= \frac{\mu_0 nid\theta}{2\sin\theta}\left(\frac{a}{r}\right)^2$$

$$= \frac{\mu_0 nid\theta}{2\sin\theta}\left(\sin^2\theta\right)\left(\therefore\left(\frac{a}{r}\right)^2 = \sin^2\theta\right)$$

$$dB = \frac{\mu_0 nid\theta\sin\theta}{2} \quad .....(2)$$

Let θ_1 & θ_2 are the angles made by the ends of solenoid at point 'p' shown in fig (3).

Take these angles as limits integration of equation yields the magnetic field due to entire solenoid length '*l*' at point 'p' on its axis when the current 'i' flows through it.

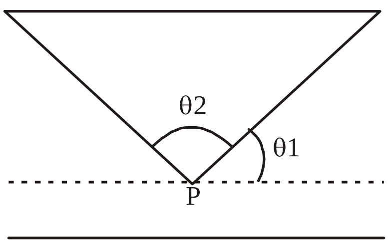


Fig (3)

$$\therefore \quad B = \int_{\theta_1}^{\theta_2} dB = \frac{\mu_0 ni}{2}\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= \frac{\mu_0 ni}{2}\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= \frac{\mu_0 ni}{2}\left[\cos\theta_1 - \cos\theta_2\right]$$

At an axial point 'p' when it is well inside a very long solenoid $\theta_1 = 0$ and $\theta_2 = \pi$,

Hence $$B = \frac{\mu_0 ni}{2}\left[\cos 0 - \cos\pi\right]$$

$$= \frac{\mu_0 ni}{2}\left[1-(-1)\right] = \mu_0 ni$$

$\boxed{\therefore B = \mu_0 ni}$ is field at the centre of solenoid.

(ii) Field at an axial end point

In this case the point 'P' lies at the second end of solenoid axis then

$$\theta_1 = 0^0, \theta_2 = 90^0$$

$$B = \frac{\mu_0 ni}{2}\left[\cos 0^0 - \cos 90^0\right]$$

$$\boxed{B = \frac{\mu_0 ni}{2}}$$

This shows that the field at either and is one half its magnitude at the centre.

(iii) Field at the centre of solenoid of finite length

Consider that the point 'P' is centre. Solenoid of length '*l*' as shown in fig (4)

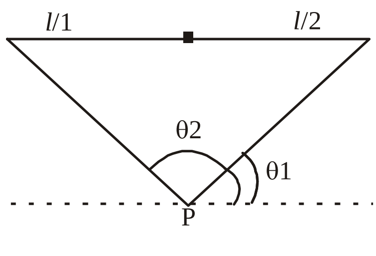


Fig (4)

$$\cos\theta_1 = \cos(180 - \theta_2) = -\cos\theta_2$$

$$\cos\theta_1 = \frac{\ell/2}{\left[a^2 + \left(\ell/2\right)^2\right]^{1/2}}$$

$$\therefore \quad \cos\theta_1 = -\cos\theta_2 = \frac{\ell}{\left(4a^2 + \ell^2\right)^{1/2}}$$

$$\therefore \quad B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$$

$$= \frac{\mu_0 ni}{\not{2}} \not{2}\cos\theta_1 = \frac{\mu_0 ni\,\ell}{\left(4a^2 + \ell^2\right)^{1/2}}$$

$$B = \frac{\mu_0 iN}{\left(4a^2 + \ell^2\right)^{1/2}} \quad \left(\because n = \frac{N}{\ell}\right)$$

This Expression gives the filed at the centre of solenoid of finite length.

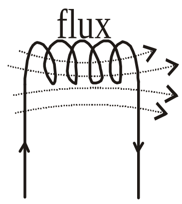
2.4 Energy Stored in Magnetic Field

Q12. Derive the Expression for energy stored in magnetic field interms of 'L' and 'i'.

Ans :

When a current flows in a coil, magnetic field is setup in it. If the current passing through the coil changes with time an induced emf is setup in the coil as shown in figure.

The emf thus induced opposes the change in current (i) that produced it. Some amount of working done during this process.



Let ‘dw’ be the workdone by the current in ‘dt’ seconds against induced emf.

$$dW = \varepsilon i dt$$

But we know that According to the defination of coefficient of self induction

The induced emf in the coil is given by

$$e = -L\frac{di}{dt}$$

Therefore $dW = \left(-L\frac{di}{dt}\right) i\ dt$

$$= -Li\ di \qquad .....(1)$$

The work should be done against emf, hence negative sign can be omitted.

$$\therefore \qquad dW = Li\ di \qquad .....(2)$$

When $t = 0$, the current $i = 0$ and as time passes $(t = t)$, the current flow becomes maximum $(i = i_0)$

The workdone against the induced emf for current to increases from $i = 0$ to $i = i_0$, is equal to the potential energy (U) stored in the coil.

$$\therefore \quad U = w = \int_{i=0}^{i=i_0} Li\ di$$

on integration

$$\therefore \quad U = L\left[\frac{i^2}{2}\right]_{e=0}^{i=i_0}$$

$$U = \frac{1}{2}\ L\ i_0^2$$

$$\boxed{\therefore U = \tfrac{1}{2}\ Li_0^2}$$

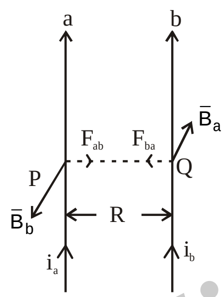
This is the expression for magnetic energy interms of current & inductance .

2.5 Magnetic Force between Two Current Carrying Conductors

Q13. Derive the Expression for the force between two parallel current carrying conductors.

Ans : **(Imp.)**

Suppose two parallel uniform conductors ‘a’ and ‘b’ of length ‘ℓ’ each carrying currents i_a and i_b respectively seperated by a distance ‘R’ are arranged as shown in figure.



According to Bio savarts law, the magnetic field ‘B’ due to current i_a in conductor at a point ‘Q’ is

$$B_a = \frac{\mu_0 ia}{2\pi R} \quad .....(1)$$

According to right hand thumb rule, the direction of B_a , on conductor ‘b’ at point Q is perpendicular to the plane of paper. Since a current i_b is flowing through the conductor ‘b’ and its direction is perpendicular to B_a , a force F_{ba} acts on the conductor.

$$\therefore \quad F_{ba} = B_a\, i_b\, \ell$$

$$\therefore \quad F_{ba} = \frac{\mu_0 i_a i_b \ell}{2\pi R} \quad .....(2)$$

$\therefore$ The force per unit length on conductor ‘b’ from the point ‘Q’ towards point ‘p’ is

$$\frac{F_{ba}}{\ell} = \frac{\mu_0 i_a i_b}{2\pi R}\ N/m$$

Similarly the current i_b in conductor ‘b’ produces magnetic field B_b at point ‘p’ on conductor ‘a’. Then direction of B_b is outward and perpendicular to the plane of paper.

$$\therefore \quad B_b = \frac{\mu_0 i_b}{2\pi R} \qquad .....(3)$$

As the conductor 'a' carrying current i_a is perpendicular to B_b , a force F_{ab} acts in the direction P to Q.

$$\therefore \quad F_{ab} = B_b i_a \ell = \frac{\mu_0 i_b i_a \ell}{2\pi R} \qquad .....(4)$$

$\therefore$ Force per unit length on conductor 'a' is

$$\frac{F_{ab}}{\ell} = \frac{\mu_0 i_a i_b}{2\pi R} N/m$$

Thus the two forces will be equal and opposite. Hence the conductors attract each other. If the direction of either current reversed, the forces will be in opposite direction. Hence parallel conductors repel each other.

2.6 Torque on a Current Loop in a Uniform Magnetic Field

Q14. Obtain the Expression for torque acting on current loop placed in a uniform magnetic field.

Ans :

Let us consider a rectangular coil PQRS of length ℓ and breadth b placed in uniform magnetic field induction B as shown in fig (1).

The side PQ and RS are always normal to the field direction. The normal to the plane of loop makes an angle 'θ' with the direction of uniform magnetic field as shown in fig (2).

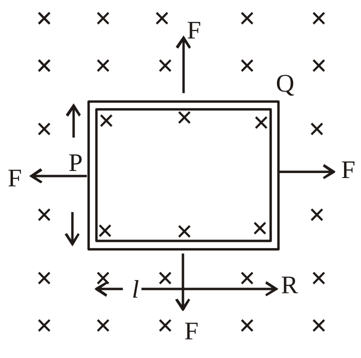


Fig (1)

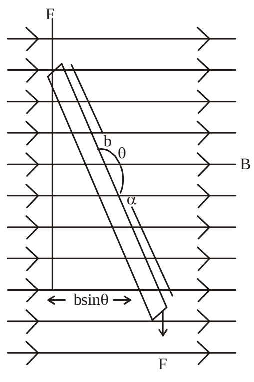


Fig (2)

The sides fig PQ and RS are perpendicular to the field equal and opposite forces of magnitude ilB acts on the m. The direction of force on PQ being upwards and on RS being downward.

This sides QR and PS make an angle α with field direction. Equal and opposite forces of magnitude ibB sin α act on them. The directions being towards right left respectively.

Thus resultant force on loop is zero, however the resultant torque is not zero. because the forces on PQ and RS constitute a torque.

The moment of torque is given by

$$\tau = i\ell \text{ B } b\sin\theta \qquad .....(1)$$

When θ = 90^0 the torque is maximum

When $\theta = 0^0$, the torque is zero.

But we know that A = lb= area of the coil

If the coil has N turns, then

$$\tau = \text{iNAB sin } \theta$$

$$\vec{\tau} = \text{Ni A} \times \text{B}$$

$$\vec{\tau} = \text{M} \times \text{B}$$

Where M = NiA = Magnetic moment of current circuit

2.7 BALLASTIC GALVANOMETER

2.7.1 Working principle of Ballastic Galvanometer

Q15. Explain the operation of a ballistic galvanometer by giving its principle of working.

(OR)

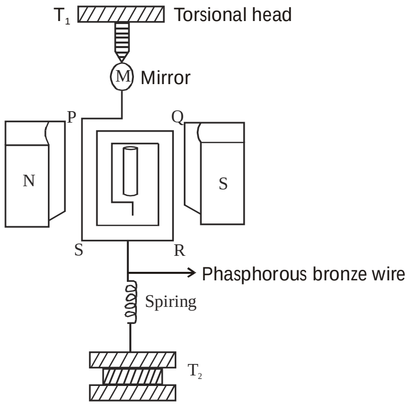
Explain the principle and working of a moving coil Ballastic galvanometer.

Ans : **(Imp.)**

Moving coil galvanometer is specially designed galvanometer suitable for measuring charge which is displaced by a varying current of short duration such as in the charging and discharging of a capacitor.

Construction :

A moving coil ballastic galvanometer as showing figure consists of a rectangular coil PQRS of large number of turns of fine insulated wire wound over a nonconducting frame of Bamboo.



This coil is suspended by means of phosphor bronze wire between the pole pieces of a powerful horse shoe magnet NS. The poles of magnet are curved to make the field radial. The lower end of the coil is attached to spring of phosphor-bronze wire. The spring and free ends of phosphor bronze wire are joined to two terminals T_2 & T_1 respectively on top of the case of the instrument & small mirror 'M' is attached on the suspend wire. Using lamp and scale arrangement, the deflection of the coil can be recorded.

The entire arrangement is placed in backlight box for the correct measurement of charge the time of swing should be greater than the time of discharge. In order (fulfil) this requirement the moment of inertia of the coil must be large torional couple must be small.

Theory :

When a charge is passed through the galvanometer coil, It gives an angular impulse to the coil.

The coil thus, set into oscillations.

Let N = number of turns in the coil

A = Area of the coil

B = Magnetic induction of radial magnetic field in which the coil is suspended

i = current in the coil at any instant.

The torque acting on the coil is given by

$$\tau = N\, i\, A\, B \qquad .....(1)$$

The torque acting for an infiniterimal time dt gives an angular impulse to the coil

Angular impulse = couple × time

$= N\, i\, AB \times dt$ .....(2)

If 't' be the total time, then total angular impulse

$$= \int_0^t Ni\; AB\; dt = NAB\int_0^t i\; dt$$

$$= N\,A\,B\,q \qquad \left(\because \int_0^t i\; dt = q\right)$$ .....(3)

But angular impulse = change of angular momentum = $I\omega$ .....(4)

Where ω is angular velocity at start and I is moment of inertia of the coil about the axis of suspension

From eq (3) & (4)

$N\,A\,B\,q = I\,\omega$ .....(5)

The force acting on the coil twists the suspension wire or utilized as rotational kinetic energy.

The K.E gives the workdone against the torsional couple .

Rotational kinetic energy of the system = $\frac{1}{2} I\omega^2$ .....(6)

This energy turns the coil by an angle 'θ'. Let the restoring couple required to produce unit twist in the wire is 'c'. Then the couple needed to turn the coil by angle (θ) is $\tau = c\,\theta$

The workdone in twisting the wire further by an angle $d\theta$ is given by $d\omega = c\theta\; d\theta$

Total workdone in twisting wire from initial rest position $(\theta = 0)$ to maximum angle $\theta = \theta_0$ is

$$w = \int dw = \int_\theta^\theta c\theta d\theta = \frac{1}{2} c\theta_0^2$$ .....(7)

$\therefore$ Total rotational kinetic energy given to twist the coil from $\theta = 0$ to $\theta = \theta_0$ is equal to total workdone

From (6) & (7)

$\therefore \quad \frac{1}{2}I\omega^2 = \frac{1}{2}c\theta_0^2$ .....(8)

$I\omega^2 = c{\theta_0}^2$

From equation (5) 7 (8) we have

$$\frac{I^2\omega^2}{I\omega^2} = \frac{B^2A^2N^2q^2}{C{\theta_0}^2}$$

$\therefore \quad I = \frac{B^2A^2N^2q^2}{C{\theta_0}^2}$ .....(9)

The time period of oscillation of the coil having momentum inertia (I) and the restoring force for unit twist (c) is given by

$$T = 2\pi\sqrt{\frac{I}{C}}$$

(or) $I = \frac{CT^2}{4\pi^2}$ .....(10)

Using equations (9) & (10)

$$\frac{CT^2}{4\pi^2} = \frac{B^2A^2N^2q^2}{C\theta_0^2}$$

$$q^2 = \frac{C^2T^2}{4\pi^2.B^2A^2N^2}.\theta_0^2$$

$$q = \frac{T}{2\pi}\left(\frac{C}{BAN}\right).\theta_0$$

$$q = K.\theta_0$$

Where $K = \frac{T}{2\pi}\cdot\frac{C}{BAN}$ is called current reduction factor. Above equation shows that the charge flowing in ballastic galvanometer is directly propotional to maximum angle of twist (θ_0) $\therefore \boxed{q \; \alpha \; \theta_0}$

2.7.2 Charge Sensitivity and Current Sensitivity

Q16. Define (i) Charge sensitivity (ii) Current sensitivity.

Ans :

(i) Charge Sensitivity

The charge sensitivity of the galvanometer is defined as the amount of charge required for unit deflection.

(ii) Current Sensitivity

The current sensitivity of the galvanometer is defined as the amount of current required for unit deflection.

Q17. Write the uses of Ballastic Galvanometer.

Ans :

- To compare the Electromotive forces of two cells
- To compare capacities of two capacitors
- To calculate self inductance and mutual inductance of Electrical circuits (or) coils.
- To know the high resistance value using capacitor
- To Estimate magnetic field intensity (or) magnetic flux between two poles.
- To find angle of dip of a place using an earth inductor.

2.8 Electromagnetic Damping Resistance

Q18. Define Electromagnetic damping.

Ans :

When a charge is passed through the galvanometer, the coil sets swinging because of induced current setup in it . This current gives rise to couple on the coil. From Lenz’s law, the direction of couple is such as to oppose the motion of coil . Now the coil comes to rest and its motion is said to be damped.

“ The damping which arises due to the induced current in the coil during its motion in permanent magnetic field is called Electromagnetic damping.

2.8.1 Critical damping Resistance

Q19. Explain briefly about critical damping resistance.

Ans :

When the External resistance of the circuit in which galvanometer is placed is smaller, the induced current is larger i.e. damping is greater.

In this case the coil makes only one swing and returns slowly to the rest position.

The particular resistance for which the motion just ceases to be oscillatory is called critical External damping resistance. The galvano meter is called as critically damped. With more resistance it is over-damped and with less outance it is order damped.

Q20. Write about the damping correction of ballastic galvanometer

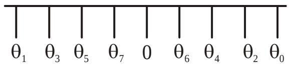
Ans :

The oscillational made by ballistic galvanometer due to rotational kinetic energy come to half after a period of time. The reasons are

(i) The movement of the coil is appeared by air, resulting frictional damping.

The motion of the coil in the permanent magnetic field is also opposed by the induced current and induced emf produced according to Lenz's law. This type of damping is called Electromagnetic damping.

Suppose $\theta_1, \theta_2, \theta_3, \theta_4$..... are the successive maximum deflections of the coil from the neutral state to the left and right sides respectively as shown in fig.



If the ration between any two successive deflections is considered, it will, be, a constant say, 'd'.

$$\therefore \quad \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \text{......... constant} = d$$

Here 'd' represents the decrement in the magnitude of the deflection. Logarithmic of 'd' is called logarthmic decrement $\log_e d$ and its value is denoted by 'λ'

$$\log_e d = \lambda$$

$$\therefore \quad d = e^{\lambda}$$

for complete rotation, the decrement is

$$\frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = e^{\lambda}, .e^{\lambda} = e^{2\lambda}$$

Due to damping, the coil can not show maximum rotation (θ_0) from its initial rest position, instead it shows maximum deflection 'θ'. Which is one fourth of its rotation

$$\therefore \quad \frac{\theta_0}{\theta_1} = e^{\lambda/2} = d^{1/2}$$

$$\therefore \quad \theta_0 = \theta_1 e^{\lambda/2} = \theta_1 \left[1 + \lambda/2 + \frac{(\lambda/2)^2}{2} + \text{........} \right]$$

Neglecting terms containing higher powers of λ

$$\theta_0 = \theta_1 \left(1 + \lambda/2\right) \qquad \therefore \quad q = \frac{T}{2\pi} \frac{C}{BAN} \theta_1 \left(1 + \lambda/2\right)$$

Problems

1. The magnetic moment of magnet (10cm×2cm×1cm) is 1 Ampr m². What is the intensity of magnetization?

Sol :

V = volume = 10×2×1 = 20 cc = 2×10^{-5} m^3

M = Magnetic moment = 1 amp × m^2

$$I = \frac{M}{V} = \frac{1}{2\times10^{-5}}\frac{amp\times m^2}{m^3} = 5\times10^4\, amp/m$$

2. A specimen of nickel is uniformly magnetized by a magnetizing field of 500 Am^{-1}. If the magnetic induction in the specimen is 0.4 wb m^{-2} final the relative permeability and the susceptibility.

Sol :

$B = \mu H = \mu_0\mu_r H$ Given

B = 0.4 wb m^{-2}

H = 500 Am^{-1}

$$\mu_r = \frac{B}{\mu_0 H} = \frac{0.4}{4\pi\times10^{-7}\times500}$$

$$\mu_r = 636.94$$

$$\chi = \mu_r - 1 = 635.9427$$

3. An iron rod of area 6 sq. cm is placed with its length parallel to a magnetic field of intensity 1200 amp/m. The flux through the rod is 4 × 10^{-4} weber what is the permeability of the rod

Sol :

$$B = \phi/A$$

Given $\phi = 4\times10^{-4}$ weber

A = 6 ×10^{-4} m^2 H = 1200 amp/m

$$B = \frac{\phi}{A} = \frac{4 \times 10^{-4}}{6 \times 10^{-4}} = 0.6 \text{ tesla.}$$

$$\mu = \frac{B}{H} = \frac{0.6}{1200} \frac{\text{weber} \times m^{-2}}{\text{amp} \times m^{-1}}$$

$$= 0.0005 \frac{\text{web}}{\text{amp} \times m}$$

4. The magnetic susceptibility of medium is 948×10^{-11}. Calculate the permeability and relative permeability.

Sol :

Given $\chi = 948 \times 10^{-11}$

The relative permeability is given by

$$\mu_r = 1 + \chi_m$$

$$= 1 + 948 \times 10^{-11}$$

Absolute permeability is given by

$$\mu = \mu_r \times \mu_0 = \left(1 + 948 \times 10^{-11}\right) \times 4\pi \times 10^{-7}$$

5. If two parallel conductors seperated by 40 cm in free space carry 40 ampere 60 ampere currents respectively. Determine magnetic induction at mid point of the line joining the two conductors.

Sol :

Given $i_1 = 40$ A , $i_2 = 60$ A

$d = 40$ cm

we know that $F = \frac{\mu_0 i_1 i_2}{2\pi d}$

$$F = \frac{\cancel{4}^{2} \cancel{\pi} \times 10^{-7} \times \cancel{40} \times 60}{\cancel{2}\pi \times \cancel{40}}$$

$$= 48 \times 10^{-6} \text{ N/m}$$

6. An infinitely long conductor carries a current of 10 mA. Find the magnetic field intensity at a point 20 cm away from it.

Sol :

Given i = 10 mA

R = 20 cm , $\mu_0 = 4\pi \times 10^{-7}$

We know that $B = \frac{\mu_0 i}{2\pi R}$

$$B = \frac{4^2 \pi \times 10^{-7} \times 10 \times 10^{-3}}{2 \times \pi \times 0.2}$$

B = 10^{-8} tesla

7. A current of 1 amp is flowing in a circular coil of radius 10 cm and 20 turns. Calculate the intensity of magnetic field at a distance 10 cm on the axis of the coil and the centre.

Sol :

Given i = 1amp a = 10 cm n = 20

$$B_{centre} = \frac{\mu_0 in}{2a} = \frac{4\pi \times 10^{-7} \times 1 \times 20}{2 \times 10}$$

$$= 4\pi \times 10^{-2} \text{ weber/m}$$

Magnetic induction B' at a distance 10 cm on.

axis from the centre is given by

$$B' = \frac{\mu_0}{2} \times \frac{nia}{(x^2 + a^2)^{3/2}}$$

$$= \frac{4\pi \times 10^{-7}}{2} \times \frac{20 \times 1 \times (10)^2}{\left[(10)^2 + (10)^2\right]^{3/2}}$$

$$= 2\pi \times 10^{-7} \times \frac{2000}{(200)^{3/2}}$$

= 0.4488×10^{-7} weber/m^2

8. A solenoid of 2000 turns is wound uniform on a tube of 20cm long and 10 cm diameter find the strength of magnetic field at the centre of solenoid when 1 amp current is flowing through it

Sol :

Given n = number of turns per meter

$$= \frac{2000}{0.2} = 10^4$$

$$B = \mu_0 H \text{ (or) } H = \frac{B}{\mu_0}$$

WKT $B = \mu_0 ni$ (At centre)

$$H = \frac{\mu_0 ni}{\mu_0} = ni$$

$H = 10^4 \times 1 = 10^4$ amp -turns/metre

9. A solenoid of length 20 cm radius 2cm is closely wound with 200 turns. Calculate magnetic field intensity at either ends of solenoid when current in the windings is 5 amp

Sol :

Given n = number of turns per meter

$$= \frac{200}{0.2} = 1000$$

i = 5 amp

The field at the end of solenoid

$$B = \frac{\mu_0 ni}{2}$$

WKT $H = \frac{B}{\mu_0} = \frac{\mu_0 ni}{2.\mu_0} = \frac{ni}{2}$

$$= \frac{1000 \times 5}{2} = 2500 \text{ amp/m}$$

10. A rectangular loop of dimensions 20cm × 10 cm having 30 turns is placed in uniform magnetic field of induction 8 × 10^{-4} wb/m^2 such that it make an angle of 60^0 with field. If 10A current is flowing through the loop find the torque acting on it when it is rotating.

Sol :

Given $\theta = 90^0 - 60^0 = 30^0$

N = 30

i = 10 Am a = 0.2 m, b = 0.1m, B = 8 × 10^{-4} wb/m^2

Torque on the coil τ = Nab Bi sin θ

τ = 30 × 10 × 0.2 × 0.1 × 8 × 10^{-4} × sin 30

= 2.4 × 10^{-3} N.m

11. A moving coil galvanometer shows 15 cm deflection for standard potential difference 0.05V. The resistance of galvanometer coil is 125 Ω. The time period is 10 sec when no damping is present calculate the charge during discharge of capacitor through the galvanometer if it shows deflection of 5 cm.

Sol :

Given Desistance = 125 Ω

standard potential difference 'v' = 0.05 V

$$i = \frac{V}{R} = \frac{0.05}{125} = 4 \times 10^{-4}\ A$$

WKT $$q = \frac{T}{2\pi}.\frac{i}{\theta_1}.\theta = \frac{10}{2\pi}\left(\frac{4 \times 10^{-4}}{15 \times 10^{-2}}\right)5 \times 10^{-2}$$

$$q = 0.21 \times 10^{-3}\ c$$

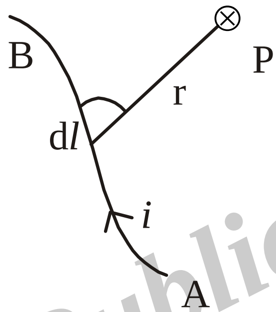
Short Question and Answers

1. Explain Bio-Savart law.

Ans :

Bio-Savart obtained a relation by means of which 'B' can be calculated at any point of space in which a current is passing. The relation is called as Biot and Savart law.

As shown fig let AB be a conductor of any arbitrary shape in which current 'i' is flowing. let P be a point at which filed is to be determined.



To evaluate 'B' at a point 'P' at a distance 'r' from point '0' divide the conductor of length 'l' into large number of small segments of lengths $\overrightarrow{dl}$.

The field at point 'P' due to flow of current 'i' through 'dl' be $\overrightarrow{dB}$.

'θ' is the angle between the vectors $\overrightarrow{dl}$ and $\vec{r}$

According to to Biot-Savart law the magnitude of $\overrightarrow{dB}$ is

(i) directly propotional to strength of current 'i' and length of the element segment dl i.e. $\overline{dB} \propto i$ and $dB \propto dl$

(ii) inversely propotional to square of the distance between P and dl i.e.

$$d\bar{B} \propto \frac{1}{r^2}$$

(iii) directly propotional to angle 'θ' between the vector $\overline{dl}$ and $\bar{r}$

i.e. $\overline{dB} \propto \sin\theta$

From the above points

$$dB \propto \frac{idl\sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2} \qquad .....(1)$$

where $\frac{\mu_0}{4\pi}$ is propotionality constant and μ_0 is permeability of free space.

2. What are the properties of magnetic field?

Ans :

Let us know some basic properties of magnetic material such as (i) magnetic induction (B), (ii) magnetising force (H) (iii) intensity of magnetization (I), (iv) Permeability (u), (v) magnitude susceptibility (x).

(i) Magnetic induction 'B'

The magnetic induction 'B' is defined as total number of lines of force per unit area due to both to the magnetizing field and to induced magnetism in the substance. units of B is weber / metre^2

(ii) Magnetic intensity (or) Magnetizing force 'H'

The degree to which a magnetic field can magnetise a material is expressed by a physical quantity called magnetizing force and it is a vector quantity.

$$B = \mu_0 H$$

(iii) Intensity of magnetization 'I'

When a magnetization is placed in a magnetic field the material acquires a magnetic moment M.

The magnetic moment per unit volume of substance is called intensity of magnetization.

Let M be the total magnetic moment of volume 'V' then

$$I = \frac{M}{V}$$

(iv) Magnetic permeability (μ)

The property of medium to allow magnetic line of force through it is called magnetic permeability. (or) Permeability is also defined as ratio between magnetic flux intensity (B) in medium to magnetising force applied on the medium.

$$H \propto B, \quad H = \frac{1}{\mu} B$$

$$\Rightarrow \qquad \mu = \frac{B}{H}$$

Relative permeability

The relative permeability is defined as permeability of medium to the permeability of free space.

$$\mu_r = \frac{\mu}{\mu_0} \text{ (or) } \mu = \mu_0 \times \mu_r$$

(v) Magnetic susceptibility 'χ' :

Magnetic susceptibility is defined as ratio of intensity of magnetization I to the magnetic intensity 'H'. Thus $\chi = \frac{I}{H}$

---

3. Show that divergence of magnetic field is zero.

Ans :

According to Biot-Savart law, the expression of B for volume current is

$$\vec{B} = \frac{\mu_0}{4\pi} \int \vec{J} \times \frac{\vec{r}}{r^3} dv$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \int \underset{A}{\vec{\nabla}} \cdot \left(\underset{B}{\vec{J}} \times \underset{C}{\frac{\vec{r}}{r^3}} \right) dv$$

We know that $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{A} \times \vec{B}) - \vec{B} \cdot (\vec{A} \times \vec{C})$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \left[\frac{\vec{r}}{r^3} \cdot (\nabla \times J) - \vec{J} \cdot \left(\vec{\nabla} \times \frac{\vec{r}}{\vec{r}} \right) \right] dv$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \int O \cdot \vec{J} \cdot \left(\vec{\nabla} \times \frac{\vec{r}}{r^3} \right) dv$$

$$= -\frac{\mu_0}{4\pi}\int \vec{J}\cdot\vec{\nabla}\times\left(\underset{\phi}{r^{-3}}\underset{A}{\vec{r}}\right)dv \qquad .....(1)$$

$$\left(\nabla\times\phi\vec{A}\right) = \nabla\phi\vec{A} + \phi\left(\vec{\nabla}\times\vec{A}\right)$$

where ϕ is scalar 'A' is vector

Applying the above formula to eq (1)

$$\vec{\nabla}\cdot\vec{B} = \frac{-\mu_0}{4\pi}\int\vec{J}\cdot\left[\nabla\left(r^{-3}\right)\times\vec{r} + ^{-3}\left(\nabla\times r\right)\right]dv$$

WKT $\vec{\nabla}\left(r^{n}\right) = nr^{n-2}\vec{r}$

$$\vec{\nabla}\left(r^{-3}\right) = -3r^{-5}\vec{r}$$

$\therefore$ $$\vec{\nabla}\cdot\vec{B} = \frac{-\mu_0}{4\pi}\int J\cdot\left[-3r^{-5}\underset{0}{\underline{\vec{r}\times\vec{r}}} + r^{-3}\cdot 0\right]dv$$

$$= \frac{-\mu_0}{4\pi}\int\vec{J}\cdot 0\ dv = 0$$

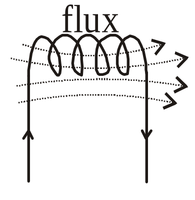
$$\boxed{\therefore\vec{\nabla}\cdot\vec{B} = 0}$$

4. Derive the expression for Energy stored in magnetic field.

Ans :

When a current flows in a coil, magnetic field is setup in it. If the current passing through the coil changes with time an induced emf is setup in the coil as shown in fig (13).

The emf thus induced opposes the change in current (i) that produced it. Some amount of working done during this process.



Let 'dw' be the workdone by the current in 'dt' seconds against induced emf.

$$dW = \varepsilon i dt$$

But we know that According to the definition of coefficient of self induction

The induced emf in the coil is given by

$$e = -L\frac{di}{dt}$$

Therefore $dW = \left(-L\frac{di}{dt}\right) i\ dt$

$$= -\,Li\ di \qquad .....(1)$$

The work should be done against emf, hence negative sign can be omitted.

$$\therefore \quad dW = Li\ di \qquad .....(2)$$

When $t = 0$, the current $i = 0$ and as time passes ($t = t$), the current flow becomes maximum $(i = i_0)$

The workdone against the induced emf for current to increases from $i = 0$ to $i = i_0$, is equal to the potential energy (U) stored in the coil.

$$\therefore \quad U = w = \int_{i=0}^{i=i_0} Li\ di$$

on integration

$$\therefore \quad U = L\left[\frac{i^2}{2}\right]_{e=0}^{i=i_0}$$

$$U = \frac{1}{2}\ L\ i_0^2$$

$$\boxed{\therefore U = \tfrac{1}{2}\ Li_0^2}$$

5. Define magnetic field (B), magnetic flux.

Ans :

(i) Magnetic Field

The intensity of magnetic field induction 'B' at a given point is equal to force acting on a test charge moving with velocity perpendicular to the direction of magneticfield induction.

It is given by $B = \frac{F_B}{q_0 V}$

where F_B is the magnetic force

q_0 is test charge

V is the velocity

- The tangent to the line of induction gives the direction of 'B' at that point.
- The magnetic lines of force are always pointed from north pole to south pole.

(ii) Magnetic flux

The magnetic flux denoted by ϕ_B can be defined as the total number of lines of induction cutting through a surface is called magnetic flux through that surface.

Then scalar product B.ds will represent i.e. magnetic flux fϕ_B over ds. Thus,

$$d\phi_B = B.ds$$

Now, the magntic flux over the entire surface will be $\phi_B = \oint B \cdot ds$

If B is uniform over the entire area, then

$$\phi_B = B\oint ds = B.S$$

6. Define force of conductor carrying current in magneticfield.

Ans :

The intensity of magnetic field induction at any point in the space is defined as the force acting on test charge q_0 moving from that point. The force on the test charge at a given point is measured by projecting particle towards the point in different directions and with different velocities.

In this, when the test charge is projected with velocity 'v' the force F_B on it given by the product two vector $\bar{V}$ and intensity of magnetic induction($\bar{B}$).

$$\bar{F}_B = q_0\left(\bar{V} \times \bar{B}\right)$$

7. Define Amperes law.

Ans :

According to Ampere's law, the line integral of magnetic field 'B' along closed curve is equal to μ_0 times the current 'i' through the area bounded by the curve.

$$\oint B \cdot dl = \mu_o i$$

where μ_0 is permeability of free space.

Amperes law can be used in finding magnetic field due to symmetrical current distributions.

8. Define energy stored in magneticfield.

Ans :

When a current flows in a coil, magnetic field is setup in it. If the current passing through the coil changes with time an induced emf is setup in the coil.

The emf thus induced opposes the change in current (i) that produced it. Some amount of working done during this process.

The workdone against the induced emf for current to increases from $i=0$ to $i=i_0$, is equal to the potential energy (U) stored in the coil.

$$\therefore \quad U = w = \int_{i=0}^{i=i_0} Li\, di$$

on integration

$$\therefore \quad U = L\left[\frac{i^2}{2}\right]_{e=0}^{i=i_0}$$

$$U = \frac{1}{2} L\, i_0^2$$

$$\boxed{\therefore U = \tfrac{1}{2} Li_0^2}$$

This is the expression for magnetic energy interms of current & inductance .

9. Define charge sensitivity & current sensitivity.

Ans :

The charge sensitivity of the galvanometer is defined as the amount of charge required for unit deflection.

Similarly the current sensitivity of the galvanometer is defined as the amount of current required for unit deflection.

10. Write the uses of Ballastic Galvanometer.

Ans :

- To compare the Electromotive forces of two cells
- To compare capacities of two capacitors
- To calculate self inductance and mutual inductance of Electrical circuits (or) coils.
- To know the high resistance value using capacitor
- To Estimate magnetic field intensity (or) magnetic flux between two poles.
- To find angle of dip of a place using an earth inductor.

11. Explain the terms electro magnetic damping and critical damping.

Ans :

When a charge is passed through the galvanometer, the coil sets swinging because of induced current setup in it . This current gives rise to couple on the coil. From Lenz's law, the direction of couple is such as to oppose the motion of coil . Now the coil comes to rest and its motion is said to be damped.

" The damping which arises due to the induced current in the coil during its motion in permanent magnetic field is called Electromagnetic damping.

Critical damping resistance

When the External resistance of the circuit in which galvanometer is placed is smaller, the induced current is larger i.e. damping is greater.

In this case the coil makes only one swing and returns slowly to the rest position.

The particular resistance for which the motion just ceases to be oscillatory is called critical External damping resistance.

Choose the Correct Answer

1. Unit of intensity of magnetic field [a]

   (a) $\frac{\text{Newton}}{\text{Ampere.metre}}$ (b) $\frac{\text{Newton} - \text{metre}}{\text{Ampere}}$

   (c) $\frac{\text{Newton} - \text{Ampere}}{\text{metre}}$ (d) None

2. According to Bio-savart law the magnitude of magnetic field is directly propational to [d]

   (a) i (current) (b) r^2

   (c) dl (d) both a&b

3. Divergence of magnetic field [c]

   (a) infinity (b) not defined

   (c) zero (d) defined

4. In Ballastic galvanometer rectangular coil is made up of [b]

   (a) conducting material (b) non conducting material

   (c) Semiconductor material (d) none

5. The magnetic field for a long straight conductor is directly propational to [c]

   (a) R (b) α

   (c) i (d) none

6. Relation between B and H is given by [b]

   (a) $B = \mu/H$ (b) $B = \mu H$

   (c) $B\mu = H$ (d) none

7. Relation between Relative permeability and susceptibility [a]

   (a) $\mu_r = 1 + x$ (b) $\chi = 1 - \mu_r$

   (c) $\mu_r = 1 + 1/x$ (d) none

8. Which one is correct Expression [c]

(a) $B = \mu_0\left(H + \frac{1}{I}\right)$ (b) $I = \frac{H}{x}$

(c) $\nabla \times B = \mu_0 J$ (d) $\mu_r = 1 + \frac{1}{x}$

9. Magnetic field of current carrying circular coil while be maximum at x = [b]

(a) $+a/2$ (b) 0

(c) $-a/2$ (d) $\frac{+a}{2}$ to $-a/2$

10. Magnetic field at an axial end point of solenoid is given by [a]

(a) $\frac{\mu_0 ni}{2}$ (b) $\mu_0 i$

(c) $\mu_0 n$ (d) $\mu_0 ni$

Fill in the blanks

1. Expression for the magnetic field at the centre of solenoid _______ .
2. Energy stored in magnetic field 'υ' _______ .
3. Magnetic field denoted by ϕ_B is given by ϕ_B = _______ .
4. The value of permeability of free space $\mu_0 =$ _______ .
5. If $\theta = 90^o$, the maximum value of force acting on point charge in magnetic field is given by F_B = _______ .
6. Rectangular coil of moving coil galvanometer is suspended by _______ type of wire.
7. Ratio of intensity of magnetisation to magnetic intensity is known as _______ .
8. If two forces of conductors are equal and opposite then the conductor _______ each other.
9. Torque acting on a current loop in a uniform magnetic field is given by τ = _______ .
10. Ballastic galvanometer is specially designed galvanometer suitable for measuring _______ .

ANSWERS

1. $\mu_0 ni$
2. $\frac{1}{2} Li_0^2$
3. $\phi_B = \int B.ds$
4. $4\pi \times 10^{-7}$
5. $q_0 \vee B$
6. Phosphor bronze
7. Magnetic susceplibility
8. Attract
9. M × B
10. Charge

One Mark Answers

1. Define magnetic permeability.

Ans :

Magnetic permeability is defined as ratio between magnetic flux density (B) in medium to magnetising force applied on the medium i.e $\mu = \frac{B}{H}$

2. Define Amperes law.

Ans :

The line integral of magnetic field 'B' along a closed curv is equal to μ_0 times the net current 'i' through the area area bounded by the curv.

i.e. $\oint B.dl = \mu_0 i$

3. Write two uses of Ballastic Galvanometer.

Ans :

(i) To compare the Electromotive forces of two cells

(ii) To know the high resistance value using capacitor.

4. Define charge sensitivity.

Ans :

The charge sensivity of galvonometer is defined as the amount of charge required for unit deflection.

5. What is meant by intensity of magnetisation.

Ans :

The magnetic moment per unit volume of substance is called intensity of magnetisation.

UNIT III	**Electromagnetic Induction and Electromagnetic waves:** Faraday's laws of induction (differential and integral form), Lenz's law, self and mutual Induction. Continuity equation, modification of Ampere's law, displacement current, Maxwell equations. Maxwell s equations in vacuum and dielectric medium, boundary conditions, plane wave equation: transverse nature of EM waves, velocity of light in vacuum and in medium. Poynting's theorem.

3.1 Faraday's Laws of Induction

3.1.1 Differential and Integral Form

Q1. State and Explain Faraday's laws of Electromagnetic Induction. Derive the differential and Integral forms of Faraday's Law.

Ans : **(Imp.)**

There are two laws of electromagnetic induction. They are

- Whenever the magnetic flux linked with a circuit is changed, an e.m.f is induced in the circuit
- The magnitude of induced emf is directly proportional to the negative rate of variation of magnetic flux linked with the circuit.

If ϕ_B be the magnetic flux linked with circuit at any instant and e be the induced e.m.f., then

$$e = -\left(\frac{d\phi_B}{dt}\right) \quad \text{... (1)}$$

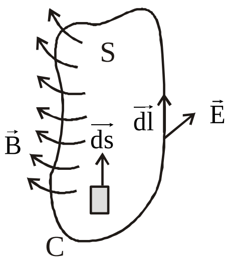
This law is also known as Neumann's's Law.

If there are N turns in the coil, then

$$e = -N\left(\frac{d\phi_B}{dt}\right)$$

i) Integral form of Faraday's Law

Consider that magnetic field is produced by a stationary magnet or a current carrying coil. Suppose there is a closed circuit C of any shape which encloses a surface S in the field as shown in figure.



Let B be the magnetic flux density in the neighbourhood of the circuit the magnetic flux through a small area ds will be B.dS.the flux through the entire circuit is

$$\phi_B = \int_S B.dS \qquad ... (2)$$

When magnetic flux is changed, an electric field is induced around the circuit. The line integral of the electric field gives the induced e.m.f in closed circuit

Thus, $$e = \oint E.dl \qquad ... (3)$$

Where E is the electric field at an element dl of the circuit.

Substituting the values of e and ϕ_B from eqns. (3) and (2) in (1), we have

$$\oint E.dl = -\frac{d}{dt}\int_S B.dS \qquad ... (4)$$

This is integral form of Faraday's Law

ii) Differential form of Faraday's Law

According to eqn (4), the line integral of the electric field around any closed circuit is equal to the negative rate of change of magnetic flux through the circuit.

By stokes theorem, we have $\oint E.dl = \int_s (\nabla \times E).ds \qquad ... (5)$

from eqns (4) and (5), we get $= \int_s (\nabla \times E).ds = \frac{-d}{dt}\int_s B.ds$

(or) $= \int_s (\nabla \times E).ds = -\int_s \frac{\partial B}{\partial t}.ds$

It follows that $\nabla \times E = \frac{-\partial B}{\partial t}$ (or) curl E = $\frac{-\partial B}{\partial t}$.

This is the differential form of Faraday's Law.

3.2 Lenz's Law

Q2. State and Explain Lenz's Law obtain an Expression for induced E.M.F.

Ans : **(Imp.)**

According to Lenz's Law, the direction of induced e.m.f (or current) in a closed circuit is such that it opposes the original cause that produces it.

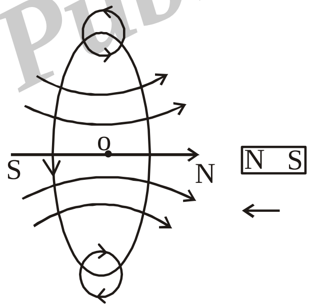
This Law is based on the principle of conservation of energy.

When the applied flux density B in a closed circuit is increasing, the e.m.f or current induced in the closed circuit is in such a direction as to produce a field which tends to decrease B.

When the applied flux density is decreasing in magnitude the current in the closed circuit is in such a direction as to produce a field which tends to increase B.

Thus, the induced current is in a direction such that it produces a magnetic flux tending to oppose the original change of flux. i.e total flux in the circuit is constant.

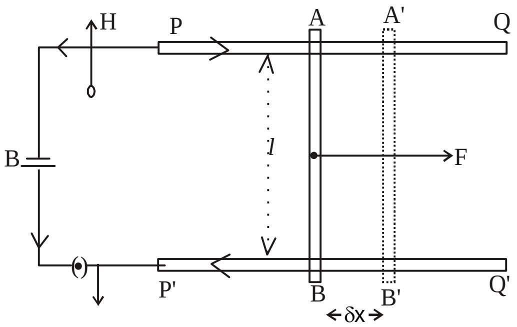
Suppose the north pole of the magnet is moved towards a coil connected to a galvanometer as shown in the figure.



As the magnet is pushed towards the circuit, an induced current is setup in the coil. The induced current produces its own magnetic field. Now coil behaves as magnet. The face of the coil towards the north pole of the magnet between them. Due to this, motion of the magnet is opposed. This causes a change of magnetic flux in the coil. Thus, the direction of induced current is such that as to oppose motion of the magnet.

Expression for Induced E.M.F.

Consider two thick copper strips PQ and 'PQ' as shown in figure which form parallel rails and connected to a battery through key. Let AB is a copper strip capable of sliding on two rails.



The rod AB now experiences a mechanical force F_0 given by $F_0 = Bi_0\ell$

where $B \rightarrow$ magnetic field

$\ell \rightarrow$ distance between two rails (i.e. PQ & P'Q')

$i_0 \rightarrow$ steady current passed around the circuit.

The force F_0 is experienced in the direction parallel to the rails towards the right. Due to this force, the rod AB moves to A'B', a small distance δx in time δt.

When the rod AB moves, flux through the rod changes due to B. and induced e.m.f is developed in the rod which causes current i_0 to change to i.

The workdone by Force F during the displacement δx

$$\delta W = F.\delta x = (iB\ell)\delta x$$

The energy required for this work is derived from the battery.

The amount is given by E δ

where E is e.m.f of battery

i is current

δt is time.

Total work done = workdone in heating + workdone in moving the rod AB

$$E\ i\delta t = i^2R\delta t + iB\ell\delta x$$

$$E = iR + B\ell\left[\frac{\delta x}{\delta t}\right]$$

$$i = \frac{E - B\ell\left(\frac{\delta x}{\delta t}\right)}{R}$$

This expression shows that the e.m.f E of the circuit is opposed by an e.m.f equal to $B\ell\left(\frac{\delta x}{\delta t}\right)$, which is equal to the induced emf 'e' in the circuit due to the motion of the rod AB.

Thus $e = -B\ell\left(\frac{\delta x}{\delta t}\right)$

$B\ell\left(\frac{\delta x}{\delta t}\right)$ denotes the rate of change of magnetic induction.

If ϕ_B denotes the magnetic induction,

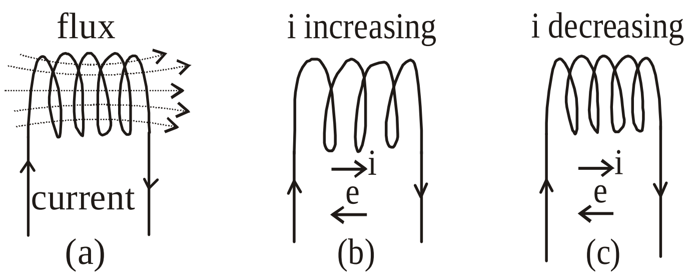
induced e.m.f = e = $-\frac{\delta\phi}{\delta t}$

3.3 Self Induction

Q3. What is self induction? Define coefficient of self induction and obtain an expression for self induction of a long solenoid.

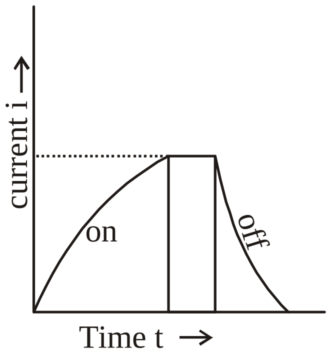
Ans : **(Imp.)**

When a current flows in a coil, magnetic field is set up in it. If the current passing through the coil changes with time, an induced e.m.f is set up in the coil. By Lenz's law, the direction of induced e.m.f. is such as to oppose the change in current



When the current is increasing, the induced e.m.f is against the current [shown in (b)]. When the current is decreasing induced e.m.f is in the direction of current [shown in (c)]. So, the induced e.m.f opposes any change of the original current. This phenomenon is called self induction.

The property of the circuit by virtue of which any change in the magnetic flux linked with it, induces an e.m.f in it, is called inductance. The induced e.m.f is called back e.m.f.



When the current in the coil is switched on, self induction opposes growth of current and when is switched off, it opposes decay of current.

Coefficient of Self Induction

The total magnetic flux ϕ_B linked with the coil is proportional to current i, flowing in it.

$\phi_B \alpha i$ or $\phi_B = Li$ ... (1)

When L is a constant called the coefficient of self induction when $i = 1$, $\phi_B = L$.

"Hence, the coefficient of self induction is numerically equal to the magnetic flux linked with the coil when unit current flows through it."

The induced e.m.f in the coil is given by

$$e = -\frac{d\phi_B}{dt}$$

$$= -\frac{d(Li)}{dt} = -L\frac{di}{dt} \quad [\because \phi_B = Li] \qquad ... (2)$$

The negative sign indicates that the induced e.m.f is in such a direction as to oppose the change.

When $\frac{di}{dt} = 1$, $e = -L$

$\therefore$ The coefficient of self induction is numerically equal to the induced e.m.f in the coil, when the rate of change of current is unity.

The current flows against the back e.m.f and does work against it.

$$dW = -e\, i\, dt$$

$$dW = L\frac{di}{dt}\, i\, dt \qquad \left[\therefore e = -L\frac{di}{dt}\right]$$

Hence, total workdone in bringing the current from zero to a steady maximum value i_0 is

$$W = L\int_0^{i_0} i\frac{di}{dt}.dt$$

$$W = L\int_0^{i_0} i di$$

$$W = \frac{1}{2}Li_0^{\,2}$$

This is stored as energy of magnetic field. When i_0 =1, L = 2W. Thus, the coefficient of self induction is twice the workdone against the induced e.m.f in establishing the unit current in the coil.

Unit of self inductance is Henry.

$$1\text{ henry} = \frac{1\text{volt}}{1\text{ampere per second}}$$

Q4. Obtain the Expression for self inductance of long solenoid.

Ans : **(Imp.)**

Consider a long air core solenoid of length ℓ metre and uniform cross - section area A metre^2 . Let n be the number of turns per meter. Suppose a current of i amp. flows through it. The magnetic field inside the solenoid is given by

$$B = \mu_0 ni \text{ weber/metre}^2$$

where μ_0 = permeability constant

$\therefore$ Magnetic flux through each turn

$$\phi_B = BA = \mu_0 niA \text{ weber}$$

Now the magnetic flux linked with all the turns of solenoid

$$= \mu_0 niA \times N \text{ weber turns}$$

where N = total number of turns in the solenoid

$$= \mu_0 niA \times nl \qquad (\because N = n\ell)$$

$$= \mu_0 n^2 iA\ell$$

The self inductance of the long solenoid is therefore

Li = total flux linked with the solenoid

$$Li = \mu_0 n^2 iA\ell$$

$$L = \mu_0 n^2 A\ell \text{ henry}$$

where n is the number of turns per unit length

In terms of total number of turns N of the solenoid

$$L = \mu_0 \left(\frac{N}{\ell}\right)^2 A\ell$$

$$L = \frac{\mu_0 N^2 A}{\ell} \text{ henry.}$$

Q5. Obtain the Expression for self inductance of toroid.

Ans :

A toroid may be regarded as a solenoid bent into circular form so that its ends are joined together.

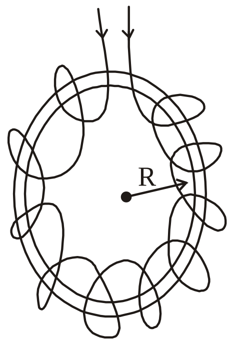


Fig (6)

The fig (6) shows toroid having circular cross section of radius R. with R. with 'N' number of turns in it. Let 'i' be current flowing in the toroid, B the magnetic field produced due to this current.

According to Amperes law

$$\oint B.dl = \mu_0 Ni$$

As the magnetic field 'B' is tangential around the circular path of radius R, we have

$$B.2\pi R = \mu_0 Ni$$

$$B = \frac{\mu_0 Ni}{2\pi R} \quad .....(1)$$

Let 'A' is area of cross reaction of each turn of toroid the magnetic flux linked will toroid is

$$\phi = BNA \quad .....(2)$$

From eq (1) & (2)

$$\phi = BNA = \frac{\mu_0 Ni}{2\pi R}.NA \Rightarrow \phi = \frac{\mu_0 N^2 iA}{2\pi R}$$

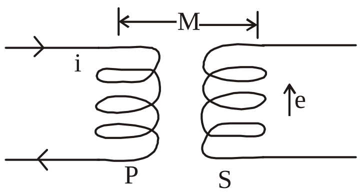
We know that $\phi = Li \Rightarrow \frac{\mu_0 N^2 iA}{2\pi R} = Li \Rightarrow L = \frac{\mu_0 N^2 A}{2\pi R}$ henry

3.4 Mutual Induction

Q6. Define Mutual Induction. Derive an expression for the coefficient of mutual induction between a pair of coils.

Ans : **(Imp.)**

Consider two coils placed near to each other as shown in (fig) When a current is passed in primary coil P, there is a change of magnetic flux linked with it and an induced emf is set in the secondary coil s. This phenomenon is called mutual inductance. Similarly, the secondary circuit also induces on e.m.f in primary.



Any two circuits in which there is mutual induction are known as mutually coupled circuit.

Let a current i amp. in primary P produces a magnetic flux ϕ_B in the secondary S. For two given coils situated in fixed relative positions, it is observed that the flux linked with the secondary is proportional to the current in primary. Thus

$$\phi_B \alpha i$$

$$\phi_B = Mi \qquad .....(1)$$

Where M is constant called the coefficient of Mutual inductance of two coils.

The e.m.f induced in the secondary is given by

$$e = -\frac{d\phi_B}{dt}$$

$$e = -\frac{d}{dt} Mi = -M\frac{di}{dt} \qquad .....(2)$$

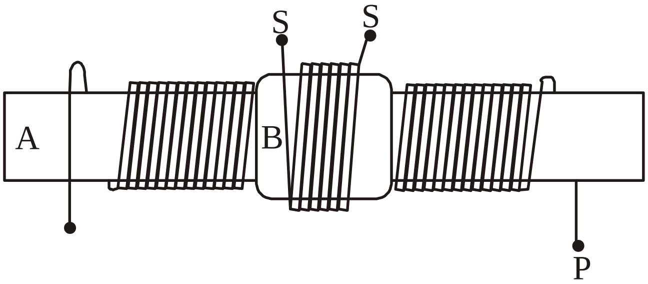
The eqns (1) & (2) enable us to define the mutually inductance in the following two ways.

1) It is the flux linked with a circuit due to a unit current flowing through the other.
2) It is the emf induced in the circuit , when the rate of decay of current in the other circuit is unity.

Units of mutual inductance is henry.

Mutual Inductance of two given coils.

Consider a long air cored solenoid with primary A and secondary B as shown in fig.



number of turns in primary coil = n_1

length of primary coil = ℓ

area of cross section = a

number of turns is secondary coil = n_2

current in primary = i

Magnetic field inside the primary = $\mu_0 \frac{n_1}{\ell} i$ weber /metre^2

Magnetic flux through each turn of primary

$$\phi = BA = \mu_0 \frac{n_1}{\ell} i \times a \text{ weber}$$

Since secondary is wound closely over the central position of primary, the same flux is also linked with each turn of secondary.

Total magnetic flux linked with secondary = $\mu_0 \frac{n_i i}{\ell} \times a \times n_2$ weber turn.

If M be mutual inductance of the two coils, total flux linked with secondary is Mi

$$Mi = \mu_0 \frac{n_1 n_2}{\ell} ia \Rightarrow M = \frac{\mu_0 n_1 n_2 a}{\ell}$$

Q7. Mention the differences between self induction and mutual induction.

Ans : **(Imp.)**

S.No.	Self Induction	S.No.	Mutual Induction
1)	When the current flowing in a coil is changed, induced current is produced in the coil itself. This phenomena is known as self induction.	1)	When current flowing in a coil is is changed then induced current is produced in another coil which is kept in vicnity of the first coil. This phenomena is called mutual induction.
2)	The induced current affects the main current flowing in the coil.	2)	The induced current flows in other coil. Thus it does not directly affect the main current of primary coil.
3)	There is only one coil in it and the other is secondary.	3)	There are two coils. One is primary

3.5 Continuity Equation

Q8. Derive continuity wave equation?

Ans :

Current density (i)

The current 'i' flowing in conductor is macroscopic quantity i. It is scalar quantity Whereas the current density 'j' is a microscopic quantity related to the conductor. and it is vector.

If 'A' is area and 'ℓ' is the length of the conductor the current density at all points of conductor is given as $j = \dfrac{i}{A}$

If we imagine a plane in conductor, the amount of current passing through the plane is equal to flux of current density expressed as $i = \int_s \bar{j}.\overline{ds}$ ..... (2)

Where $\overline{ds}$ represents infiniterimal area in plane of conductor.

(2) can also written as i = jA

"The differential equation that gives relationship between the current density (j) and volume charge density (ρ) at a point in a closed circuit is known as equation of continuity".

Imagine a infinerimal arbitrary volume 'dv' in a plane. If 'ρ' is the volume charge density then the charge in the entire volume can be written as

$$Q = \int \rho dv \qquad \text{... (3)}$$

The rate of decrease of charge at any instant of time going out of volume is equal to amount of current passing through conductor at that instant.

$$\therefore \quad -\frac{dQ}{dt} = i \qquad \text{... (4)}$$

$$i = -\frac{d}{dt}\int \rho dv \quad \text{[from eq (3) \& (4)]}$$

$$i = -\int_v \frac{\partial \rho}{\partial t} dv \qquad \text{... (5)}$$

$$i = -\int_s i.ds \qquad \text{... (6)}$$

From eq (5) & (6)

$$\int_S \overline{j}.\overline{ds} = \int_v -\frac{\partial \rho}{\partial t}.dv \qquad \text{... (7)}$$

using Gauss divergence theorem

$$\int_S j.ds = \int_V \nabla \cdot j dv$$

$$\int_s \overline{j}.\overline{ds} = \int_v \left(-\frac{\partial \rho}{\partial t}\right) dv$$

$$\therefore \quad \int_v \nabla.j dv = \int_v \left(-\frac{\partial \rho}{\partial t}\right) dv$$

$$\therefore \quad \nabla . j = \frac{-\partial \rho}{\partial t}$$

(or) $\boxed{\nabla . j + \frac{\partial \rho}{\partial t} = 0}$ ... (8)

Known as equation of continuity and used for varying Electric currents. When current is static i.e. no change in value with time

$$\frac{\partial \rho}{\partial t} = 0 \qquad \therefore \quad \nabla . j = 0$$

3.6 Displacement Current (or) Modification of Ampere's Law

Q9. Discuss the concept of displacement current.

(or)

Derive the Maxwell correction to Ampere's law.

Ans : **(Imp.)**

The current in the conductor produces the magnetic field.

Maxwell stated that a changing electric field in vacuum or in dielectric also produces a magnetic field. So, a changing electric field is equivalent to a current which flows as long as the electric field is changing and produces the same magnetic effect as an ordinary conduction current. This is known as displacement current.

Modification of Ampere's Law

Ampere's law in vector form can be expressed as

$$\vec{\nabla} \times B = \mu_0 j \qquad \text{... (1)}$$

where j is current density.

Divergence of above eqn.

$$\vec{\nabla} . \left(\vec{\nabla} \times B\right) = \text{div curl B} = \text{div } \mu_0 j = \mu_0 \text{divj}$$

Divergence of curl of a vector is always zero and hence

$$\text{divj} = 0 \qquad \left[\therefore \vec{\nabla} \cdot (\mu_0 j) = 0\right] \qquad \text{... (2)}$$

$\therefore$ The total flux of current out of any closed surface is zero It means that current is always closed and there are no sources and no sinks.

Eq (2) is in contradiction with the equation of continuity

which states that

$$\text{div } j + \frac{\partial \rho}{\partial t} = 0 \qquad \text{... (3)}$$

where ρ represents the charge density.

Ampere's law in the form $\vec{\nabla} \times B = \mu j$ is valid only for a steady state condition and is insufficient for the case of time varying electric field in which the charge density varies with times i.e. $\frac{\partial \rho}{\partial t} \neq 0$

To modify Ampere's law so that it may be valid for both steady state and time-varying electric field "something" must be added in j of eqn (1) such that the divergence of both side is same.

Thus, curl B = $\mu_0 j$ + something ... (4)

In vector form, the Gauss Law is expressed as $\vec{\nabla}.D = \rho$ (where D $= \varepsilon_0 E$)

Differentiating with respect to t, we get

$$\vec{\nabla} \cdot \frac{\partial D}{\partial t} = \frac{\partial \rho}{\partial t}$$

Adding $\vec{\nabla}$.j on both sides and rearranging, we get

$$\vec{\nabla}.j + \frac{\partial \rho}{\partial t} = \vec{\nabla}.j + \vec{\nabla}.\frac{\partial D}{\partial t}$$

$$= \vec{\nabla}.\left(j + \frac{\partial D}{\partial t} \right)$$

Adding to equation of continuity

$$= \vec{\nabla}.j + \frac{\partial \rho}{\partial t} = \vec{\nabla}.\left(j + \frac{\partial D}{\partial t} \right) = 0$$

Thus $\vec{\nabla}.j = 0$ for steady current

$$\vec{\nabla}.\left[j+\frac{\partial D}{\partial t}\right]=0 \text{ every where} \qquad ...(5)$$

In this way j in Ampere's law is replaced by $\left(j+\frac{\partial D}{\partial t}\right)$

Thus the Ampere's law becomes

$$\text{Curl B} = \mu_0\left(j+\frac{\partial D}{\partial t}\right) \qquad ...(6)$$

The term $\frac{\partial D}{\partial t}$ is called as displacement current density

Eqn (6) can also be written as $[\because D=\varepsilon_0 E]$

$$\text{Curl B} = \mu_0\left[j+\varepsilon_0\frac{\partial E}{\partial t}\right] \qquad ...(7)$$

3.7 Maxwell Equations

Q10. Write Maxwell's equations in differential and integral forms.

Ans : **(Imp.)**

There are basic laws formulated by Maxwell for electricity and magnetism. These equations are known as Maxwell's equations.

The integral forms of these equations are

$$\oint E.ds=\left(\frac{q}{\varepsilon_0}\right) \qquad ...(1)$$

$$\oint B.ds=0 \qquad ...(2)$$

$$\oint E.dl=\frac{-d\phi_B}{dt} \qquad ...(3)$$

$$\oint B.dl=\mu_0\left(j+\varepsilon_0\frac{\partial E}{\partial t}\right) \qquad ...(4)$$

Maxwell's equations can also be stated in the differential forms as follows

$$\text{div } E = \frac{\rho}{\varepsilon_0} \qquad \text{... (5)}$$

$$\text{div } B = 0 \qquad \text{... (6)}$$

$$\text{curl } E = \frac{-\partial B}{\partial t} \qquad \text{... (7)}$$

$$\text{curl } B = \mu_0\left[j + \varepsilon_0 \frac{\partial E}{\partial t}\right] \qquad \text{... (8)}$$

Derivations.

The above differential forms can be obtained from the integral forms as follows.

1. $\oint E.dS = \frac{q}{\varepsilon_0}$ [Gauss Law of electricity]

If ρ be the charge density and dV, the small volume considered, then

$$q = \int_v \rho dV$$

$$\oint E.ds = \frac{1}{\varepsilon_0}\int_v \rho dv$$

$$\oint \varepsilon_0 E.ds = \int_v \rho dv \qquad (\because \varepsilon_0 E = D)$$

$$\oint D.ds = \int_v \rho dv$$

According to the divergence theorem.

$$\oint A.ds = \int_v (\vec{\nabla}.A) dv$$

$$\oint D.ds = \int_v (\vec{\nabla}.D) dv$$

So, $\int_v \vec{\nabla}.D dv = \int_v \rho dv$

$$\vec{\nabla}.D = \rho \text{ (or) } \vec{\nabla}.E = \frac{\rho}{\varepsilon_0}$$

$$\text{div.}E = \frac{\rho}{\varepsilon_0} \qquad \text{... (a)}$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0}$$

2. $\oint B.ds = 0$ (Group Law for magnetism)

Transforming the surface integral into volume integral, we get

$$\oint_s B.ds = \int_v \vec{\nabla}.Bdv$$

$$\int_v \vec{\nabla}.Bdv = 0$$

As the volume is arbitrary, the integral must be zero.

$$\vec{\nabla}.B = 0$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

3. $\oint E.dl = \frac{-d\phi_B}{dt} = -\frac{\partial}{\partial t}\int_s B.ds = -\int_s \frac{\partial B}{\partial t} ds$

Applying stoke's theorem

$$\oint E.dl = \int_s \left(\vec{\nabla} \times E\right).ds$$

$$\therefore \quad \int_s \left(\vec{\nabla} \times E\right).ds = -\int_s \frac{\partial B}{\partial t}.ds$$

$$\vec{\nabla} \times E = -\frac{\partial B}{\partial t} \text{ (or) curl } E = \frac{-\partial B}{\partial t} \qquad \text{... (c)}$$

$$\text{i.e } i\left[\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}\right]+j\left[\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right]+k\left[\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right]=\frac{-\partial}{\partial t}\left[iB_x+jB_y+kB_z\right]$$

$$\therefore\left[\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}\right]=\frac{\partial B_x}{\partial t},\quad\left[\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right]=\frac{-\partial B_y}{\partial t}\quad\left[\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right]=\frac{-\partial B_z}{\partial t}$$

4. $\oint B.dl=\mu_0 i$ [$\therefore$ Ampere's Law]

using stokes theorem

$$\oint B.dl=\int_s\left(\vec{\nabla}\times B\right).ds$$

$$\int_s\left(\vec{\nabla}\times B\right).ds=\mu_0\int_s j.ds$$

$$\vec{\nabla}\times B=\mu_0 j$$

$$\vec{\nabla}\times B=\mu_0\left[j+\varepsilon_0\frac{\partial E}{\partial t}\right]\left\{\therefore \text{Replacing } j \text{ by}\left[j+\varepsilon_0\frac{\partial E}{\partial t}\right]\right\}.$$

3.8 MAXWELL'S EQUATIONS IN VACUUM AND DIELECTRIC MEDIUM

Q11. Derive the Maxwell's Electromagnetic wave equation for E & B in dielectronic medium and vacuum (or) free space.

Ans : **(Imp.)**

Case (i)

Dielectric medium

Maxwell's electromagnetic equations for a homogeneous, isotropic dielectric medium. The dielectric medium is one which offers infinite resistance to the current and hence its conductivity is zero i.e $j=0$. In homogeneous isotropic medium, there is no volume distribution of charge, thus the charge density ρ is zero.

Hence $j=0$, $\rho=0$, $D = K\varepsilon_0 E = \varepsilon E$, $B = \mu_0\mu_r H$, $B = \mu H$

Now Maxwell's equations for a dielectric becomes

$$\vec{\nabla} \cdot E = 0 \qquad .....(1)$$

$$\vec{\nabla} \cdot B = 0 \qquad .....(2)$$

$$\vec{\nabla} \times E = \frac{-\partial B}{\partial t} \qquad .....(3)$$

$$\vec{\nabla} \times B = \mu\varepsilon \frac{\partial E}{\partial t} \qquad .....(4)$$

(A) We can obtain the equation of propagation of a wave in dielectric medium by eliminating E from eqns (3) and (4)

Taking curl of eq (4), we get

$$\vec{\nabla} \times \vec{\nabla} \times B = \vec{\nabla} \times \mu\varepsilon \frac{\partial E}{\partial t} = \mu\varepsilon\left[\vec{\nabla} \times \frac{\partial E}{\partial t}\right]$$

{$\therefore$ μ & ε remains constant throughout the medium]

$$= \mu\ \varepsilon\ \frac{\partial}{\partial t}\left(\vec{\nabla} \times E\right)$$

$$= \mu\ \varepsilon\ \frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right) \qquad \left\{\because \vec{\nabla} \times E = \frac{-\partial B}{\partial t}\right\} \text{ from eq (3)}$$

$$= -\mu\ \varepsilon\ \frac{\partial^2 B}{\partial t^2}$$

Thus $\vec{\nabla} \times \vec{\nabla} \times B = -\mu\ \varepsilon\ \dfrac{\partial^2 B}{\partial t^2}$ .....(5)

We know that $\vec{\nabla} \times \vec{\nabla} \times B = \vec{\nabla}\left(\vec{\nabla} \cdot B\right) - \nabla^2 B$

$$= \vec{\nabla}(0) - \nabla^2 B \quad \text{\{from eqn (2)\}}$$

$$\vec{\nabla} \times \vec{\nabla} \times B = -\nabla^2 B \qquad .....(6)$$

Substituting the value of $\nabla \times \nabla \times B$ from eq (6) in eq (5), we get

$$-\vec{\nabla}^2 B = -\mu\varepsilon \frac{\partial^2 B}{\partial t^2}$$

$$\boxed{\nabla^2 B = \mu\varepsilon \frac{\partial^2 B}{\partial t^2}} \quad .....(7)$$

Similarly, from eqn (3) we can show that

(B) Let us take curl on both sides of eq(3) , we have

$$\vec{\nabla} \times \left(\vec{\nabla} \times E\right) = \vec{\nabla} \times \left(-\frac{\partial B}{\partial t}\right)$$

$$= \frac{-\partial}{\partial t}\left(\vec{\nabla} \times B\right)$$

$$= -\mu\varepsilon \frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t}\right) \quad \{\text{ from eqn (4)}\}$$

$$= -\mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

So, $\vec{\nabla} \times \vec{\nabla} \times E = -\mu\varepsilon \frac{\partial^2 E}{\partial t^2}$ .....(8)

We know that $\vec{\nabla} \times \vec{\nabla} \times E = \vec{\nabla}\left(\vec{\nabla} \cdot E\right) - \nabla^2 E$

$= \vec{\nabla}(0) - \nabla^2 E$ ....(9) {from eq (1)}

From eqns (8) and (9), we get

$$-\nabla^2 E = -\mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E = \mu\varepsilon \left(\frac{\partial^2 E}{\partial t^2}\right) \quad .....(10)$$

Case (ii)

Free Space

In homogeneous medium, we have no charge and no conduction current,

i.e. $\rho_V = 0$, $\sigma = 0$ and J = 0

For free space $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$

The Maxwell's equations in free space are

$$\vec{\nabla} \times E = -\frac{\partial B}{\partial t} = -\mu_0 \frac{\partial H}{\partial t} \qquad (\because\ B = -\mu_0 H) \qquad .....(1)$$

$$\vec{\nabla} \times H = \frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t} \qquad (\because\ D = \varepsilon_0 E) \qquad .....(2)$$

$$\vec{\nabla} \cdot D = 0, \quad \vec{\nabla} \cdot E = 0 \qquad .....(3)$$

$$\vec{\nabla} \cdot B = 0, \quad \vec{\nabla} \cdot H = 0 \qquad .....(4)$$

Wave equation in terms of E

Taking curl of both sides of eq(1), we get

$$\vec{\nabla} \times \vec{\nabla} \times E = -\mu_0 \vec{\nabla} \times \frac{\partial H}{\partial t}$$

$$\vec{\nabla} \times \vec{\nabla} \times E = -\mu_0 \frac{\partial}{\partial t}(\vec{\nabla} \times H) \qquad .....(5)$$

We know that the vector identity

$$\vec{\nabla} \times \vec{\nabla} \times E = \vec{\nabla} \times (\vec{\nabla} \cdot E) - \nabla^2 E$$

$$= \vec{\nabla} \times (0) - \nabla^2 E \quad \text{[using eqn (3) i.e. } \vec{\nabla} \cdot E = 0\text{]}$$

$$\vec{\nabla} \times \vec{\nabla} \times E = -\nabla^2 E \qquad .....(6)$$

from eqns (5) & (6)

$$-\nabla^2 E = -\mu_0 \frac{\partial}{\partial t}(\vec{\nabla} \times H)$$

$$= -\mu_0 \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{\partial E}{\partial t}\right) \qquad \{\text{Using eqn (1)}\}$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\therefore \quad \nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad .....(7)$$

This expression is the wave equation for propagation of electric field E in free space.

In cartesian coordinates, eq (7) can be expressed as

$$\frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial y^2}=\mu_0\varepsilon_0\frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial y^2}+\frac{\partial^2 E_y}{\partial z^2}=\mu_0\varepsilon_0\frac{\partial^2 E_y}{\partial t^2} \qquad .....(8)$$

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\frac{\partial^2 E_z}{\partial z^2}=\mu_0\varepsilon_0\frac{\partial^2 E_z}{\partial t^2}$$

Wave equation in terms of B.

We know that the wave equation in terms of H is

$$\nabla^2 H=\mu_0\varepsilon_0\frac{\partial^2 H}{\partial t^2}$$

We know that $B=\mu_0 H$ (or) $H=\frac{B}{\mu_0}$

$$\frac{\nabla^2 B}{\mu_0}=\mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\left(\frac{B}{\mu_0}\right)=\varepsilon_0\frac{\partial^2 B}{\partial t^2}$$

$$\nabla^2 B=\mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2} \qquad .....(9)$$

3.9 Boundary Conditions

Q12. Derive Electromagnetic waves in bounded media

(or)

Derive Boundary conditions for D, B, E, and H.

Ans : **(Imp.)**

1. Boundary conditions for D. D is Electric displacement vector Maxwell's first equation is given by

$$\nabla.E=\frac{\rho}{\varepsilon} \text{ (or) } \nabla.D=\rho \qquad ... (1)$$

Construct a pill box of volume V and height h across boundry as shown in fig(1)

Integrating eqn (1) over the pill box of volume V.

we get $\int_v \nabla.D\, dv = \int_v \rho\, dv$ ... (2)

Using Gauss divergence theorem, converting volume integral on LHS into surface integral, we get

$\int_v \nabla.D\, dv = \int_s D\, \hat{n}.\, ds$ ... (3)

from eqn's (2) and (3), we have

$\int_s D\hat{n}.ds = \int_v \rho\, dv$

Where LHS $\int_S D\hat{n}.ds = \int_{S_1} D_1 \hat{n}_1 ds + \int_{S_2} D_2 \hat{n}_2.ds + \int_{S_3} D_1^1 n_1^1.ds + \int_{S_4} D_2^1 \hat{n}_2^1 ds$

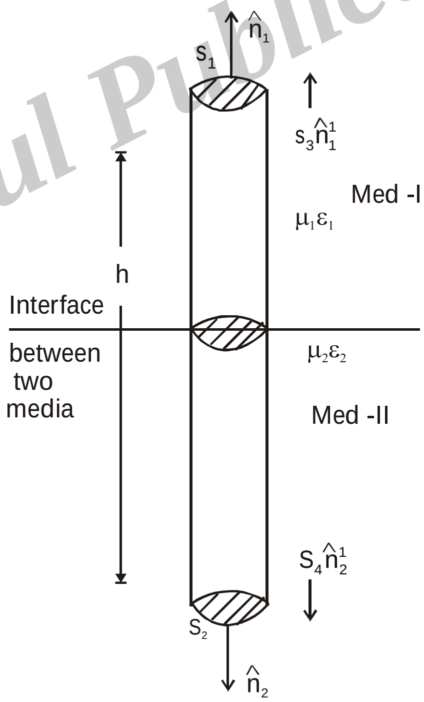


Fig (1)

So, $\int_{S_1} D_1 \hat{n}_1 ds + \int_{S_2} D_2 \hat{n}_2 ds + \int_{S_3} D_1^1 \hat{n}_1^1 ds + \int_{S_4} D_2^1 \hat{n}_2^1 ds = \int_V \rho dv$ .....(4)

If the height of the pill box $h \to 0$, then the contribution from S_3 and S_4 becomes zero and the S_3 and S_4 merges with each other and form a surface A.

$$\lim_{h\to 0}\left[\int_{S_1} D_1 \hat{n}_1 \, ds + \int_{S_2} D_2 \hat{n}_2 \, ds\right] = \lim_{h\to 0}\int_V \rho \, dv$$

The volume charge density ρ can be replaced by a surface charge density σ

i.e. $\int_V \rho \, dv \to \int_S \sigma \, ds$ $\left\{\because \int_{S_1} ds = \int_{S_2} ds = A\right\}$

$$\lim_{h\to 0}\left[\int_{S_1} D_1 \hat{n}_1 ds + \int_{S_2} D_2 \hat{n}_2 .ds\right] = \lim_{h\to 0}\int_S \sigma ds$$

(or) $D_1 \hat{n}_1 A + D_2 \hat{n}_2 A = \sigma A$

(or) $D_1 \hat{n}_1 + D_2 \hat{n}_2 = \sigma$

(or) $D_1 \hat{n}_1 = -D_2 \hat{n}_2 + \sigma$

Where $D_1 \hat{n}_1$ is the normal component of electric displacement vector in medium 1.

This equation shows that the normal component of electric displacement vector is not continuous at the boundary (interface) and it changes by 'σ'

Let $n_1 = n$ and $n_2 = -n$

So $D_1 n = D_2 n + \sigma$

$$\boxed{D_1 n - D_2 n = \sigma}$$

(or) $\boxed{D_1 \perp - D_2 \perp = \sigma}$ $\{\because n = \perp\}$

2. Boundary Condition for B

B $\to$ magnetic induction vector

Maxwell's second equation is given by $\nabla . B = 0$ .....(1)

Integrating eqn (1) over the volume of pill box V

we get $\int_V \nabla.B\ dV = 0$ .....(2)

Using Gauss divergence theorem, converting volume integral into surface integral , we get

$$\int_V \nabla.B\ dV = \int_S B\hat{n}\ ds = 0 \quad .....(3)$$

In eqn (3), we have

$$\int_S B\hat{n}\ ds = \int_{S_1} B_1\hat{n}_1 ds + \int_{S_2} B_2\hat{n}_2^1 ds + \int_{S_4} B_2\hat{n}_2^1 ds \quad .....(4)$$

From eqn (3) and (4)

$$\int_V \nabla.B\ dV = \int_{S_1} B_1\hat{n}_1 ds + \int_{S_2} B_2\hat{n}_2\ ds + \int_{S_3} B_1^1\hat{n}_1^1 ds + \int_{S_4} B_2^1\hat{n}_2^1\ ds = 0$$

If the height of pill box $h \to 0$ then the contribution from S_3 to S_4 becomes zero

Now above equation becomes $\lim_{h\to 0}\left[\int_{S_1} B_1\hat{n}_1 ds + \int_{S_2} B_2\hat{n}_2\ ds\right] = 0$

(or) $B_1\hat{n}_1 A + B_2\hat{n}_2 A = 0$

$$B_1\hat{n}_1 + B_2\hat{n}_2 = 0$$

$$B_1\hat{n}_1 = -B_2\hat{n}_2$$

Let $n_1 = n$ and $n_2 = -n$ then $B_1\hat{n}_1 = B_2\hat{n}_2$

(or) $\boxed{B_1 \perp = B_2 \perp}$ (or) $\boxed{B_1 \perp - B_2 \perp = 0}$

This shows that normal or perpendicular component of magnetic induction vector B is continuous at the interface

3. Boundary condition for E

Maxwell's third equation is given by $\nabla \times E = \frac{-\partial B}{\partial t}$ .....(1)

Constant a rectangular loop ABCD of length '*l*' and height 'h' across the boundary as shown in fig (2).

Integrating eqn (1) over the surface of the loop ABCD

We get $\int\limits_{ABCD} (\nabla \times E)\hat{n}\, ds = -\int\limits_{ABCD} \frac{\partial B}{\partial t}\hat{n}\, ds$ .....(2)

Converting surface integral on LHS into integral using stokes theorem, we have

$\int\limits_{S} (\nabla \times E)\hat{n}\, ds = \int\limits_{C} E.dl$ .....(3)

from eqn (2) & (3) we have

$$-\int \frac{\partial B}{\partial t}\hat{n}\, ds = \int\limits_{C} E.dl$$

where $\int\limits_{C} E.dl = \int\limits_{AB} E_1.dl + \int\limits_{CD} E_2.dl + \int\limits_{BC\,\&\,AD} E_3.dl$

So, $\int\limits_{AB} E_i dl + \int\limits_{CD} E_2.dl + \int\limits_{BC\,\&\,AD} E_3.dl = -\int \frac{\partial B}{\partial t}\hat{n}\, ds$

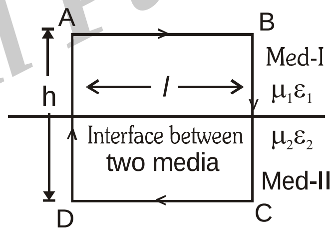


Fig (2)

If the height of the loop $h \to 0$ then contribution from BC & AD becomes zero

So, $\int\limits_{AB} E_1.dl + \int\limits_{CD} E_2.dl = -\int \frac{\partial B}{\partial t}\hat{n}\, ds$

(or) $\int\limits_{AB} E_1 dl + \int\limits_{CD} E_2.dl = 0$ $\left(\therefore \int \frac{\partial B}{\partial t}\hat{n}\, ds = 0\right)$

(or) E_1 AB + E_2 CD = 0

But AB = – CD

So $E_1\,AB - E_2\,AB = 0$

$E_1 - E_2 = 0$

$\boxed{E_{1||} - E_{2||} = 0}$ (or) $\boxed{E_{1||} = E_{2||}}$

$E_{1t} = E_{2t}$

This shows that the parallel component of electric field vector E is continuous at the surface

4. Boundary condition for Magnetic field vector H

From the Maxwell's fourth equation, we have

$$\nabla \times B = \mu_0\left(J + \frac{\partial D}{\partial t}\right) \qquad \{\because B = \mu_0 H\}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad .....(1)$$

Integrating eqn (1) over the surface of loop ABCD

$$\int_S (\nabla \times H)\hat{n}.ds = \int_S \left(J + \frac{\partial D}{\partial t}\right)\hat{n}\ ds$$

(or) $\int_S (\nabla \times H)\hat{n}\ ds = \int_S J\hat{n}\ ds + \int_S \frac{\partial D}{\partial t}\hat{n}\ ds$

(or) $\int_S (\nabla \times H)\hat{n}\ ds = \int_S J\hat{n}\ ds + \int_S \frac{\partial D}{\partial t}\hat{n}\ ds$

$\left\{\because \int_S \frac{\partial D}{\partial t}\hat{n}\ ds = 0 \text{ As D is arbitary \& bounded}\right\}$ (or) $\int_S (\nabla \times H)\hat{n}ds = \int_S J\hat{n}ds$ .....(2)

Converting surface integral on LHS into line integral using stokes theorem. We get

$$\int_S (\nabla \times H)\hat{n}\ ds = \int_C H.dl \qquad .....(3)$$

from eqn (2) and (3) we have

$$\int_C H.dl = \int_S J.\hat{n}\ ds$$

(or) $\int_{AB} H_1 dl + \int_{CD} H_2.dl + \int_{AD\&BC} H.dl = \int_S J\hat{n}\ ds$ .....(4)

As $h \to 0$, $\lim_{h \to 0} \int_{AD\&BC} H.dl = 0$

Now eqn (4) becomes $\int_{AB} H_1.dl + \int_{CD} H_2.dl = \int_S J\hat{n}\ ds$

(or) $H_1\ AB + H_2\ CD = J_{S\perp} \cdot S$

The $J_{S\perp}$ is known as perpendicular component of surface current density

Here AB = – CD

$H_1 AB - H_2 AB = J_{S\perp} AB$

or $\boxed{H_{1||} - H_{2||} = J_{S\perp}}$

$\boxed{H_{1t} - H_{2t} = J_{S\perp}}$

This shows that the tangential (or) parallel component of magnitude field vector is not continuous at the interface but it changes by $J_{S\perp}$.

Finally we may write

1. $D_{1n} - D_{2n} = \sigma$ (or) $D_{1\perp} - D_{2\perp} = \sigma$
2. $E_{1t} - E_{2t} = 0$ (or) $E_{1||} - E_{2||} = 0$
3. $B_{1n} - B_{2n} = 0$ (or) $B_{1\perp} - B_{2\perp} = 0$
4. $H_{1t} - H_{2t} = J_{S\perp}$ (or) $H_{1||} - H_{2||} = J_{S\perp}$

3.10 Plane Wave Equation

Q13. Derive plane wave equation?

Ans :

A uniform plane wave is a particular case of wave equation for which the electric field is independent of y and z and is a function of x and t only. Such a wave is called uniform plane wave

The plane wave equation can be written as

$$\frac{\partial^2 E}{\partial x^2} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad .....(1)$$

In terms of component of E, we have

$$\frac{\partial^2 E_x}{\partial x^2} = \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2}, \quad \frac{\partial^2 E_y}{\partial x^2} = \mu\varepsilon \frac{\partial E_y}{\partial t^2}, \quad \frac{\partial^2 E_z}{\partial x^2} = \mu\varepsilon \frac{\partial^2 E_z}{\partial t^2}$$

In a region in which there is no change density $(\rho = 0)$

$\therefore \quad \vec{\nabla}.E = 0$

i.e. $\dfrac{\partial E_x}{\partial x} + \dfrac{\partial E_y}{\partial y} + \dfrac{\partial E_z}{\partial z} = 0 .$ .....(2)

For a uniform plane wave in which E is independent of y & z $\dfrac{\partial E_y}{\partial y}$ and $\dfrac{\partial E_z}{\partial z} = 0$

$\therefore \quad \dfrac{\partial E_x}{\partial x} = 0$ or E_x = constant .....(3)

Equation (3) shows that there is variation of E_x in x-direction. Therefore, a uniform plane wave propagating in x direction has no component of E. Similarly, there is no component of H or B in X-direction. Therefore, the uniform plane electromagnetic waves are transverse and have components of E and H only in the directions perpendicular to the direction of propagation.

3.11 Transverse Nature of EM Waves

Q14. Show that Electromagnetic waves are transverse in nature.

Ans : **(Imp.)**

Consider the electromagnetic wave in which the components of E and B vary with one coordinate only (say x) and also with time t, i.e.

E = E (x, t) and B = B (x, t)

But $\vec{\nabla}.E = 0$ or $\dfrac{\partial E_x}{\partial x} + \dfrac{\partial E_y}{\partial y} + \dfrac{\partial E_z}{\partial z} = 0$

$$\frac{\partial E_x}{\partial x} = 0 \text{ or } E_x = \text{constant} \qquad .....(1)$$

Further $\vec{\nabla}.B = 0$ or $\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$

$$\frac{\partial B_x}{\partial x} = 0 \text{ or } B_x = \text{constant} \qquad .....(2)$$

Eqns (1) & (2) are obtained on the fact that the derivative of E and B with respect to y and z are zero

$$\text{curl } E = \frac{-\partial B}{\partial t}$$

i.e. $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Ex & Ey & Ez \end{vmatrix} = -\frac{\partial}{\partial t}\left[iB_x + jB_y + kB_z\right]$

Now $i\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] = -i\frac{\partial B_x}{\partial t} = 0 \qquad .....(3)$

3.12 VELOCITY OF LIGHT IN VACUUM AND IN MEDIUM

Q15. Derive the velocity of light in vacuum and in medium.

Ans :

The following are the represents the relation between the space and time variation of magnetic field B and electric field E.

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad ... (1)$$

This expression is the wave equation for propagation of electric field E in free space.

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \qquad ... (2)$$

These are called wave equations for B and E. The general wave equation is represented by

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{... (3)}$$

Where v is the velocity of wave, On comparing eqns (3) and (4) of case(ii), we find that the variations of E and B are propagated in homogeneous, isotropic medium with a velocity given by

$$\frac{1}{v^2} = \mu\varepsilon$$

$$v^2 = \frac{1}{\mu\varepsilon}$$

$$v = \sqrt{\frac{1}{\mu\varepsilon}}$$

$$v = \sqrt{\frac{1}{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{.....(4)}$$

Where μ and ε are permeability and permittivity of the medium.

Therefore, electric field vector E and magnetic field vector B are propagating in space according to the wave equation with velocity $v = \frac{1}{\sqrt{\mu\varepsilon}}$. These waves are commonly referred to as electromagnetic waves.

For free space $v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$\mu_0 = 4\pi\times10^{-7}$ and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

$$\varepsilon_0 = \frac{1}{4\pi\times9\times10^9}$$

$$v = \frac{1}{\sqrt{\left[\left(4\pi\times10^{-7}\right)\times\left(\frac{1}{4\pi\times9\times10^{9}}\right)\right]}}$$

$$v = 3 \times 10^8 \text{ m/sec}$$

Thus, the velocity of propagation of variation of E and B is the same as the velocity of light.

3.13 Poynting's Theorem

Q16. Write about poynting vector.

Ans :

The rate of energy travelled through per unit area i.e., the amount of energy flowing through per unit area in the perpendicular direction to the incident energy per unit time is called "Poynting vector".

Mathematically poynting vector is

Represented as $\vec{P} = \vec{E} \times \vec{H} \Rightarrow \frac{\vec{E}\times\vec{B}}{\mu}$

- Here the direction of poynting vector is perpendicular tot he plane containing $\vec{E}$ and $\vec{H}$.
- Poynting vector is also called as "instantaneous energy flux density".
- Here rate of energy transfer $\vec{P}$ is perpendicular to both $\vec{E}$ and $\vec{H}$. Since it represents the rate of energy transfer per unit area, its unit is w/m^2.

Q17. State and prove poynting's theorem?

(OR)

Explain energy conservation law in electromagnetism?

Ans : **(Imp.)**

Statement

The net power flowing out of a given volume 'V' is equal to the time rate of decrease of stored electromagnetic energy in that volume decreased by the conduction losses that volume decreased by the conduction losses.

i.e., Total power leaving the volume.

= Rate of decrease of stored electromagnetic energy – Ohmic power dissipated due to motion of charge.

Proof :

The energy density carried by the electromagnetic wave can be calculated using maxwell's equations.

as

$$\text{div } \vec{D} = 0 \qquad \text{... (i)}$$

$$\text{div } \vec{B} = 0 \qquad \text{... (ii)}$$

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{... (iii)}$$

and curl $\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

Taking scalar product of (iii) with 'H' and (iv) with $\vec{e}$

i.e., $\vec{H} \text{ curl } \vec{E} = -\vec{H} \cdot \dfrac{\partial \vec{B}}{\partial t}$ ... (v)

$\vec{E} \text{ curl } \vec{H} = \vec{E} - \vec{J} + \vec{E} \dfrac{\partial \vec{D}}{\partial t}$ ... (vi)

doing (vi) – (v) i.e.,

$$\vec{H} \text{ curl } \vec{E} - \vec{E} \text{ curl } \vec{H} = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \left[\vec{E}.\vec{J} + \vec{E}.\frac{\partial \vec{D}}{\partial t}\right]$$

$$= -\vec{H}\frac{\partial \vec{B}}{\partial t} - \vec{E} \cdot \vec{J} - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$= -\left[\vec{H}\frac{\partial \vec{B}}{\partial t} + \vec{E}\frac{\partial \vec{D}}{\partial t}\right] - \vec{E} \cdot \vec{J}$$

as div $(\vec{A} \times \vec{B}) = \vec{B}$. curl $\vec{A} - \vec{A}$ curl B

So, div $(\vec{E} \times \vec{H}) = -\left[\vec{H}.\dfrac{\partial \vec{B}}{\partial t} + \vec{E}.\dfrac{\partial \vec{D}}{\partial t}\right] - \vec{E} \cdot \vec{J}$ ... (vii)

But $\vec{B} = \mu\vec{H}$ and $\vec{D} = \varepsilon\vec{E}$

Substituting $\vec{B}$ and $\vec{D}$ values in the equation

$$\vec{H} \cdot \frac{\partial\vec{B}}{\partial t} = \vec{H} \cdot \frac{\partial}{\partial t}(\mu\vec{H}) = \tfrac{1}{2} \quad \mu\frac{\partial}{\partial t}(H^2)$$

and $\vec{E} \cdot \frac{\partial\vec{D}}{\partial t} = \vec{E} \cdot \frac{\partial}{\partial t}(\varepsilon\vec{E}) \Rightarrow \frac{1}{2}\ \varepsilon\ \frac{\partial}{\partial t}(\in)^2 \Rightarrow \frac{\partial}{\partial t}\left[\frac{1}{2}\vec{E}.\vec{D}\right]$

So from equation (vii)

$$\text{div } (\vec{E}\times\vec{H}) = -\frac{\partial}{\partial t}\left[\frac{1}{2}(\vec{H}\,.\,\vec{B} + \vec{E}\,.\vec{D})\right] - \vec{E}\,.\,\vec{J}$$

(or)

$$\vec{E}\,.\,\vec{J} = -\frac{\partial}{\partial t}\left[\frac{1}{2}(\vec{H}\ .\ \vec{B} + \vec{E}\ .\vec{D})\right] - \text{div } (\vec{E}\times\vec{H}) \qquad \text{... (viii)}$$

Integrating equation (viii) over a volume (v) enclosed by a surface 'S'.

$$\int_V \vec{E}\,.\,\vec{J}\,dV = -\int_V \left|\frac{\partial}{\partial t}\left\{\frac{1}{2}(\vec{H}.\vec{B} + \vec{E}.\vec{D})\right\}\right| dV \ -\int_V \text{div}(\vec{E}\times\vec{H})\,dV$$

$$\int_V \vec{E}\,.\,\vec{J}\,dV = -\int_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right] dV - \int_S (\vec{E}\times\vec{H})\,.dS$$

as $\vec{B} = \mu\vec{H}$, $\vec{D} = \varepsilon\vec{E}$ and $\int_V \text{div}\left[\vec{E}\times\vec{H}\right]dV = \int_S (\vec{E}\times\vec{H}).dS$

$$\int_V \left[\vec{E}\times\vec{H}\right]dV = -\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\in H^2\right]dV - \int_S (\vec{E}\times\vec{H})dS$$

$$\int_S \left[\vec{E}\times\vec{H}\right]dS = \int_V \frac{\partial U_{em}}{\partial t}dV - \int_V (\vec{E}.\vec{J})dV$$

(or) $$\boxed{\int_S \vec{P}.d\vec{S} = -\int_V \frac{\partial U_{em}}{\partial t} dV - \int_V (\vec{E}.\vec{J}) dv}$$ ... (ix)

(as $\vec{P} = \vec{E} \times \vec{H}$)

i.e., Total power leaving the volume

= Rate of decrease of stored electromagnetic energy - Ohmic power dissipated due to charge motion.

- This equation (ix) represents the poynting theorem according to which the net power flowing out of a given volume is equal to the rate of decrease of stored electromagnetic energy in that volume minus the conduction losses.
- In equation (ix) $\int_S \vec{P}.ds$ represents the amount of electromagnetic energy of crossing the closed surface per second (or) the rate of flow of outward energy thought he surface "S" enclosing volume "V" i.e., it is poyinting vector.
- The term $\int_V \frac{\partial U_{em}}{\partial t}$ dV (or) $\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right] dV$, here, $\frac{1}{2}\mu H^2$ and $\frac{1}{2}\varepsilon H^2$ represent the energy stored in electric and magnetic fields reprectively and their sum denotes the total energy stored in electromagnetic field.
- So, total terms gives the ratio of decrease of energy stored in volume 'V' due to electric and magnetic fields.

  $\int_V (\vec{E}.\vec{J}) dV$ gives the rate of energy transferred into the electromanetic field.
- This is also known as work - energy theorem

  (or)

  The energy conservation law in electromagnetism.

Problems

1. A conducting loop of 4 ohm is in plane of paper. It has a uniform induction B over its area of 0.002 m² . The direction of B is normal to the plane of loop. Calculate induced current is B is decreasing at the rate of 0.1 weber/m²

Sol :

Given B =0.1 R = 4 Ω

Area 0.002 m^2

Rate of change of magnetic flux linked with the loop.

$$d\phi = B \times \text{area} = 0.1 \times 0.002$$

$$= 2 \times 10^{-4} \text{ weber}$$

Induced emf $e = \dfrac{-d\phi}{dt} = \dfrac{-2 \times 10^{-4}}{1} = -2 \times 10^{-4}$ volt

Induced current $i = \dfrac{e}{R} = \dfrac{2 \times 10^{-4}}{4} = 0.5 \times 10^{-4}$ ampere

2. A coil of 5 lurns has dimension 9cm × 7 cm. It rotates at the rate of 15 π rad /sec in uniform magnetic field whose flux density is 0.8 weber/m² what is the maximum e.m.f induced in the coil.

Sol :

Let at any instant during rotation the normal to the plane of coil makes an angle 'θ' with field B.

Then the flux through each turn of coil is $\phi_B = B.A = BA\cos\theta$

If the coil has N number of turns the induced emf is given by

$$e = -N\frac{d\phi_B}{dt}$$

$$= -N\frac{d}{dt}(BA\cos\theta)$$

$$= NBA \sin\theta \frac{d\theta}{dt}$$

$$e = NBA\ \omega \sin\theta$$

e_{max} = NBA ω [$\therefore$ max. value of $\sin\theta$ = 1]

Given N = 5 B = 0.8 A = 9cm × 7cm

$$\omega = 15\pi$$

$$\therefore \quad e_{max} = 5 \times 0.8 \times 9 \times 7 \times 10^{-4}\, m^2 \times 15\pi$$

= 1.188 volt

3. A coil has 600 turns. Its self inductance is 50 m.H. Find the self inductance of same type of coil having 200 turns.

Sol :

Given N_1 = 600 N_2 = 200

L_1 = 50 mH L_2 = ?

We know that $L \alpha N^2$

$$\frac{L_1}{L_2} = \frac{N_1^{\ 2}}{N_2^{\ 2}}$$

$$L_2 = \frac{L_1 N_2^{\ 2}}{N_1^{\ 2}} = \frac{50 \times 200 \times 300}{600 \times 600}$$

= 5.5 mH

4. What is the self inductance of a 100 cm long solenoid with 1 cm diameter and having 50 turns

Sol :

Given l = 100 cm, d = 1 cm = 10^{-2} m

l = 1m

N = 50 turns

$$a = \pi r^2 = \pi \times \left(\frac{10^{-2}}{2}\right)^2 = \frac{\pi}{4} \times 10^{-4}$$

WKT self inductance of solenoid $L = \frac{\mu_0 N^2 a}{\ell}$

$$L = \frac{4\pi \times 10^{-7} \times 50 \times 50 \times 10^{-4} \times \pi}{1 \times 4}$$

$= 246.49 \times 10^{-9}$ henry

5. A 20 henry inductor carries a steady current of 2 amp. How can a 200 volt self induced emf be made to appear in the inductor.

Sol :

Given L = 20 henry

i = 2 amp e = 200 volt

WKT $e = -L\frac{di}{dt}$

$\frac{di}{dt} = \frac{e}{L} = \frac{200}{20} = 10$ amp /sec

6. A coil has an inductance 100 mH and 200 turns. Calculate the flux linked with it when 20×10^{-3} A current is passed through it.

Sol :

Given L = 100 mH = 100 × 10^{-3} H

i = 20 × 10^{-3} N = 200 turns

We know that $Li = N\phi$

$$\phi = \frac{Li}{N}$$

$$\phi = \frac{100 \times 10^{-3} \times 20 \times 10^{-3}}{200}$$

$= 10^{-7}$ weber

7. Calculate the self inductance of an air cored toroid of mean radius 20 cm and circular cross section of area 5 cm². The total number of turns of the toroid is 3000

Sol :

Given N = 3000 A = 5 cm^2 = 5 × 10^{-4} m^2

R = 20 cm = 0.2 m

We know that the self inductance of an air core toroidal solenoid is given by L

$$= \frac{\mu_0 N^2 A}{2\pi R}$$

$$L = \frac{4\pi \times 10^{-2} \times 3000 \times 3000 \times 5 \times 10^{-4}}{2\pi \times 0.2}$$

= 4. 5 × 10^{-3} henry

8. The current in the primary circuit of coils changes from 10 amp to 8 amp in time of also. Find the induced emt in recondary coil. The mutual inductance between two coils is given to be 2H.

Sol :

Given M = 2

i_1 = 10 amp i_2 = 8 amp t = 0.1 sec

WKT $e = -M\left(\frac{di}{dt}\right)$

$$e = 2 \times \frac{(10-8)}{0.1} = 40 \text{ volt}$$

9. A coil of 100 turns and 1 cm radius is kept coaxially within a long solenoid of 8 turns per cm and 5 cm radius. Find the mutual inductance.

Sol :

Magnetic field 'B' in primary coil of solenoid is given by B = $\mu_0 N_p . i$

Magnetic flux linked with the secondary coil is given by

$$N_S\phi_B = N_S B A_S = N_S.(\mu_0 N_P i) \times A_S$$

Where A_S is cross - sectional area of secondary coil of solenoid.

$$\therefore \quad \text{Mutual inductance } M = \frac{N_S\phi_B}{i}$$

$$= N_S \frac{(\mu_0 N_P i) \times A_S}{i}$$

$$= \mu_0 N_P N_S.A_S$$

Substituting the given values

$$M = 4\pi \times 10^{-7} \times 800 \times 100 \times \pi \times 10^{-4}$$

$$= 3.15 \times 10^{-5} \text{ henry.}$$

10. Calculate the mutual inductance between two coils when current of 4 amp changes to 12 amp in 0.5 sec. and induces an emf of 50 milli-volt in secondary. Also calculate the induced emf in the secondary if current primary changes from B amp to 9 amp in 0.02 sec.

Sol :

(i) $\frac{di}{dt} = \frac{4-12}{0.5} = -16$ amp/sec

$$e = -M\frac{di}{dt} \Rightarrow M = {-e}\Big/{\frac{di}{dt}}$$

$$M = \frac{50 \times 10^{-3}}{16} = 3.125 \times 10^{-3} \text{ amp/sec}$$

(ii) $\frac{di}{dt} = \frac{3-9}{0.02}$

$= -300$ amp/sec

$e = -(3.125 \times 10^{-3})(-300)$

$= 0.9375$ volt

Short Question and Answers

1. State and Explain Lenz's law?

Ans :

According to Lenz's Law, the direction of induced e.m.f (or current) in a closed circuit is such that it opposes the original cause that produces it.

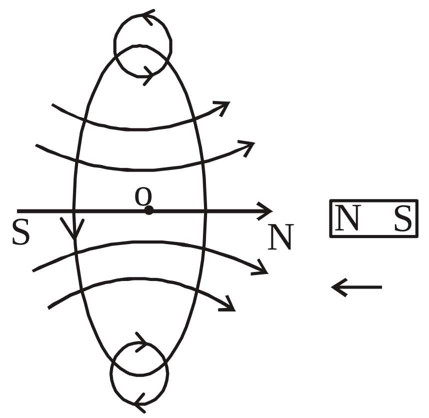
This Law is based on the principle of conservation of energy.

When the applied flux density B in a closed circuit is increasing, the e.m.f or current induced in the closed circuit is in such a direction as to produce a field which tends to decrease B.

When the applied flux density is decreasing in magnitude the current in the closed circuit is in such a direction as to produce a field which tends to increase B.

Thus, the induced current is in a direction such that it produces a magnetic flux tending to oppose the original change of flux. i.e total flux in the circuit is constant.

Suppose the north pole of the magnet is moved towards a coil connected to a galvanometer as shown in the Figure.

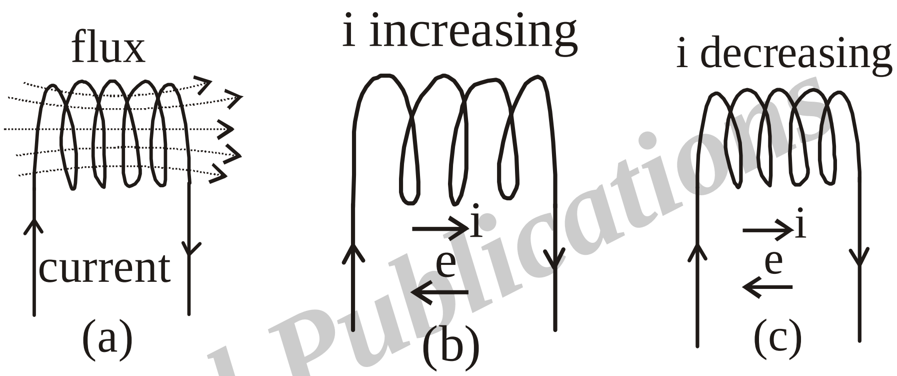


As the magnet is pushed towards the circuit, an induced current is setup in the coil. The induced current produces its own magnetic field. Now coil behaves as magnet. The face of the coil towards the north pole of the magnet between them. Due to this, motion of the magnet is opposed. This causes a change of magnetic flux in the coil. Thus, the direction of induced current is such that as to oppose motion of the magnet.

2. Derive the coefficient of self induction?

Ans :

By Lenz's law, the direction of induced e.m.f. is such as to oppose the change in current



When the current is increasing, the induced e.m.f is against the current [shown in (b)]. When the current is decreasing induced e.m.f is in the direction of current [shown in (c)]. So, the induced e.m.f opposes any change of the original current. This phenomenon is called self induction.

The total magnetic flux ϕ_B linked with the coil is proportional to current i, flowing in it.

$\phi_B \alpha i$ or ϕ_B = Li ....(1)

When L is a constant called the coefficient of self induction when i =1, ϕ_B = L.

"Hence, the coefficient of self induction is numerically equal to the magnetic flux linked with the coil when unit current flows through it."

3. Mention the differences between self induction and mutual induction?

Ans :

S.No.	Self Induction	S.No.	Mutual Induction
1)	When the current flowing in a coil is changed, induced current is produced in the coil itself. This phenomena is known as self induction.	1)	When current flowing in a coil is is changed then induced current is produced in another coil which is kept in vicnity of the first coil. This phenomena is called mutual induction.
2)	The induced current affects the main current flowing in the coil.	2)	The induced current flows in other coil. Thus it does not directly affect the main current of primary coil.
3)	There is only one coil in it and the other is secondary.	3)	There are two coils. One is primary

4. What is displacement current?

Ans :

The current in the conductor produces the magnetic field.

Maxwell stated that a changing electric field in vacuum or in dielectric also produces a magnetic field. So, a changing electric field is equivalent to a current which flows as long as the electric field is changing and produces the same magnetic effect as an ordinary conduction current. This is known as displacement current.

Q5. State and Explain Faraday's laws of Electromagnetic Induction.

Ans :

There are two laws of electromagnetic induction. They are

(1) Whenever the magnetic flux linked with a circuit is changed, an e.m.f is induced in the circuit.

(2) The magnitude of induced emf is directly proportional to the negative rate of variation of magnetic flux linked with the circuit.

If ϕ_B be the magnetic flux linked with circuit at any instant and e be the induced e.m.f., then

$$e = -\left(\frac{d\phi_B}{dt}\right) \quad .....(1)$$

This law is also known as Neumann's's Law.

If there are N turns in the coil, then

$$e = -N\left(\frac{d\phi_B}{dt}\right)$$

Q6. Define Mutual Induction.

Ans :

Consider two coils placed near to each other as shown in (fig) When a current is passed in primary coil P, there is a change of magnetic flux linked with it and an induced emf is set in the secondary coil s. This phenomenon is called mutual inductance. Similarly, the secondary circuit also induces on e.m.f in primary.

Let a current i amp. in primary P produces a magnetic flux ϕ_B in the secondary S. For two given coils situated in fixed relative positions, it is observed that the flux linked with the secondary is proportional to the current in primary. Thus

$$\phi_B \alpha i$$

$$\phi_B = Mi \quad .....(1)$$

Where M is constant called the coefficient of Mutual inductance of two coils.

Q7. Define continuity equation write its expression.

Ans :

"The differential equation that gives relationship between the current density (j) and volume charge density (ρ) at a point in a closed circuit is known as equation of continuity".

$$\boxed{\nabla.j + \frac{\partial\rho}{\partial t} = 0}$$

Known as equation of continuity and used for varying Electric currents. When current is static i.e. no change in value with time

$$\frac{\partial \rho}{\partial t} = 0 \qquad \therefore \quad \nabla \cdot j = 0$$

Q8. Write Maxwell's equations in differential and integral forms.

Ans :

There are basic laws formulated by Maxwell for electricity and magnetism. These equations are known as Maxwell's equations.

The integral forms of these equations are

$$\oint E.ds = \left(\frac{q}{\varepsilon_0}\right) \qquad .....(1)$$

$$\oint B.ds = 0 \qquad .....(2)$$

$$\oint E.dl = \frac{-d\phi_B}{dt} \qquad .....(3)$$

$$\oint B.dl = \mu_0\left(j + \varepsilon_0 \frac{\partial E}{\partial t}\right) \qquad .....(4)$$

Maxwell's equations can also be stated in the differential forms as follows

$$\text{div } E = \frac{\rho}{\varepsilon_0} \qquad .....(5)$$

$$\text{div } B = 0 \qquad .....(6)$$

$$\text{curl } E = \frac{-\partial B}{\partial t} \qquad .....(7)$$

$$\text{curl } B = \mu_0\left[j + \varepsilon_0 \frac{\partial E}{\partial t}\right] \qquad .....(8)$$

9. Explain the transverse nature of em waves?

Ans :

Consider the electromagnetic wave in which the components of E and B vary with one coordinate only (say x) and also with time t, i.e.

E = E (x, t) and B = B (x, t)

But $\vec{\nabla}.E = 0$ or $\dfrac{\partial E_x}{\partial x} + \dfrac{\partial E_y}{\partial y} + \dfrac{\partial E_z}{\partial z} = 0$

$$\frac{\partial E_x}{\partial x} = 0 \text{ or } E_x = \text{constant} \qquad .....(1)$$

Further $\vec{\nabla}.B = 0$ or $\dfrac{\partial B_x}{\partial x} + \dfrac{\partial B_y}{\partial y} + \dfrac{\partial B_z}{\partial z} = 0$

$$\frac{\partial B_x}{\partial x} = 0 \text{ or } B_x = \text{constant} \qquad .....(2)$$

Eqns (1) & (2) are obtained on the fact that the derivative of E and B with respect to y and z are zero

$$\text{curl } E = \frac{-\partial B}{\partial t}$$

$$\text{i.e} \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Ex & Ey & Ez \end{vmatrix} = -\frac{\partial}{\partial t}\left[iB_x + jB_y + kB_z\right]$$

$$\text{Now } i\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] = -i\frac{\partial B_x}{\partial t} = 0 \qquad .....(3)$$

10. Explain the types of polarisation.

Ans :

Polarization

The light which has acquired the property of one sidedness is known as polarization.

In the case of an Electromagnetic wave, the electric field vector E, magnetic field vector H and propagation wave vector K are mutually perpendicular to each other. Such EM waves are known as plane polarized EM waves.

There are three different types of polarised waves. They are

(i) Linearly polarized wave

(ii) Elliptically polarized wave

(iii) Circularly polarized wave

Linearly Polarized wave

In an EM wave, if electric field vector E maintains a fixed direction with respect to the direction of wave propagation then it is said to be linearly polarized wave.

Elliptically Polarized wave

When E_1 and E_2 have different phases and different magnitudes then the wave is said to be Elliptically polarized because the Electric field vector E will trace out an Elliptical path If the wave is propagating along the z-axis.

Circularly Polarized Wave

When E_1 and E_2 have same magnitudes and different phases then the wave is said to be circularly polarised because the electric field vector E will trace out a circular path.

11. Derive the expression for velocity of light in vacuum and medium?

Ans :

The general wave equation is represented by

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \qquad .....(1)$$

Where v is the velocity of wave on comparing eqns (10) and (11), of article (1) we find that the variations of E and B are propagated in homogeneous, isotropic medium with a velocity given by

$$\frac{1}{v^2} = \mu\varepsilon$$

$$v^2 = \frac{1}{\mu\varepsilon}$$

$$v = \sqrt{\frac{1}{\mu\varepsilon}}$$

$$v = \sqrt{\frac{1}{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad .....(2)$$

Where μ and ε are permeability and permittivity of the medium.

Therefore, electric field vector E and magnetic field vector B are propagating in space according to the wave equation with velocity $v = \frac{1}{\sqrt{\mu\varepsilon}}$. These waves are commonly referred to as electromagnetic waves.

For free space $v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\mu_0 = 4\pi \times 10^{-7} \text{ and } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$$

$$\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9}$$

$$v = \frac{1}{\sqrt{\left[\left(4\pi \times 10^{-7}\right) \times \left(\frac{1}{4\pi \times 9 \times 10^9}\right)\right]}}$$

$$v = 3 \times 10^8 \text{ m / sec}$$

Q12. Write Maxwells em wave equation for E&B in dielectric medium vaccum.

Ans :

Indielectric medium

$$\boxed{\nabla^2 B = \mu\varepsilon \frac{\partial^2 B}{\partial t^2}}$$

$$\nabla^2 E = \mu\varepsilon\left(\frac{\partial^2 E}{\partial t^2}\right)$$

In free space

$$\nabla^2 E = \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 B = \mu_0\varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

Q13. Write boundary conditions for D, E, B, H.

Ans :

1. $D_{1n} - D_{2n} = \sigma$ (or) $D_{1\perp} - D_{2\perp} = \sigma$
2. $E_{1t} - E_{2t} = 0$ (or) $E_{1||} - E_{2||} = 0$
3. $B_{1n} - B_{2n} = 0$ (or) $B_{1\perp} - B_{2\perp} = 0$
4. $H_{1t} - H_{2t} = J_{s\perp}$ (or) $H_{1||} - H_{2||} = J_{s\perp}$

Q14. Explain the terms polarization reflection & transmission.

Ans :

Polarization

The light which has acquired the property of one sidedness is called polarised light.

When the vibrations are confined along a single direction at right angles to the direction of propagation, the light is said as plane - polarised. If the vibrations are along a circle or an ellipse lying in the plane normal to the direction of propagation, the line is said to be circularly or elliptically polarised respectively.

Reflection

When a beam of light is incident on a surface, a part of it is returned back into the same medium. The part of light which is returned back into the same medium is called reflection of light.

Laws of reflection.

(i) Angle of incidence is equal to angle of reflection

(ii) The incident ray, reflected say and the normal at the point of incidence must lie on the same plane.

Transmission

Transmission is the passage of electromagnetic radiation through a medium.

Choose the Correct Answer

1. Differential form of Faraday's law (b)

   (a) $\nabla \times B = \frac{-\partial E}{\partial t}$ (b) $\nabla \times E = \frac{-\partial B}{\partial t}$

   (c) $\nabla \times E = \frac{\partial B}{\partial t}$ (d) None

2. According to Lenz law the total flux in the circuit remains (a)

   (a) constant (b) changes

   (c) increases (d) decreases

3. 1 henry = ______ (b)

   (a) $\frac{\text{volt} - \text{ampere}}{\text{second}}$ (b) $\frac{\text{volt}}{\text{amp.second}}$

   (c) $\frac{\text{second}}{\text{volt.amp}}$ (d) $\frac{\text{second. Amp}}{\text{volt}}$

4. Expression for self inductance of toroid (c)

   (a) $\frac{\mu_0 NA}{2\pi R}$ (b) $\frac{2\pi R}{\mu_0 N^2 A}$

   (c) $\frac{\mu_0 N^2 A}{2\pi R}$ (d) $\frac{\mu_0 NA}{2\pi R^2}$

5. Which one is microscopic quantity (b)

   (a) Current (i) (b) Current density 'j'

   (c) a + b (d) None

6. Equation of continuity is given by (c)

   (a) $\nabla.j = 0$ (b) $\nabla.j - \frac{\partial \rho}{\partial t} = 0$

   (c) $\nabla.j + \frac{\partial \rho}{\partial t} = 0$ (d) $\frac{\partial \rho}{\partial t} = 0$

7. The Extra term added in modification of Amperes law (d)

(a) $\mu_0\varepsilon_0 \frac{\partial E}{\partial t}$ (b) $\varepsilon_0 \frac{\partial E}{\partial t}$

(c) $\mu_0 \frac{\partial D}{\partial t}$ (d) a & c

8. Which one is incorrect (b)

(a) div E = ρ/ε_0 (b) div B = ρ

(c) cur E = $\frac{-\partial B}{\partial t}$ (d) curl B = $\mu_0\left[j+\varepsilon_0 \frac{\partial E}{\partial t}\right]$

9. Which one is correct expression for Maxwell equation in dielectric [c]

(a) $\nabla \times E = \rho$ (b) $\nabla \times B = \varepsilon_0$

(c) $\nabla \times E = 0$ (d) $\nabla \times E = \rho/\varepsilon_0$

10. The general wave equation is represented by [a]

(a) $\nabla\psi = v^2 \frac{\partial^2\psi}{\partial t^2}$ (b) $\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$

(c) $\nabla^2\psi = \frac{\partial^2\psi}{\partial t^2}$ (d) None

11. Boundary condition for 'H' is given by [b]

(a) $H_1 \perp - H_2 \perp = J_{S\perp}$ (b) $H_1 || - H_2 || = J_{S\perp}$

(c) $H_1 || - H_2 || = J_{S||}$ (d) None

12. Perpendicular component of 'B' is ________ at the interface [a]

(a) continuous (b) dis continuous

(c) more dis continuous (d) none

13. Condition for circularly polarised wave [d]

(a) $E_x{}^2 + E_y{}^2 = 0$ (b) $E_x{}^2 + E_y{}^2 = E_z{}^2$

(c) $E_z{}^2 + E_y{}^2 = E_x{}^2$ (d) $E_x{}^2 + E_y{}^2 = E_0{}^2$

14. The polarising angle for air - glass is [c]

(a) 58^0 (b) 90^0

(c) 57^0 (d) 56^0

Fill in the blanks

1. According Faradays law magnetic flux linked with circuit is changed and ______ induced in the circuit.
2. $e = \frac{-d\phi_B}{dt}$ is also known as ______ law
3. Lenz law is based on the principle of ______
4. Expression for self inductance of solenoid is given by ______
5. In Mutual induction phenomena flux linked with the secondary is propotional to current ______ in coil
6. Equation of continuity is used for only ______ Electric currents.
7. Expression for Modification of Amperes law ______
8. Differential form of Maxwells 3[rd] equation ______
9. In homogeneous, isotropic medium value of the charge density ______
10. Velocity of light in vaccum is inversly propational to square root of ______
11. Electric magnetic waves ______ in nature.
12. EM waves are also known as ______ EM waves.
13. In Maxwell equations for dielectrics $\nabla \times B =$ ______

Answers

1. emfs
2. Neumann's law
3. Conservation of energy
4. $\frac{\mu_0 N^2 A}{\ell}$
5. Primary
6. Varying
7. $\nabla \times B = \mu_0 \left[j + \varepsilon_0 \frac{\partial E}{\partial t} \right]$
8. div B = 0
9. Zero
10. Refractive index
11. Transverse
12. Polarized
13. 0 (zero)

One Mark Answers

1. Define Displacement current ?

Ans :

Changing Electric field is equivalent to current which flows as long as the Electric field is changing & produces the same magnetic effect as an ordinary conduction current known as displacement current.

2. Define self induction.

Ans :

When the current flowing in a coil is changed induced current is produced in the coil itself. This phenomenon is known as self induction.

3. Define Mutual induction

Ans :

When the current flowing in a coil is changed then induced current is produced in another coil which is kept in the viclinity of first coil. This phenomenon is called Mutual induction.

4. Define equation of continuity?

Ans :

The differential equation that gives the relation ship between the current density (j) and volume density (ρ) at a point in closed circuit is known as equation of continuity.

5. Mention one difference between self Mutual induction.

Ans :

In self induction only one coil will be used. There are two coils in Mutual induction

(i) Primary coil

(ii) Secondary coil

Q6. Write the Boundary conditions for E & D.

Ans :

The boundary condition for ‘D’ is

$$D_1 \perp - D_2 \perp = \sigma$$

Thus Normal component of displacement vector is not continuous at boundary

The boundary condition for ‘E’ is

$$E_1 \,||\, - E_2 \,||\, = 0$$

The Parallel component of Electric field vector E is continuous at surface.

Q7. Write Maxwell wave equations for a dielectric.

Ans :

$$\nabla \cdot E = 0\,, \quad \nabla \cdot B = 0\,, \quad \nabla \times E = \frac{-\partial B}{\partial t}\,, \quad \nabla \times B = \mu\varepsilon \frac{\partial E}{\partial t}$$

Q8. Define Brewsters angle.

Ans :

The angle of incidence at which light with a particular polarisation is perfectly transmitted through a transperent dielectric surface, with no reflection.

Q9. What is meant by reflection

Ans :

When a beam of light is incident on a surface a part of it is returned back into the same medium. The part of light which is returned back into same medium is called reflection of light.

Q10. What is meant by polarisation.

Ans :

The light which has acquired the property of one sided ness is called polarisation.

UNIT IV	**Varying and alternating currents:** Growth and decay of currents in LR. CR and LCR circuits - Critical damping. Alternating current, relation between current and voltage in pure R, C and L-vector diagrams - Power in ac circuits'. LCR series and parallel resonant circuit - Q-factor. AC & DC motors-single phase, three phase (basics only). **Network Theorems:** Passive elements, Power sources, Active elements, Network models: T and n Transformations, Superposition theorem, Thevenin's theorem, Norton's theorem. Reciprocity theorem and Maximum power transfer theorem (Simple problems).

4.1 Growth and Decay of Currents in LR. CR and LCR Circuits

Q1. Write a brief note on Growth of current in LR circuit?

Ans : **(Imp.)**

Consider a circuit consisting of a battery of a steady emf E, inductance L and a resistance R as shown in figure (a).

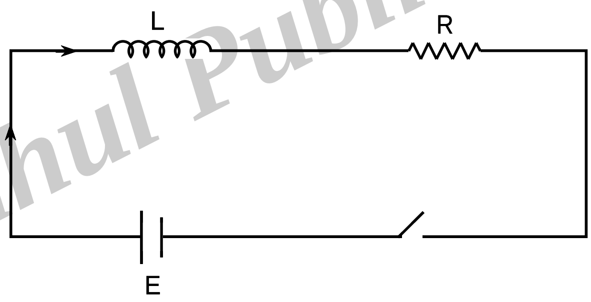


Figure (a)

When the key is suddenly pressed, there is growth of current in the circuit and a back emf is induced. Let I be the current at any instant of time "t" then,

$$E = RI + L\frac{dI}{dt} \qquad ... (1)$$

When the current reaches maximum value I_0, the back emf $L\frac{dI}{dt} = 0$

$$\therefore E = RI_0 \qquad .... (2)$$

From equations (1) and (2) we have

$$RI_0 = RI + L\frac{dI}{dt}$$

$$R(I_0 - I) = L\frac{dI}{dt} \quad \text{... (3)}$$

Taking $(I_0 - I) = x$

Differentiating with respect to time, we get

$$-\frac{dI}{dt} = \frac{dx}{dt}$$

Substituting this is equations (3), we get

$$Rx = -L\frac{dx}{dt}$$

$$\frac{dx}{x} = -\frac{R}{L}dt$$

$$\log_e^x = -\frac{R}{L}t + k$$

where "k" is a constant

$$\therefore \quad \log_e(I_0 - I) = -\frac{R}{L}t + k$$

when t = 0, I = 0,

$$\therefore \quad \log_e I_0 = k$$

$$\therefore \log_e(I_0 - I) = -\frac{R}{L}t + \log_e I_0$$

$$\log_e(I_0 - I) - \log_e I_0 = -\frac{R}{L}t$$

$$\log_e\left(\frac{I_0 - I}{I_0}\right) = -\frac{R}{L}t$$

$$\frac{I_0 - I}{I_0} = e^{-R/L\,.\,t}$$

$$1 - \frac{I}{I_0} = 1 - e^{-\frac{R}{L}.t}$$

$$\boxed{I = I_0(I - e^{-\frac{R}{L}.t})}$$

The quantity L/R is called the time constant of the circuit.

Time Constant

The quantity $\frac{L}{R}$ has the dimension of time and is called the time constant (λ) of the L–R circuit.

If $\frac{L}{R} = t$, then $I = I_0\,(1 - e^{-1})$

$$I = I_0\,(1 - 1/e)$$

$$\boxed{I = 0.632\, I_0}$$

Thus,

The time constant I/R of a L–R circuit is the time token by the current to grow from zero to 0.632 time the maximum value of current $-I_0$ in the circuit.

The graph between current and time at the time of the growth of current is shown in figure (b).

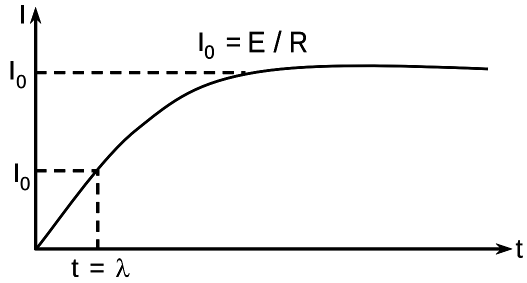


Figure (b)

Q2. Explain about decay of current in L - R circuit.

Ans :

When the circuit is broken, an induced emf, equal to $-L\frac{dI}{dt}$ is again produced in the inductance L and and its slow down and decay to zero. The current in the circuit decays from maximum value I_0 to zero. During the decay, let I be the current at time t. In this case E = 0.

The emf equation for the decay of current is

$$0 = RI + L\frac{dI}{dt} \qquad \text{... (1)}$$

$$\therefore \quad \frac{dI}{I} = -\frac{R}{L}dt$$

Integrating, $\log_e I = -\frac{R}{L}t + k,$ where k is a constant.

When t = 0, I = I_0

$$\therefore \quad \log_e I_0 = k$$

$$\therefore \quad \log_e I = -\frac{R}{L}t + \log_e I_0$$

$$\text{Log}\ \frac{I}{I_0} = -\frac{R}{L}t$$

$$\frac{I}{I_0} = e^{-R/L\, t}$$

$$\boxed{I = I_0\, e^{-R/L\, t}}$$

Equations (2) represents the current at any instant “t” dwing decay. A graph between current and time during decay is shown in figure (a).

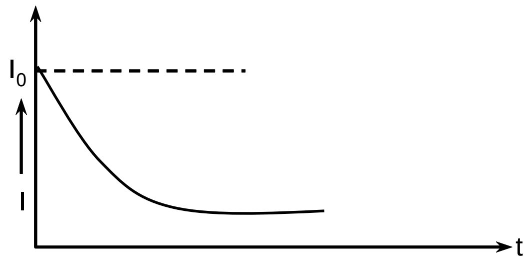


Figure (a)

Time Constant

$t = 1/R$, $\therefore$ $I = I_0 e^{-1}$

$= I_0\, 1/e$

$$\boxed{I = 0.365\, I_0}$$

$\therefore$ The time constant $\frac{L}{R}$ of a L – R circuit may also be defined as the time in which the current in the circuit falls to 1/e of its maximum value when emf is removed.

Figure (b) shows that the growth and decay curves are complementary with each other.

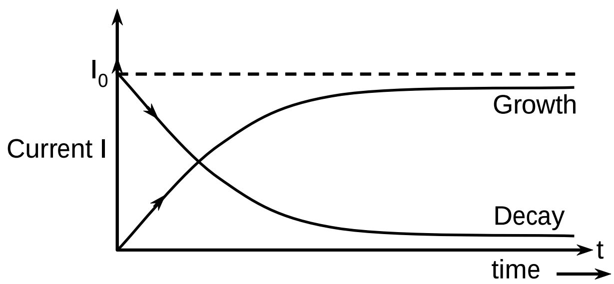


Figure (a)

Q3. Explain about growth and decay of charge in RC-circuit?

Ans : **(Imp.)**

Growth of Charge

Consider a circuit consisting of a cell of emf E, a key K, a capacitor “C” and a resistance R as shown Figure (a).

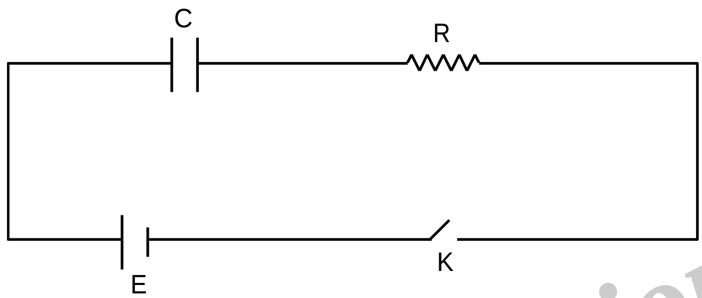


Figure (a)

When the current is started, let q be the instantaneous charge on the condenser and I be the instantaneous current.

∴ The emf equation for the RC circuit is,

$$RI + q/C = E$$

But $I = \frac{dq}{dt}$ = rate of flow of charge,

Hence, $R\frac{dq}{dt} + q/c = E$

(or) $R\frac{dq}{dt} = E - q/c$

and $E = \frac{q_0}{c}$

$$\therefore \quad R\frac{dq}{dt} = \frac{q_0}{c} - \frac{q}{c}$$

$$R\frac{dq}{dt} = \frac{(q_0 - q)}{c}$$

$$\frac{dq}{(q_0 - q)} = \frac{dt}{CR}$$

After

Integrating above equation we get

$$-\log_e(q_0 - q) = \frac{t}{CR} + k$$

where k is a constant

Applying the initial condition,

When $t = 0, q = 0, -\log_e q_0 = k$

$$\therefore -\log_e(q_0 - q) - \frac{t}{CR} - \log_e q_0$$

$$\log_e(q_0 - q) - \text{loge } q_0 = -\frac{t}{CR}$$

$$\log_e\left(\frac{(q_0 - q)}{q}\right) = -\frac{t}{CR}$$

$$\frac{q_0 - q}{q_0} = e^{-t/CR}$$

$$1 - q/q_0 = e^{-t/CR}$$

$$q/q_0 = 1 - e^{-t/CR}$$

$$\boxed{q = q_0(1 - e^{-t/CR})} \quad \text{... (1)}$$

This is called instantaneous value of the charge at time t. The term CR is called time constant of the circuit.

Time Constant

At the end of time t = CR,

$\therefore$ Equation (1) becomes $q = q_0(1 - e^{-t/CR})$

$$= q_0 (1 - e^{-1})$$

$$= q_0(1 - 1/e)$$

$$\boxed{q = 0.632q_0}$$

(ii) Decay of Charge

Let the capacitor having charge q_0 be now discharged by opened the key "K". The charge flows out of the capacitor.

In this case E = 0

The emf equation is

$$R\frac{dq}{dt} + \frac{q}{c} = 0$$

$$\frac{dq}{dt} = -\frac{1}{CR}\,dt$$

Integrating, $\log_e q = \frac{-t}{CR} + K,$

where k is a constant

when t = 0, $q = q_0$

$\log_e q_0 = k$

$\therefore \quad \log_e q = \frac{-t}{CR} + \log_e q_0$

$$\log_e q - \log_e q_0 = \frac{-t}{CR}$$

$$\therefore \quad \log_e \frac{q}{q_0} = \frac{-t}{CR}$$

$$\frac{q}{q_0} = e^{-t/CR}$$

$$\boxed{q = q_0 e^{-t/CR}} \qquad \text{... (2)}$$

This is called the instantaneous value of the charge during the discharge. The graph for decay of charge is shown figure (b).

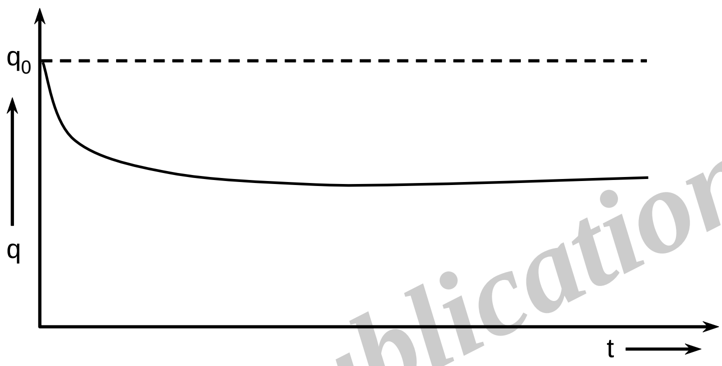


Figure (b)

Time Constant

If we put t = CR in equation (2)

We get $q = q_0 e^{-1}$

$$q = q_0 \cdot \frac{1}{e}$$

$$\boxed{q = 0.368\ q_0}$$

Q4. Explain about growth of charge in LCR – Circuit?

Ans :

Consider a circuit containing an inductance L, capacitance c and resistance R joined in series to a cell of emf E. When the key is pressed, the capacitor is charged. Let Q be the charge on the capacitor and I the current in the circuit at an instant "t" during charging.

Then, the potential difference across the capacitor is Q/c and the self induced emf in the inductance coil is L(dI/dt), both being opposite to the direction of E. The potential difference across the resistance R is RI.

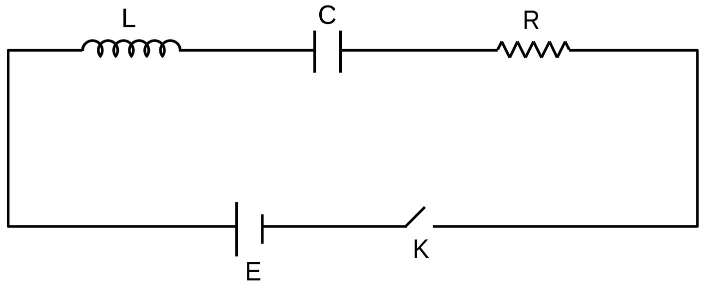


Figure (a)

The equation of emf is

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = E \quad \text{... (1)}$$

But

$$I = \frac{dQ}{dt} \text{ and } \frac{dI}{dt} = \frac{d^2Q}{dt^2}$$

$$\therefore \quad L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E$$

or $$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q - CE}{LC} = 0$$

Putting

$$\frac{R}{L} = 2b \text{ and } \frac{1}{LC} = k^2$$

We have

$$\frac{d^2Q}{dt^2} + 2b\frac{dQ}{dt} + k^2(Q - CE) = 0 \quad \text{... (2)}$$

Let $x = Q - CE$, Then $\frac{dx}{dt} = \frac{dQ}{dt}$ and $\frac{d^2x}{dt^2} = \frac{d^2Q}{dt^2}$

$\therefore$ equation (2) becomes

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + k^2x = 0 \qquad ... (3)$$

Hence the general solution of equation (3) is

$$x = Ae^{[-b+\sqrt{b^2-k^2}]t} + Be^{[-b-\sqrt{b^2-k^2}]t}$$

Now $CE = Q_0$ = final steady charge on the capacitor.

$\therefore \quad x = Q - CE = Q - Q_0$

Hence $\quad Q - Q_0 = Ae^{[-b+\sqrt{b^2-k^2}]t} + Be^{[-b-\sqrt{b^2-k^2}]t}$

$$Q = Q_0 + Ae^{[-b+\sqrt{b^2-k^2}]t} + Be^{[-b-\sqrt{b^2-k^2}]t} \qquad ... (4)$$

Using initial conditions

at $t = 0$, $Q = 0$

$\therefore 0 = Q_0 + (A + B)$

$$A + B = -Q_0 \qquad ... (5)$$

$$\frac{dQ}{dt} = A^{[-b+\sqrt{b^2-k^2}]e}e^{[-b+\sqrt{b^2-k^2}]t} + B^{[-b-\sqrt{b^2-k^2}]}\ e^{t[-b-\sqrt{b^2-k^2}]}$$

At $t = 0$, $\frac{dQ}{dt} = 0$

$$0 = A[-b + \sqrt{b^2 - k^2}] + B[-b - \sqrt{b^2 - k^2}]$$

$$\sqrt{b^2 - k^2}[A - B] = b[A + B]$$

$$= -b\,Q_0$$

(or)

$$A - B = -\frac{Q_0 b}{\sqrt{b^2 - k^2}} \qquad \text{... (6)}$$

Solving (5) and (6)

$$A = -\tfrac{1}{2}\, Q\left(1 + \frac{b}{\sqrt{b^2 - k^2}}\right) \qquad \text{... (7)}$$

$$B = -\tfrac{1}{2}\, Q_0\left(1 - \frac{b}{\sqrt{b^2 - k^2}}\right) \qquad \text{... (8)}$$

Substituting the values A and B in equation (4) we have,

$$Q = Q_0 - \tfrac{1}{2}\, Q_0\, e^{-bt}\left[\left(1 + \frac{b}{\sqrt{b^2 - k^2}}\right)e^{\sqrt{(b^2 - k^2)}t} + \left(1 - \frac{b}{\sqrt{b^2 - k^2}}\right)e^{-\sqrt{(b^2 - k^2)}t}\right]$$

Case (i)

If $b^2 > k^2$, $\sqrt{b^2 - k^2}$ is real . The charge on the capacitor grows experientially with time and attains the maximum value Q_0 asymptotically. The charge is known as over damped (or) head beat (curve (1) in figure (b)).

Case (ii)

If $b^2 = k^2$, the charge rises to a maximum value Q_0 in a short time (curve (2) in figure (b)).

Such a charge is called critically damped.

Case (iii)

$b^2 < k^2$, $\sqrt{b^2 - k^2}$ is imaginary

Let $\sqrt{b^2 - k^2} = i\omega$, where $i = \sqrt{-1}$ and $\omega = \sqrt{k^2 - b^2}$

Equation (9) may be written as

$$Q = Q_0 - \frac{1}{2}\, Q_0 e^{-bt}\left[\left(1 + \frac{b}{i\omega}\right)e^{i\omega t} + \left[1 - \frac{b}{j\omega}\right]e^{-i\omega t}\right]$$

$$Q = Q_0 - Q_0 e^{-bt}\left[\left(\frac{e^{i\omega t}+e^{i\omega t}}{2}\right)+\frac{b}{\omega}\left[\frac{e^{i\omega t}+e^{-i\omega t}}{2}\right]\right]$$

$$Q = Q_0 - Q_0 e^{-bt}\left(\cos\omega t+\frac{b}{\omega}\sin\omega t\right)$$

$$Q = Q_0\left[1-\frac{e^{-bt}}{\omega}(\omega\cos\omega t+b\sin\omega t)\right]$$

Let $\omega = k \sin\alpha$ and $b = K\cos\alpha$

So that $\tan\alpha = \omega/b$

$$Q = Q_0\left[1-\frac{c^{-bt}}{\omega}(k\sin\alpha\cos\omega t+k\cos\alpha\sin\omega t)\right]$$

$$Q = Q_0\left[1-\frac{ke^{-bt}}{\omega}(\sin(\omega t+\alpha)\right]$$

$$Q = Q_0\left[1-\frac{e^{\frac{-R}{2L}.t}\sqrt{\frac{1}{LC}}}{\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}}\sin\left[\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\right)t+\alpha\right]\right]$$

This equation represents a damped oscillatory charge as shown by the curve (figure b). The frequency of the oscillation in the circuit is given by.

$$\gamma = \frac{\omega}{2\pi}\rightarrow\frac{\sqrt{k^2-b^2}}{2\pi}\rightarrow\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$$

When

$$R = 0,$$

$$\gamma = \frac{1}{2\pi\sqrt{LC}}$$

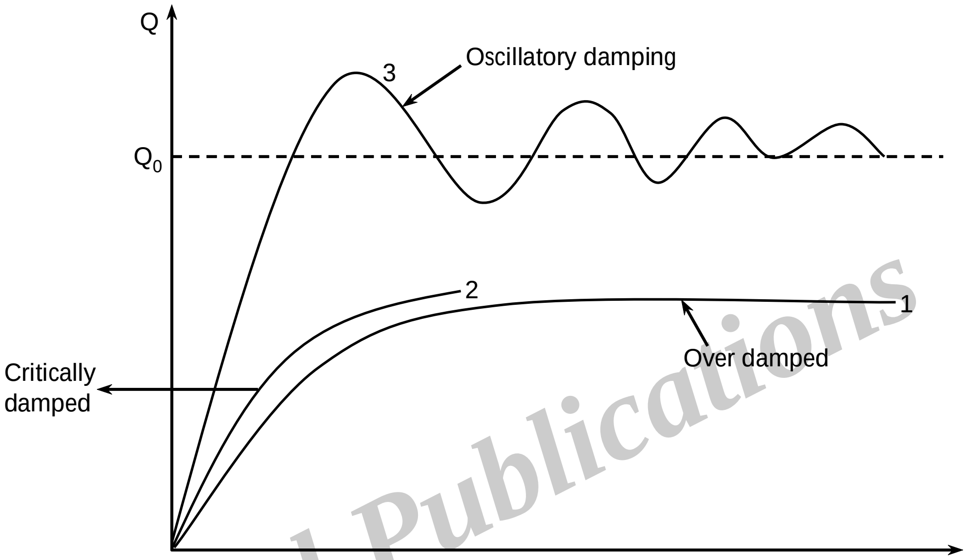


Figure (b)

4.2 Critical Damping

Q5. Explain the phenomenon of critical damping.

Ans : **(Imp.)**

"A circuit with a value of resistor that causes it to be just on the edge of ringing is called critically damped".

Consider a series LCR circuit (one that has a resistor, an inductor and a capacitor) with a constant driving electromotive force (emf) "E". The current equation of the circuit is

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = E$$

This is equivalent : $L\frac{di}{dt} + Ri + \frac{1}{C}q = E$

Differentiating we have

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

This is a second order linear homogeneous equation the corresponding auxiliary equation is,

$$Lm^2 + RM + \frac{1}{C} = 0$$

with roots

$$m_1 = -\frac{R}{2L} + \sqrt{\frac{(R^2 - 4L/C)}{2L}} \; ; \qquad M_2 = -\frac{R}{2L} - \frac{\sqrt{(R^2 - 4L/C}}{2L}$$

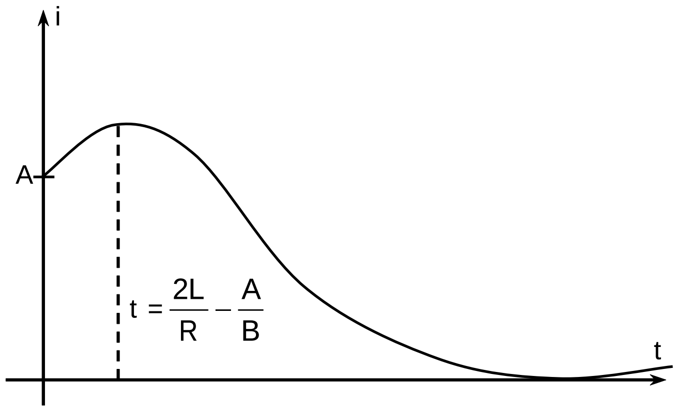
$$= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad\qquad = -\infty - \sqrt{\alpha^2 - \omega_0^2}$$

Now

$\alpha = \frac{R}{2L}$ is called the damping coefficient of the circuit.

Here,

If $R^2 = 4L/C$ then the circuit is said to be critically damped.



Graph of Critically Damped Case

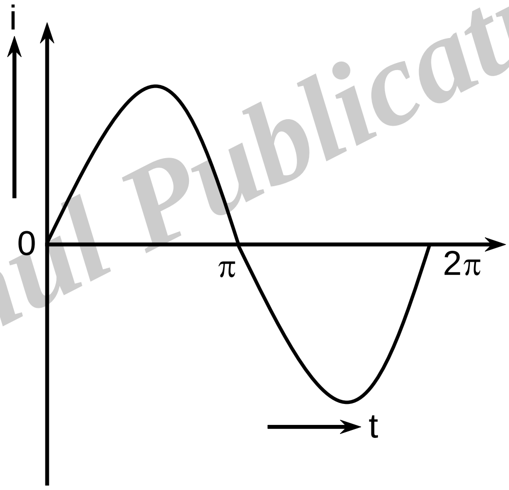
4.3 Alternating Current

Q6. Write about Alternating current.

Ans :

The current that changes its magnitude and polarity at regular intervals of time is called an alternating current.

- The major advantage of using the alternating current instead of direct current is that the alternating current is easily transformed from higher voltage to lower level voltage.
- The wave shape of the source voltage and the current flow thought he circuit (i.e., load resistor) is shown in the figure below.



Wave shape of alternating current

- The graph which represents the manner in which an alternating current changes with respect to time is known as wave shape or waveform usually the alternating value is taken along the y-axis and the time taken to the x-axis.
- An alternating current which varies according to the sine of angle 'θ' is known as sinusoidal alternating current.
- The alternating supply is always used for domestic and industrial applications.

4.4 Relation between Current and Voltage in Pure R, C and L-Vector Diagrams

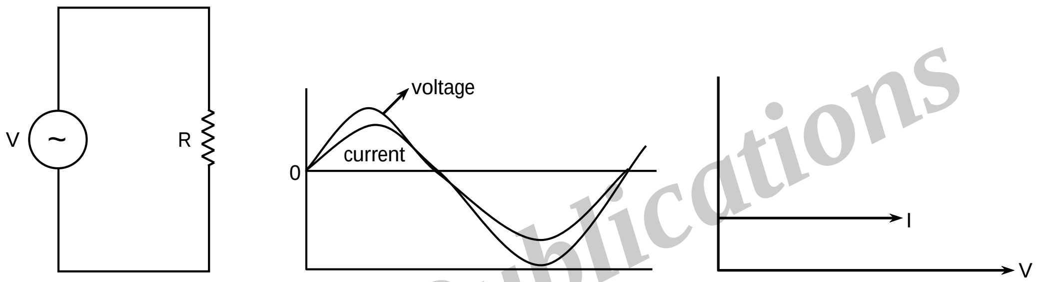
Q7. Explain about the AC circuit which is having pure resistance?

Ans :

When an alternating voltage is applied across a pure ohmic resistance it produces an alternating current through the resistance.

i) Which is in phase with the voltage.

ii) Whose r.m.s value is given by I = V/R.



(a) (b) (c)

If the expression of applied voltage is

$$V = V_0 \sin \omega t \qquad \text{... (1)}$$

Then the equation of current is

$$I = I_0 \sin \omega t \qquad \text{... (2)}$$

Comparing (1) and (2) equations it is obvious that in a pure resistor the current is always in the same phase as the applied voltage which is graphically represented figure (b). The power dissipated in the circuit in the form of heat is I^2R.

Q8. Explain the relation of current and voltage in pure capacitance and Inductances?

Ans :

1. In Pure Capacitance

When an alternating voltage $v = v_0 \sin \omega t$ is applied across a pure capacitor it produces an alternating current through the circuit whose magnitude is given by,

$$I = \frac{V}{X_C}$$

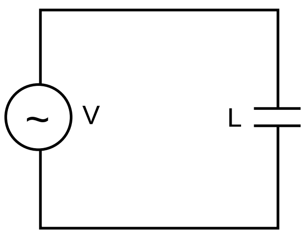


Figure (a)

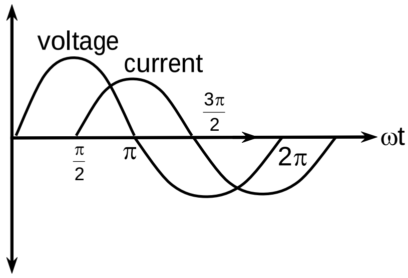


Figure (b)

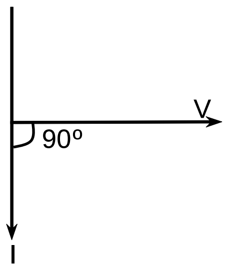


Figure (c)

Where $X_C - \frac{1}{\omega C} - \frac{1}{2\pi fc}$ called capacitive reactance.

For direct current (DC)

$$f = 0, \text{ hence } X_C = \infty$$

The current through the circuit leads the applied voltage by 90º as shown in figure (b) hence the equation of current is given by,

$$I = I_0 \sin(\omega t + \pi/2)$$

$$= I_0 \cos \omega t$$

Instantaneous power dissipated by this circuit is zero because.

$$p = VI = V_0 \sin \omega t \, I_0 \cos \omega t$$

$$= V_0 I_0 \sin \omega t \cos \omega t$$

Average power dissipated by this circuit is zero because

$$< \sin \omega t \cos \omega t > = \frac{1}{2} < \sin 2\omega t > = 0$$

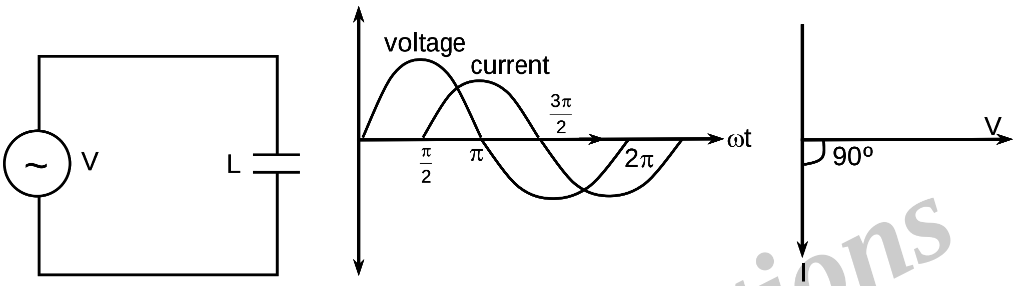
$$P = 0.$$

2. In Pure Inductance

When an alternating voltage is applied across a pure inductee coil of inductance L if produces an alternating current through the circuit. As the current in the coil varies continuously an opposite back voltage is setup in the coil whose magnitude is $L\frac{dI}{dt}$ where I is instantaneous current.

The net instantaneous voltage is,

$$V_0 \sin \omega t - 1 \frac{dI}{dt}$$



(a) (b) (c)

Since there is no resistance in the circuit,

Hence instantaneous voltage should be zero, thus

$$V_0 \sin \omega t - L\frac{dI}{dt} = 0 \quad \text{... (1)}$$

$$V_0 \sin \omega t - L \frac{dI}{dt} \quad \text{... (2)}$$

Solving above equation

$$I = -\frac{V_0}{\omega L}\cos \omega t$$

$$I = \frac{V_0}{\omega L} \sin(\omega t - \pi/2)$$

$$I = I_0 \sin(\omega t - \pi/2)$$

Where $I_0 = \frac{V_0}{\omega L}$ is the maximum current comparing the above current equation with voltage equation it is clear that for a pure inductive circuit current lags behind the voltage by π/2 as shown in figure (b).

Instantaneous power dissipation in this circuits is given by,

$$P = VI = V_0 \sin \omega t \, I_0 \cos \omega t$$

$$P = V_0 I_0 \sin \omega t \cos \omega t$$

Average power dissipated by the circuit is zero because.

$$\sin \omega t \text{ cost } \omega t = 1/2 \sin 2 \omega t$$

$$\langle \sin 2 \omega t \rangle = 0$$

$$\Rightarrow \quad p = 0$$

In the expression $I_0 = \frac{V_0}{\omega L}$, ωL has the dimension of resistance and it is called inductive reactance and denoted by XL.

Thus $X_L = \omega L = 2\pi f L$

Where f is in hertz and L is in Henry then X_L is in Ohm.

4.5 Power in AC Circuits

Q9. Explain about power in AC circuit.

Ans : **(Imp.)**

Consider an AC circuit containing resistance, inductance and capacitance, E and I vary continuously with time. Therefore power is calculated at any instant and then its mean is calculated over a completed cycle.

The instantaneous values of the voltage and current are given by,

$$E = E_0 \sin \omega t$$

$$I = I_0 \sin (\omega t - \phi)$$

where ϕ is the phase difference between current and voltage.

Hence power at any instant is,

$$E \times I = E_0 \, I_0 \sin \omega t \sin (\omega t - \phi)$$

$$= \frac{1}{2} E_0 \, I_0 [\cos \phi - \cos(2\omega t - \phi)]$$

Average power consumed over one complete cycle is,

$$P = \frac{\int_0^T EI\,dt}{\int_0^T dt}$$

$$= \frac{\int_0^T \tfrac{1}{2} E_0 I_0 [\cos\phi - \cos(2\omega t - \phi)]dt}{T}$$

$$= \tfrac{1}{2}\frac{E_0 T_0}{T}\left[(\cos\phi)t - \frac{\sin(2\omega t - \phi)}{2\omega}\right]_0^T$$

$$= \frac{1}{2}\frac{E_0 T_0}{T}\left[(\cos\phi)t - 0 - \frac{\sin(2\omega t - \phi)}{2\omega} + \frac{\sin(-\phi)}{2\omega}\right]$$

Now $T = \dfrac{2\pi}{\omega}$ and $\sin(4\pi - \phi) = \sin(-\phi)$

$$P = \frac{1}{2}\frac{E_0 I_0 \omega}{2\pi}\left[(\cos\phi)\frac{2\pi}{\omega} - \frac{\sin(-\phi)}{2\omega} + \frac{\sin(-\phi)}{2\omega}\right]$$

$$= \frac{1}{2} E_0 I_0 \cos\phi$$

$$= \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos\phi$$

$$= E_{rms} \times I_{rms} \cos\phi$$

Average power = (Virtual volts) × (Virtual ampere) × cos ϕ. The term (virtual volts × virtual ampere) is called apparent power and cos ϕ is called power factor.

Thus

True power = Apparent power × Power factor.

4.6 LCR Series and Parallel Resonant Circuit

Q10. Write about comparison between series and parallel resonant circuit?

Ans :

Series resonant circuit	Parallel Resonant Circuit
1. An acceptor circuit	1. A rejector circuit.
2. Resonant frequency $\gamma_r = \frac{1}{2\pi\sqrt{LC}}$	2. Resonant frequency $\gamma_r = \frac{1}{2\pi\sqrt{LC}}$
3. At resonance the impedance is a minimum equal to the resistance in the circuit	3. At resonance the impedane is maximum nearly equal to infinity.
4. Selective	4. Selective
5. Used in the turning circuit to separate the wanted frequency from the incoming frequencies by offering low impedance at that frequency	5. Used to present a maximum impedance to the wanted frequency, usually in the plate circuit of value.

Q11. Explain the LCR circuit in series and parallel resonant condition?

Ans : **(Imp.)**

1. Series Resonance Circuit

Consider a circuit containing an inductance L, a capacitance c and a resistance R joined in series (Figure (a)).

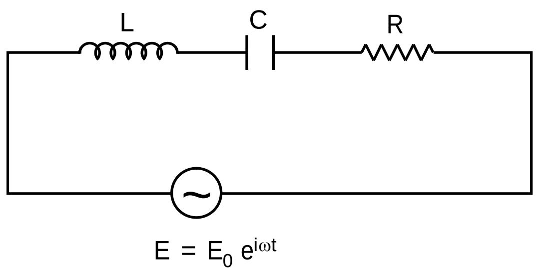


Figure (a)

The series circuit is connected to an AC supply given by $E = E_0 e^{j\omega t}$ ... (1)

The total complex impedance is

$$Z = Z_R + Z_L + Z_C$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \; e^{j\phi} \quad \text{... (2)}$$

where $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$

Using Ohm's law in complex form, the complex current in the circuit is,

$$I = \frac{E}{Z} = \frac{E_0 e^{j\omega t}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 e^{j\phi}}$$

$$I = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{... (3)}$$

But

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\therefore \quad I = I_0 e^{j(\omega t - \phi)} \quad \text{... (4)}$$

The actual emf is the imaginary part of the equivalent complex emf. Hence the actual current in the circuit is obtained by taking the imaginary part of the above complex current.

$$\therefore \quad i = I_m(I) = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t - \phi)$$

The equivalent impedance of the series LCR circuit $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

The current "lags" behind the voltage by an angle $\phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$.

2. Parallel Resonant Circuit

In this circuit, capacitor C is connected in parallel to the series combination of resistance R and inductance L. The combination is connected across the AC source (figure (b)).

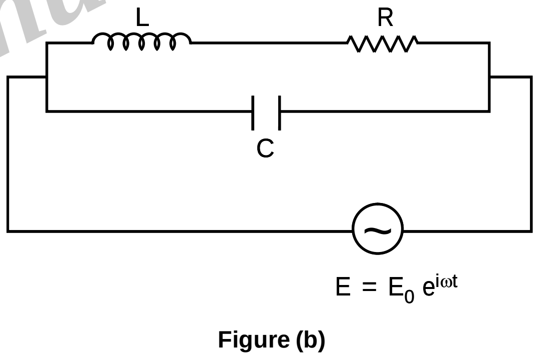


Figure (b)

The applied voltage is

$$E = E_0 e^{j\omega t}$$

The complex impedance of L - branch

$$Z_1 = R + J\omega L$$

Complex impedance of C – branch

$$Z_2 = \frac{1}{j\omega C}$$

Z_1 and Z_2 are parallel

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \Rightarrow \frac{1}{R + j\omega L} + \frac{1}{1/j\omega c}$$

$$= \frac{1}{R + j\omega L} = j\omega c$$

$$= \frac{(R - j\omega L)}{(R + j\omega L)(R - j\omega L)} + j\omega c$$

$$= \frac{R}{R^2 + (\omega L)^2} + j\left[\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right]$$

The current I = E/Z $\Rightarrow$ E × 1/Z

$$\therefore \quad I = E\left[\frac{R}{R^2 + (\omega L)^2} + j\left[\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right]\right]$$

Let

$$A \cos \phi = \frac{R}{R^2 + (\omega L)^2} \; ; A \sin \phi = \omega C - \frac{\omega L}{R^2 + (\omega L)^2}$$

$$\therefore \quad I = E(A \cos \phi + j A \sin \phi)$$

$$= E\, Ae^{\phi j}$$

$$= E_0\, Ae^{j(\omega t + \phi)}$$

Where

$$\phi = \tan^{Y}\left(\frac{\omega C - \dfrac{\omega L}{(R^2 + (\omega L)^2}}{\left(\dfrac{R}{R^2}\right) + (\omega L)^2}\right)$$

$$A^2 = \frac{R^2}{(R^2+\omega^2L^2)^2} + \left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right)$$

The magnitude of admittance,

$$y = \frac{1}{Z} = \frac{\sqrt{\left[R^2 + (\omega CR^2 + \omega L^2C - (\omega L)^2\right]}}{R^2+\omega^2L^2}$$

The admittance will be maximum when

$$\omega CR^2 + \omega^3L^2C - \omega L = 0$$

$$\omega = \omega_0 = \sqrt{\left(\frac{1}{1C} - \frac{R^2}{1^2}\right)}$$

$$\gamma_0 = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R_2}{L^2}\right)}$$

This is the resonant frequency of the circuit. If R is very small so that $\frac{R^2}{L^2}$ is negligible compared to $\frac{1}{LC}$.

$$\gamma_0 = \frac{1}{2\pi(\sqrt{1C})}$$

At such a minimum admittance, i.e., maximum impedance, the circuit current is minimum. The graph between current and frequency is shown in figure (c).

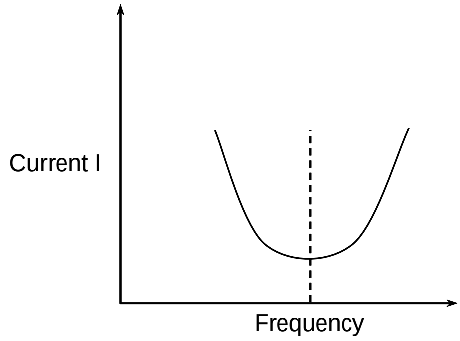


Figure (c)

4.7 Q-FACTOR

Q12. Explain about Q-factor (or) Qualify factor?

Ans :

The current in the LCR series circuit becomes maximum at resonance. Due to the increase in current, the voltage across L and C are also increased. This magnification of voltages at series resonance is termed as Q-factor.

➢ It is defined as the ratio of voltage across L or C to the applied voltage.

$$Q - \text{factor} - \frac{\text{Voltage across L or C}}{\text{Applied Voltage}}$$

➢ At resonance, the circuit is purely resistive. Therefore, the applied voltage is equal to the voltage across R.

$$Q\text{–factor} = \frac{I_m X_L}{I_m R} = \frac{X_L}{R}$$

$$= \frac{\omega_r L}{R} \quad (\because X_L = \omega_r L)$$

$$= \frac{L}{R\sqrt{LC}} \quad \left(\because \omega_r = \frac{1}{\sqrt{LC}}\right)$$

$$\therefore Q\text{-factor} = \frac{1}{R}\sqrt{\frac{1}{C}}$$

➢ The physical meaning is that Q-factor indicates the number of times the voltage across L or C is grater than the applied voltage at resonance.

➢ Conditions for the large value of Q-factor.

(i) Value of $\frac{L}{C}$ should be large

(ii) Value of R should be less.

4.8 AC & DC Motors-Single Phase, Three Phase (Basics Only)

Q13. Explain about 3-phase AC-motors?

Ans : **(Imp.)**

A three phase alternator is shown in the diagram below. It consists of three similar rectangular coils displaced equally from each ones, i.e., 120º. Each oil is provided with its own brushes and slip rings.

Three emfs are generated when they are rotated at a constant velocity in a uniform magnetic field. They are of the same frequency and of equal values. Each of the three sources of voltage is called a phase. Each phase voltage lags 120º behind that of the one preceding it [Figure (b), (c)]

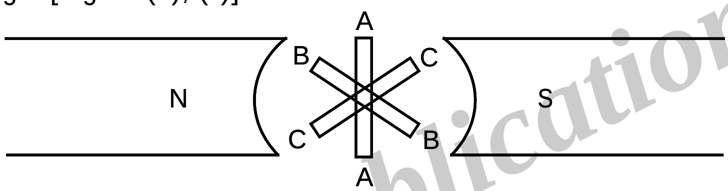


Figure (a)

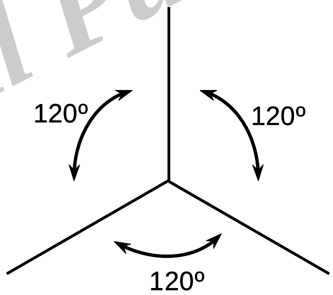


Figure (b)

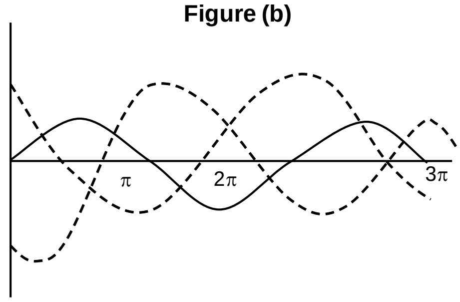


Figure (c)

- The instantaneous values of emf in each coil may be written as:

$$E_1 = E_0 \sin \omega t$$

$$E_2 = E_0 \sin \left(\omega t + \frac{2\pi}{3}\right)$$

$$E_3 = E_0 \sin \left(\omega t + \frac{4\pi}{3}\right)$$

- It can be used to supply a three phase system of thee single phase circuits.

Advantages of 3-phase System

i) In 3-phase alternators the total power does not fluctuate. While in a single phase generator the current fluctuates.

ii) The output power of a 3-phase alternator is always greater than that of a single phase generator of the same size.

iii) Three phase system is superior for transmission of electrical energy. It involves lot of saving.

Q14. What is a single phase motor?

Ans :

A single phase motor is an electrically powered rotary machine that can Aurn electric energy into mechanical energy.

- It works by using a single-phase power supply. They contain two types of wiring hot and neutral.
- Their power can reach 3 kw and supply voltages vary in unison.
- They only have a single alternating voltage. The circuit works with two wires and the current that runs across them is always the same.
- In most cases these are small motors with a limited torque. However, there are single phase motors with a power of up to 10 hp that can work with connections of upto 440 V.
- This type of motor is used mainly in homes, offices, stores and small non-industrial companies. Their most common uses include home appliance.
- These do not generate a rotating magnetic field they can only generate an alternate field. Which means that they need a capacitor for startup.

Q15. What do you know about a DC motor? Explain.

Ans :

- A DC motor is any of a class of rotary electrical motors that converts direct current electrical energy into mechanical energy.
- The most common types rely on the forces produced by magnetic fields. Nearly all types of DC motors have some internal mechanism, either electromechanical or electronic, to periodically change the direction of current in part of the motor.
- DC motor were the first form of motor widely used, as they could be powered from existing direct-current lighting power distribution systems.
- A DC motor's speed can be controlled over a wide range, using either a variable supply voltage or by changing the strength of current in its field windings. Small DC motors are used in tools, toys, and appliances.
- Larger DC motors are currently used in propulsion of electric vehicles, elevator and hoists, and in drives for steel rolling mills.
- The advent of power electronics has made replacement of DC motors with AC motors possible in many applications.

4.9 Network Elements

Q16. Define the following terms.

(i) Networks

(ii) Linear and Non Linear Networks

(iii) Node and Branch

(iv) Loop and Mesh

(v) Active and Passive Networks

Ans :

(i) Networks

An electrical circuit containing impedances (or elements like, resistance, inductance, capacitance etc.) and generators (sources of e.m.f. of power) is known as electrical network. Thus, an electrical network is nothing but a combination of circuit elements and generators.

(ii) Linear and Non-linear Networks

A network is said to be linear when the current in all branches is directly proportional to the driving voltage.

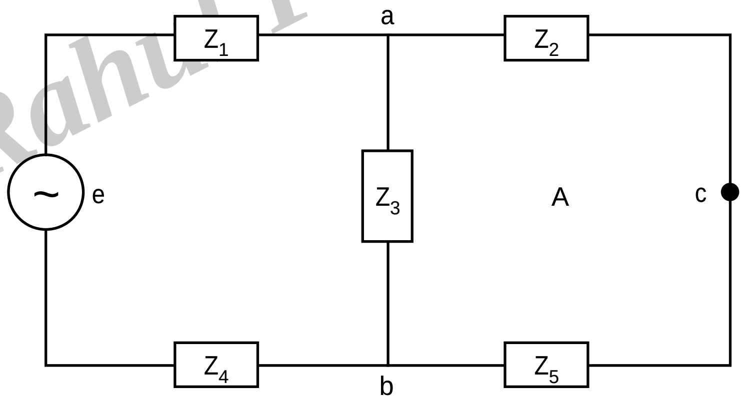
If relation between voltage and current in any branch of network is non-linear, the network is said to be non-linear.

(iii) Node and Branch

The circuit arrangement of an electrical network. The junction points where the elements in a network meet, such as the points a and b in Figure are called nodes or junctions. An element joining two nodes, such as a to b in Figure is called a branch. Thus any group of elements in series, having two terminals, is called a branch.

(iv) Loop and Mesh

Any closed path in a network such as abce is called a loop. The space that the loop encloses is called a mesh, such as A in Figure. Which is the space enclosed by the loop abca'.



Figure

(v) Active and Passive Networks

If a network contains energy sources as well as other circuit elements, it is called an active network while a network containing circuit elements without any energy source is known as the passive network.

4.9.1 Passive Elements

Q17. Explain briefly about Passive elements.

Ans :

(i) Passive elements ulilizes power (or) energy in the circuit.

(ii) Resistors, Capacitors, inductors are passive elements

(iii) These devices stores energy in the form of voltage (or) current

(iv) They can not control flow of current

(v) They do not require any external source for operation.

(vi) These elements are energy acceptor

4.9.2 Power Sources

Q18. Explain the types of power sources.

Ans :

Power sources can be classified as independent sources and dependent sources.

(i) Independent Sources

An independent source maintains the same voltage (or) current regardless of other elements present in circuit. Its value is either DC or AC. The strength of the voltage (or) current is not changed by any variation in the connected network.

(ii) Dependent Sources

Dependent sources depend upon particular element of the circuit for delivering voltage or current depending upon type of source it is.

4.9.3 Active Elements

Q19. Explain briefly about Active elements.

Ans :

(i) Active elements deliver power (or) energy to the circuit.

(ii) Diodes, transistors SCR, Integrated circuits are Active elements.

(iii) These elements produce energy in the form of voltage (or) current.

(iv) They control the flow of current.

(v) They require external source for operation.

(vi) These elements are energy donor.

4.10 Network Models

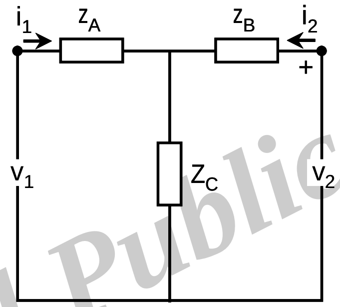
4.10.1 T-Network

Q20. Express the elements of T-Network interms of Z and ABCD parameters.

Ans :

Now we shall study the use of two-port network parameters, discussed in earlier sections, in the analysis of T and π networks.

(i) The T-network : Any two-port network may be represented by an equivalent T-network, as shown in Fig. (a). The elements z_A, z_B and z_C of this equivalent T-network may be Represented in terms of any of the four types of parameters of the two-port network say z- or ABCD parameter.



The T-network

Fig. (a)

Conversely, these parameters of the network may be represented in terms of the elements of the equivalent T-network.

(a) Using z-parameters : The z- parameters are given by

$$z_{11} = \left.\frac{\upsilon_1}{i_1}\right|_{i_2=0}$$

i.e., when output is open-circuited, the input impedance is,

$$z_{11} = z_A + z_c. \qquad ...(1)$$

The forward transfer impedance,

$$z_{21} = \left.\frac{\upsilon_2}{i_1}\right|_{i_2=0}$$

when $i_2 = 0$, p.d. across $z_B = 0$, so that i_1 flows through z_A and z_C, and the p.d. υ_2 develops across z_C. Thus

$$z_{21} = z_C \quad \text{... (2)}$$

The output impedance,

$$z_{22} = \left|\frac{\upsilon_2}{i_1}\right|_{i_1=0}$$

with $i_1 = 0$, the current i_2 flows through z_B and z_C, so that,

$$z_{22} = z_B + z_C \quad \text{... (3)}$$

The reverse transfer impedance,

$$z_{12} = \left|\frac{v_1}{i_2}\right|_{i_1=0}$$

Again, with $i_1 = 0$, the p.d. v_1 appears across z_C and i_2 flows through z_B and z_C, so that

$$z_{12} = z_C \quad \text{... (4)}$$

Solving eqns. (1) to (4), we find,

$$z_A = z_{11} - z_{21} \quad z_B = z_{22} - z_{12}, \; z_C = z_{12} - z_{21} \quad \text{... (5)}$$

(b) Using ABCD parameters : The ABCD parameters are given by,

$$A = \left|\frac{\upsilon_1}{\upsilon_2}\right|_{i_2=0}$$

when $i_2 = 0$, υ_2 appears across z_C , so that

$$\upsilon_1 = i_1 (z_A + z_C)$$

and $\upsilon_2 = i_1 z_C$

Thus $A = \dfrac{\upsilon_1}{\upsilon_2} = \dfrac{i_1(z_A + z_C)}{i_1 z_C}$

$$A = 1 + z_A y_C \quad \text{... (6)}$$

where

The parameter B is given by

$$B = \left|\frac{\upsilon_1}{-i_1}\right| \upsilon_2 = 0$$

with $\upsilon_2 = 0$, i.e., output short-circuited, applying Kirchoff's laws (KVL) to the two meshes in Fig. 1.12

$$\upsilon_1 = i_1 (z_A + z_B) + i_2 z_C$$
$$0 = i_1 z_C + i_2 (z_B + z_C)$$

This gives, $i_1 = -\dfrac{(z_B + z_C) i_2}{z_C}$ ... (7)

so that, $\upsilon_1 = -\dfrac{(z_B + z_C)}{z_C}(z_A + z_B)\, i_2 + i_2 z_C$

or $\dfrac{\upsilon_1}{-1_1} = B = \dfrac{(z_B + z_C)(z_A + z_B) - z_C^2}{z_C}$ ... (8)

The parameter C is given by

$$C = \left|\frac{i_1}{\upsilon_2}\right|_{i_2 = 0}$$

with $i_2 = 0$, υ_2 appears across z_C, so that

$C = \dfrac{i_1}{\upsilon_2} = \dfrac{i_1}{i_1 z_C} = \dfrac{1}{z_C} = Y_C$ ... (9)

and the parameter D is given by, $D = \left|\dfrac{i_1}{-i_2}\right|_{\upsilon_2 = 0}$

when $\upsilon_2 = 0$, we find from eqn. (7)

$$-\frac{i_1}{i_2} = D = \frac{z_B + z_C}{z_C}$$

$= 1 + z_B Y_C$ ... (10)

Equations (6) to (10) can be solved to find

$$z_A = \frac{A - 1}{C}$$

$$z_B = \frac{D - 1}{C}$$

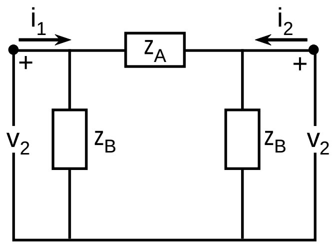
$$z_C = \frac{1}{C}$$

4.10.2 The π-network

Q21. Express the elements of π-network interms of Y and ABCD parameter.

Ans : **(Imp.)**

The π-network : Any two-port network may also be represented by an equivalent π network, as shown in Fig. 1. The elements of the equivalent π-network may be expressed in terms of the four types of parameters of the two-port network, say the y- and ABCD parameters of the two-port network may be expressed in terms of the elements of the equivalent π-network.



The π-network

Fig. 1

(a) Using y-parameters: The y-parameters of a two-port network are given by

$$y_{11} = \left|\frac{i_1}{\upsilon_1}\right|_{\upsilon_2=0}$$

with $\upsilon_2 = 0$, i,e., output short-circuited,

$$\upsilon_1 = i_1 . \frac{z_A z_B}{z_A + z_B}$$

or $$\frac{i_1}{\upsilon_1} = y_{11} = \frac{z_A + z_B}{z_A - z_B} = y_B + y_A \qquad \text{... (1)}$$

where $y_A = 1/z_A$ and $y_B = 1/z_B$. then forward transfer admittance parameter y_{21} is.

$$y_{21} = \left|\frac{i_2}{\upsilon_1}\right|_{\upsilon_2=0}$$

when $\upsilon_2 = 0$, z_B and z_A appear in parallel, so that current through z_A is,

$$i_2 = -\,i_1 . \frac{z_B}{z_B + z_A}, \text{ and } \upsilon_1 = i_1 . \frac{z_A \cdot z_B}{z_A + z_B}$$

Therefore, $$y_{21} = \frac{i_2}{\upsilon_1} = \frac{-i_1 \cdot z_B / (z_A + z_B)}{i_1 z_A \cdot z_B / (z_A + z_B)}$$

$$y_{21} = -\frac{1}{z_A} = -y_A \qquad \text{... (2)}$$

Again, the output admittance parameter is,

$$y_{22} = \left|\frac{i_2}{\upsilon_2}\right|_{\upsilon_1=0}$$

when input is short-circuited, z_A and z_C appear in parallel, so that,

$$\upsilon_2 = i_2 \cdot \frac{z_A + z_C}{z_A + z_C}$$

or $$y_2 = \frac{i_2}{\upsilon_2} = \frac{z_A + z_C}{z_A \cdot z_C} = y_C + y_A \qquad \text{... (3)}$$

The reverse transfer admittance,

$$y_{12} = \left|\frac{i_1}{\upsilon_2}\right|_{\upsilon_1=0}$$

when $\upsilon_1 = 0$, z_A and z_C become in parallel and current through z_A becomes,

$$i_1 = -\frac{z_C}{z_A + z_C} \cdot i_2 \text{ and } \upsilon_2 = i_2 \frac{z_A z_C}{z_A + z_C}$$

so that $$y_{12} = \frac{-i_1}{\upsilon_2} = \frac{-i_2 \cdot z_C / (z_A + z_C)}{i_2 \cdot z_A z_C / (z_A + z_C)}$$

or $$y_{21} = \frac{1}{z_A} = -y_A \qquad \text{... (4)}$$

Equations (12) to (15) can be solved to yield

$$y_A = y_{21} = -y_{12},\ y_B = y_{11} + y_{21}, y_C = y_{22} + y_{21} \qquad \text{... (5)}$$

(b) Using ABCD parameters : The ABCD parameters are given by.

$$A = \left|\frac{\upsilon_1}{\upsilon_2}\right|_{i_1=0}$$

$$= \frac{\upsilon_1}{\upsilon_1 \cdot z_C / (z_A + z_C)} = \frac{z_A + z_C}{z_C}$$

$$= \frac{y_C + y_A}{y_A} \quad \text{... (6)}$$

$$C = \left|\frac{i_1}{\upsilon_2}\right|_{i_2=0}$$

$$= \frac{\upsilon_1 / \left[z_B \cdot (z_A + z_C) / (z_B + z_A + z_C)\right]}{\upsilon_1 \cdot z_C / (z_A + z_C)}$$

$$C = \frac{(z_B + z_C + z_A)}{z_B \cdot z_B} = \frac{1}{z_C} + \frac{1}{z_B} + \frac{z_A}{z_B z_C} \quad \text{... (7)}$$

$$= y_C + y_B + \frac{y_B y_C}{y_A}$$

$$B = \left|\frac{\upsilon_1}{-i_1}\right|_{\upsilon_2=0} = \frac{\upsilon_1}{\upsilon_1 / z_A} = z_A = \frac{1}{y_A} \quad \text{... (8)}$$

$$D = \left|\frac{i_1}{-i_2}\right|_{\upsilon_2=0}$$

$$= \frac{i_1}{i_1 \cdot z_B / (z_A + z_B)} = \frac{z_A + z_B}{z_B}$$

$$= 1 + \frac{y_B}{y_A} = \frac{y_B + y_A}{y_A} \quad \text{... (9)}$$

Conversely,

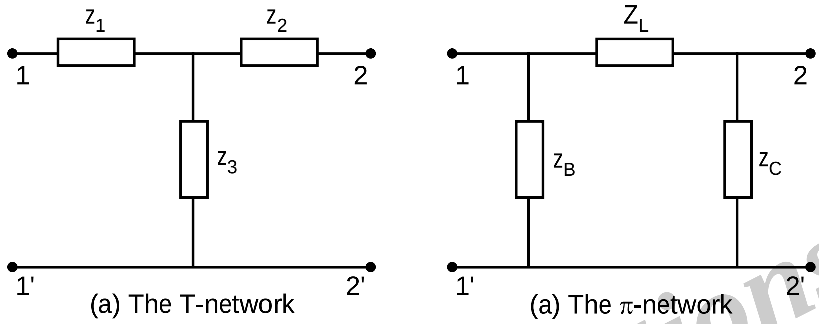
$$y_A = \frac{1}{z_A} = \frac{1}{z_B}, y_B = \frac{(D-1)}{B} = \frac{1}{z_B}, y_C = \frac{1}{z_C} = \frac{(A-1)}{B} \quad \text{... (10)}$$

Q22. Derive the conversion formulae between T and π network.

Ans :

Let the figure (1) (a) represents T-network and its equivalent π network is shown in figure 1 (b)

Referring to the figures the impedance between 1-1' terminals, with 2-2' open, is,



(a) The T-network (a) The π-network

Fig 1

$$z_1 + z_3 = \frac{z_B(z_A + z_C)}{z_A + z_B + z_C} \quad \text{... (1)}$$

The impedance between terminals 2–2' with 1–1' open is,

$$z_2 + z_3 = \frac{z_C(z_A + z_B)}{z_A + z_B + z_C} \quad \text{... (2)}$$

The impedance between terminals 1-2, with terminals 1' 2' left free, is,

$$z_1 + z_2 = \frac{z_A(z_B + z_C)}{z_A + z_B + z_C} \quad \text{... (3)}$$

Adding eqns. (1) and (3),

$$z_1 + z_2 + z_3 = \frac{z_A z_B + z_A z_C + z_B z_C}{z_A + z_B + z_C} \quad \text{... (4)}$$

Substituting eqns. (1), (2) and (3) from eqn. (4) one by one, we get

$$z_1 = \frac{z_A \cdot z_B}{z_A + z_B + z_C} \quad \text{... (5)}$$

$$z_2 = \frac{z_A \cdot z_C}{z_A + z_B + z_C} \quad \text{... (6)}$$

and $$z_3 = \frac{z_B \cdot z_C}{z_A + z_B + z_C} \qquad ...(7)$$

Thus if the π network elements z_1, z_2, z_2 given the equivalent T network elements can be computed from eq(7).

If the T network of Fig. 1(a) is given then the elements z_A, z_B and z_C of the equivalent π network shown in Fig. 1(b) can be obtained from eqns. (1) to (4) to yield,

$$z_A = (z_1z_2 + z_1z_3 + z_2z_3)/z_3$$

$$z_B = (z_2z_3 + z_1z_3 + z_1z_2)/z_2$$

and $$z_C = (z_2z_3 + z_1z_3 + z_1z_2)/z_1 \qquad ...(8)$$

4.11 NETWORK THEOREMS

Q23. What is meant by Network Theorems?

(OR)

What do you understand by Network Theorem.

Ans :

Network problems can very often be solved by the application of Ohm's law and Kirchoffs Laws. But in complicated circuits, calculations based on these laws are very lengthy and tedious. Therefore, some short cuts in calculations are badly needed. Some theorems provide these short cuts and by applying them the complicated networks can be easily solved. These theorems are called the network theorems. These theorems are applicable for both AC and DC networks.

In AC networks, however, the impedances, voltages and currents are treated as phasors. Therefore, it is necessary to know the phase angle of each source and the amplitude of the voltage or current generated by the source.

4.11.1 Superposition Theorem

Q24. State and prove superposition theorem.

Ans : **(Imp.)**

Statement

In a network containing linear impedances (impedance of which value does not change with the flow of current) and energy sources (generators), the current flowing at any point is the vector sum of the currents which would exist if each source of e.m.f. Were considered separately, all the other sources being replaced on that time by their impedances.

Proof :

To verify the theorem, let us consider the simple network with two generators of e.m.f.'s E_1 and E_2 with internal impedances Z_1 ar.t Z_2 respectively (Fig. 1).

Let the current due to E_1 and E_2 acting together be I_1 and I_2 Fig. 1 (a) and let the current due to e.m.f. E_1 acting alone be I_1' and I_2' (Fig 1 (b)) and those due to E_2 acting alone be I_1'' and I_2'' (Fig. 2)

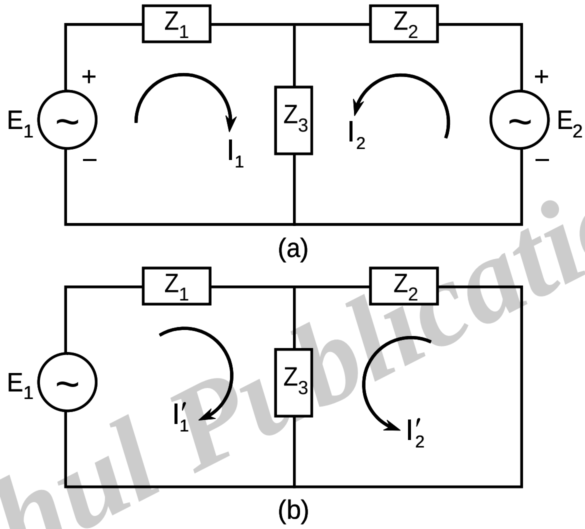


Fig. 1

Applying Kirchoff's second law to the mesh of Fig. 1(a), we have

$$E_1 = I_1 (Z_1 + Z_3) + I_2Z_3 \quad ... (1)$$

and $$E_2 = I_2 (Z_2 + Z_3) + I_1Z_3 \quad ... (2)$$

When E_1 is considered to act alone, mesh of Fig. 1(b) gives

$$E_1 = I_1'(Z_1 + Z_3) + I_2'Z_3 \quad ... (3)$$

and $$0 = I_2' (Z_2 + Z_3) + I_1'Z_3 \quad ... (4)$$

When E_2 is considered alone, mesh of Fig. (2) gives

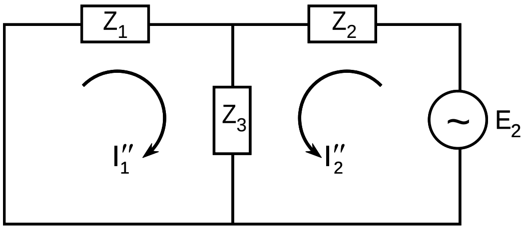


Fig. 2

$$0 = I_1''(Z_1 + Z_3) + I_2'' Z_3 \quad \text{... (5)}$$

$$E^2 = I_2''(Z_2 + Z_3) + I_1''Z_3 \quad \text{... (6)}$$

Adding eqns. (3) and (5), we get

$$E_1 = (I_1' + I_1'')(Z_1 + Z_3) + (I_2' + I_2'')Z_3 \quad \text{... (7)}$$

Addition of eqns. (4) and (6) gives

$$E_2 = (I_2' + I_2'')(Z_2 + Z_3) + (I_1' + I_1'')Z_3 \quad \text{... (8)}$$

Equations (7) and (8) will be identical with equations (1) and (2) respectively, if

$$I_1 = I_1' + I_1''$$

and $I_2 = I_2' + I_2''$.

This proves the truth of the superposition theorem.

4.11.2 Thevenin's Theorem

Q25. State and prove Thevenin's theorem.

Ans : **(Imp.)**

Any combination of batteries and resistances with two terminals can be replaced by voltage source and single series resistor.

The voltage source is the open circuit voltage at the terminals A & B and the value of single series resistor is open circuit voltage divided by current with the terminals short circuited.

Referring to Fig. (a), N is a network containing a number of generators and linear impedances with output terminals A and B Let E' be the open-circuit voltage across A and B when all the generators have been replaced by their respective internal impedances.

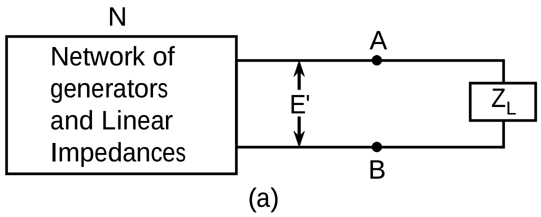


Fig (a)

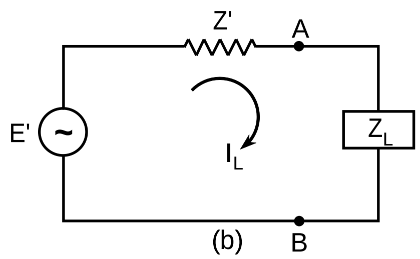


Fig (b)

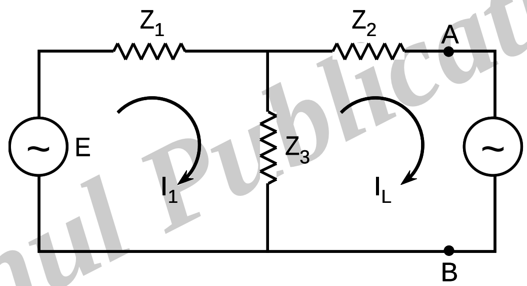


Fig (c)

The network N will produce the same current in an external load impedance connected across A and B as a single voltage generator of e.m.f. E', and internal impedance Z' would do.

Proof : To establish the theorem network may be reduced to an equivalent T-section arrangement of impedances Z_1 Z_2 and Z_3 as shown in Fig. (c). Here E is the e.m.f. of the source, I_1 current supplied by the source and I_L the current flowing through the load impedance Z_L. Then according to Thevenin's theorem, the circuit of Fig. (b) with voltage source E' and impedance Z' will be equivalent to that of Fig. (c) with identical voltages and current at Z_L

To find the expression for load current I_2, applying Kirchoff's second law to the mesh of Fig. (c); we have

$$E = I_1(Z_1 + Z_3) - I_L Z_3 \quad \text{... (1)}$$

and $$0 = I_L(Z_2 + Z_3 + Z_L) - I_1 Z_3 \quad \text{... (2)}$$

From equn. (2), we have

$$I_1 = \frac{I_L(Z_2 + Z_3 + Z_L)}{Z_3}$$

Substituting this value of I_1 in eqn (1), we get

$$E = \frac{I_L(Z_2 + Z_3 + Z_L)}{Z_3}(Z_1 + Z_3) - I_L Z_3$$

or $$I_L = \frac{E}{\frac{(Z_2 + Z_3 + Z_L)(Z_1 + Z_3)}{Z_3} - Z_3}$$

or $$I_L = \frac{EZ_3}{Z_2(Z_1 + Z_3) + Z_1 Z_3 + Z_L(Z_1 + Z_3)}$$

$$= \frac{E\left(\frac{Z_3}{Z_1 + Z_3}\right)}{Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3} + Z_L} \quad \text{... (3)}$$

Again, from Fig. (c) the open-circuit voltage at terminals (A, *B) i.e.,* when load is disconnected, is given by

$$E' = \frac{E}{Z_1 + Z_3} \cdot Z_3 = E\left(\frac{Z_3}{Z_1 + Z_3}\right) \quad \text{... (4)}$$

The impedance between terminals A, *B* after disconnecting the load Z_L in Fig. (c) is given by

$$Z' = Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3} \quad \text{... (5)}$$

Substituting the values of E' and Z' from (4) and (5) in eqn. (3), we have

$$I_L = \frac{E'}{Z' + Z_L}$$

which is the current expression for network in Fig. (b). The truth of Thevenin's theorem is thus, established.

4.11.3 Norton's Theorem

Q26. State and explain Norton's network theorem.

Ans :

The theorem may be stated as follows :

Any linear circuit containing several energy sources and resistances can be replaced by a single constant current generator in parallel with a single resistor

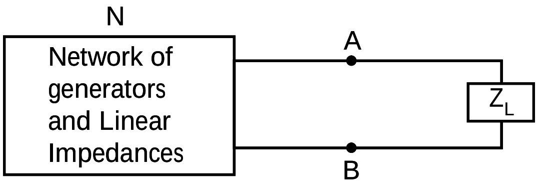


Fig. (a)

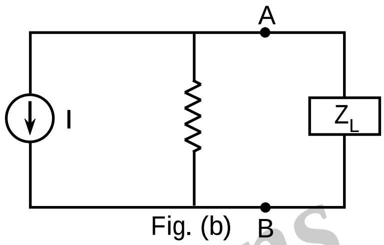


Fig. (b)

In Fig. (a), N is a network containing a number of generators and linear impedances with output terminals A and B. Let the current through AB when short-circuited be I and let the impedance of the network when looked back into the terminal, A, B with all the generators being replaced by their internal impedances be Z. Then the network will produce the same current in an external impedance Z_L connected across the terminals A and B as a constant current generator generating a current I and placed in parallel of an impedance Z would do [Fig. (b)].

The Norton's circuit may be obtained from the Thevenin's equivalent and the current across the load impedance Z_L given by two equivalent circuits may be seen to be exactly equal. If E' is the open- circuit voltage across A and B, then the Thevenin's equivalent circuit is as shown in Fig. (c). The short-circuit current on shorting the terminals A and B is found to be

$$I' = \frac{E'}{Z'}$$

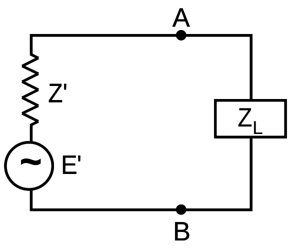


Fig. (c)

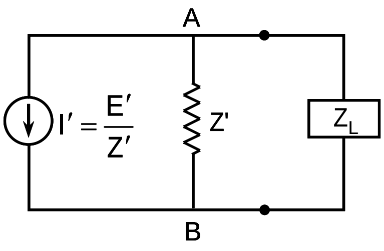


Fig. (b)

Fig. (d) depicts the Norton's equivalent circuit in which the voltage generator *E'* has been replaced by a current generator I' = $\left(\frac{E'}{Z'}\right)$ and the impedance Z' has been replaced by an equal parallel impedance.

The current through Z_L in case (a) is

$$I_L = \frac{E'}{Z' + Z_L}$$

In case (b), the current through Z_L is

$$I_L' = \frac{Z'}{Z' + Z_L} I' = \frac{Z'}{Z' + Z_L} \cdot \frac{E'}{Z'}$$

$$= \frac{E'}{Z' + Z_L} = I_L$$

which proves the validity of Norton's theorem.

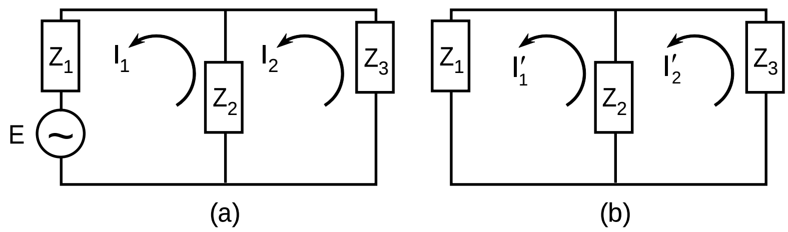
4.11.4 Reciprocity Theorem

Q27. State and prove Reciprocity Theorem.

Ans :

If an e.m.f. applied in one mesh of network of linear impedance produces a certain current in the second mesh, then the same e.m.f. acting in the second mesh will give an identical current in the first mesh.

Proof : To establish the theorem, let us consider the arrangement, shown in Fig. (a), where the source of e.m.f. E is in the first mesh. Let the current in the first and second mesh be I_1 and I_2 respectively.



Figure

Applying Kirchoff's second law to the two meshes, we get

$I_1(Z_1 + Z_2) - I_2Z_2 = E$ ...(1)

and $I_2(Z_2 + Z_3) - I_1Z_2 = 0$ ...(2)

Substituting the value of I_1 fro eqn. (2) into (1), we get

$$I_2\left[\frac{(Z_1+Z_2)(Z_2+Z_3)}{Z_2} - Z_2\right] = E$$

or $$I_2 = \frac{EZ_2}{(Z_1+Z_2)(Z_2+Z_3)-Z_2^2} \quad ...(3)$$

Again considering the network of Fig. (b) in which the source of e.m.f. is in the second mesh. Let the current in the first and second mesh be I_1' and I_2' respectively. Application of Kirchoff's second laws to the two meshes now gives,

$I_1'(Z_1 + Z_2) - I_2'Z_2 = E$ ...(4)

and $I_2'(Z_2 + Z_3) - I_1'Z_2 = 0$ ...(5)

Substituting the value of I_2' from eqn. (4) into (5) we get

$$I_1'\left[\frac{(Z_1+Z_2)(Z_2+Z_3)}{Z_2} - Z_2\right] = E$$

or $$I_1' = \frac{EZ_2}{(Z_1+Z_2)(Z_2+Z_3)-Z_2^2} \quad ...(6)$$

By equations (3) and (6), we have,

$$I_2 = I_1'$$

which proves the reciprocity theorem.

4.11.5 Maximum Power Transfer Theorem

Q28. State and prove Maximum Power transfer theorem.

Ans :

Statement

"This theorem states that, "A two terminal network will absorb maximum power from a generator if the load impedance is the complex conjugate of the internal impedance of the generator."

Proof :

To establish the theorem, consider the circuit diagram of Fig. (a). Here, the generator impedance

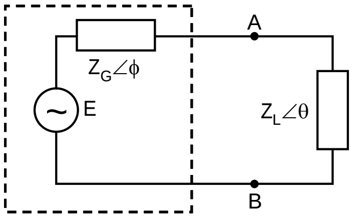


Figure (a)

$$Z_G\angle\phi = R_G + jX_G$$

where phase angle is given by

$$\phi = \frac{X_G}{R_G}$$

And, the load impedance

$$Z_L\ \angle\theta = R_L + jX_L$$

where $\tan\theta = \frac{X_L}{R_L}$

The current I flowing in the circuit is given by

$$I = \frac{E}{(R_L + R_G) + j(X_L + X_G)} \quad ...(1)$$

The power P in the load is

$$P = I^2R_L = \frac{E^2R_L}{(R_L + R_G)^2 + (X_L + X_G)^2} \quad ...(2)$$

Now let us consider the variation of P with X_L. The power P will be maximum when.

$$\frac{dP}{dX_L} = \frac{-2E^2R_L(X_L + X_G)}{[(R_L + R_G]^2 + (X_L + X_G)^2]^2} = 0$$

or, the condition of maximum power is

$$X_L = -X_G \quad \text{... (3)}$$

Then, the power in the load becomes

$$P_{max} = \frac{E^2 R_L}{(R_L + R_G)^2} \quad \text{... (4)}$$

Again, considering the variation of P with R_L. For it, differentiating eq. (2) with respect to R_L and equating derivative to zero. Hence power will be maximum when,

$$\frac{dP}{dR_L} = \frac{E^2(R_L + R_G)^2 - 2E^2 R_L (R_L + R_G)}{(R_L + R_G)^4} = 0$$

or $\quad (R_L + R_G)^2 - 2R_L^2 - 2R_L R_G = 0$

or $\quad R_L = R_G \quad$ ... (5)

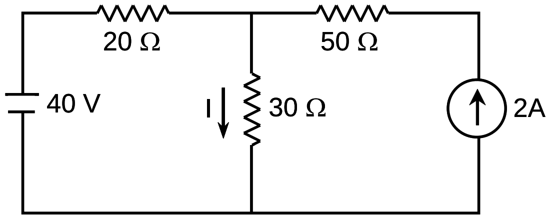
The expression for the maximum power may then be written from eqn. (4), as

$$P_{max} = \frac{E^2 R_L}{(2R_L)^2} = \frac{E^2}{4R_L}$$

Thus the power absorbed by the load will be maximum when the resistive components of both the load and generator impedances are equal and also the reactance of the load is equal but opposite in sign to the reactance of the generator, i.e., when the load impedance is the conjugate of the internal impedance of the generator. Hence if we write the generator impedance in the form $R_G + jX_G$, then the value of load impedance for maximum power is $R_G - jX_G$.

PROBLEMS

1. Find the current I in the circuit given below using superposition theorem.



Sol:

Considering first the voltage source alone, the circuit is reduced to Fig. (a) when current source is open circuited. Then

$$I_1 = \frac{40}{50} = 0.8 \text{ Amp.}$$

Next considering the 2 Amp. current source alone (short circuiting the voltage source), Fig.(b) is obtained.

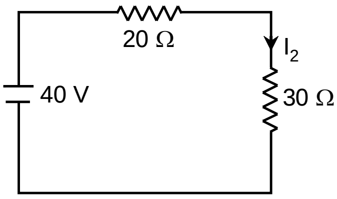


Fig. (a)

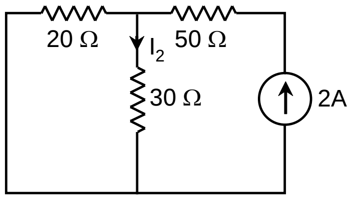


Fig. (b)

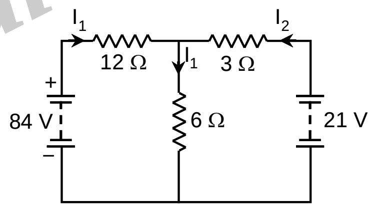
The current flowing through 20 Ω. resistor,

$$I_2 = 2 \times \frac{20}{(20+30)} = 0.8 \text{ Amp.}$$

Applying principle of superposition, the total current through 20 Ω branch

$$I = I_1 + I_2 = 0.8 + 0.8 = 1.6 \text{ Amp.}$$

2. Find the branch currents in the following circuit.



Sol:

Considering first only the 84 volt battery in the circuit by replacing 21 volt battery by its internal resistance (zero), then circuit

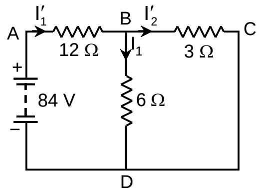


Fig. (a)

reduces to that shown in Fig. (a) The various branch currents are shown in Fig. (a). Then

$$R_{BD} = 6 || 3 = 2\Omega$$

Hence total circuit resistance

$$R = 12 + 2 = 14\Omega$$

Therefore, $I_1 = \frac{84}{14} = 6A$

At point B this current divides in two parts in the inverse ratio of the resistance of two parallel paths i.e. in the ratio 1 : 2. Thus

$$I' = 2\text{ A} \quad \text{and} \quad I_2 = 4A$$

Next considering 21 volts battery in the circuit and replacing 84 volt battery by short circuit (zero internal resistance). Then circuit reduces to that shown in Fig. (b)

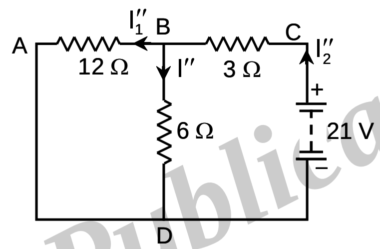


Fig.: (b)

In this case

$$R_{BD} = 6 || 12 = 4\Omega.$$

Total resistance of circuit

$$R' = 3 + 4 = 7\,\Omega.$$

$$I_2'' = \frac{21}{7} = 3A.$$

This current divides at point B in the inverse ratio of the resistances of two parallel paths. Thus

$$I_1'' = \frac{6}{12+6} \times 3 = 1A,\ I'' = \frac{12}{12+6} \times 3 = 2A$$

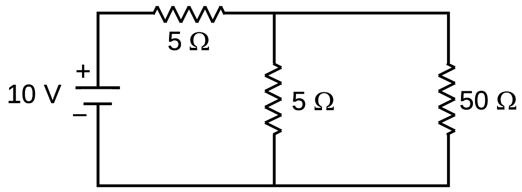
According to superposition theorem, combining the result of Figs. (a) and Fig. (b), we get

$$I_1 = I_1' - I_1'' = 6 - 1 = 5A$$

$$I_2 = I_2' - I_2'' = 4 - 3 = 1A$$

$$I = I' + I'' = 6 + 2 = 8A$$

3. Find the current in 10 Ω resistor as shown in fig. Thevenin's theorem.



Sol:

The open circuit voltage or Thevenin voltage V_{th} after disconnecting the load Fig. (a) is given by

$$V_{th} = I \times 5\Omega$$

$$= \frac{10}{5+5} \times 5 = 5 \text{ volt.}$$

The Thevenin resistance is found by removing load and short circuiting voltage source Fig. (b).

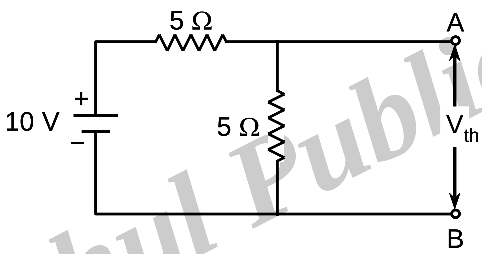


Fig.: (a)

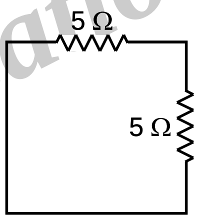


Fig.: (b)

Thus,

$$R_{th} = 5\,||\,5 = 2.5\ \Omega$$

According to Thevenin's theorem, the given circuit can be replaced by the equivalent circuit shown in Fig. (d). Therefore, current through 10Ω load

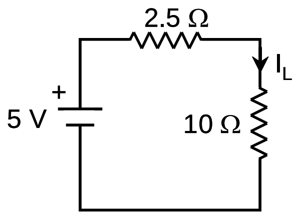
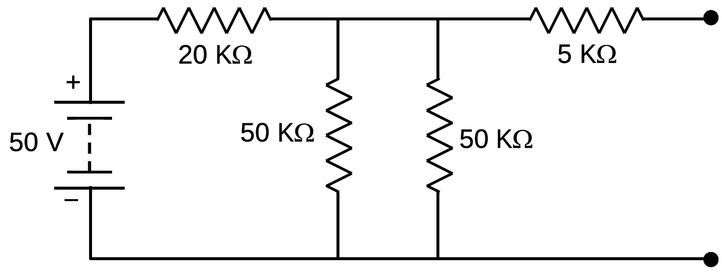


Fig.: (c)

$$I_L = \frac{5}{2.5+10} = 0.4 \text{ Amp}$$

4. Show the Thevenin's equivalent of the following circuit as shown in figure.



Sol:

Since 50 kΩ and 50 kW are in parallel (equivalent resistance 25 25 kΩ) and is in series to 20 kΩ, the source current is

$$I = \frac{50}{20+25} = \frac{10}{9} \text{ Amp.}$$

Therefore, Thevenin voltage

$$V_{th} = I \times 25 = \frac{10}{9} \times 25 = 27.78 \text{ volt.}$$

On short circuiting the voltage source by above Figure reduces to Figure (1). The 5 kΩ and 25 kΩ are in parallel and are in series to 20 kΩ. Then Thevenin resistance

$$R_{th} = 20 \text{ k}\Omega + \frac{25 \times 5}{25+5} \text{k}\Omega$$

$$= 20 + \frac{25}{6} = 24.17 \text{ k}\Omega.$$

Accordingly Thevenin equivalent circuit of Fig. is shown in Fig.

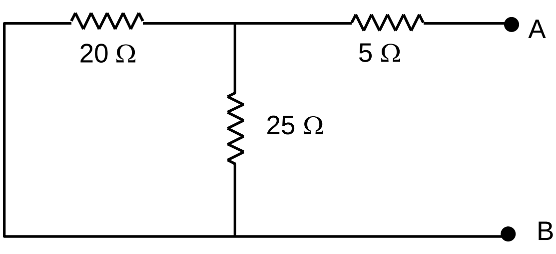


Fig.: (1)

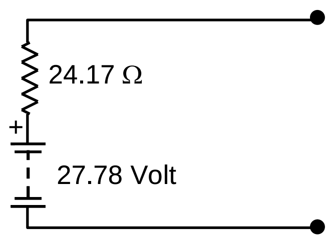
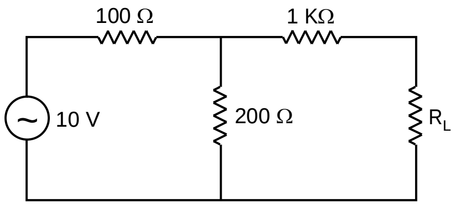


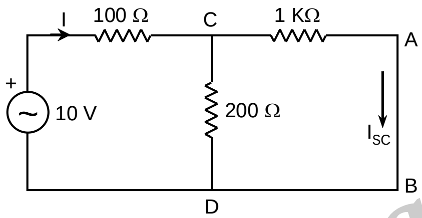
Fig.: (2)

5. What is the Norton's equivalent to the network shown below.



Sol:

(i) On removing load R_L and then putting a short circuit in its place, as shown in above figure, the current flowing through 1 kΩ is also



(a)

the short circuit current I_{sc} (written as I_N). For finding this current, we first find I by simplifying the circuit. As seen total circuit resistance

$$= 100 + (200 \,||\, 1000)$$

$$= 100 + \frac{1000}{6}$$

$$= \frac{1600}{6}\,\Omega = 266.7\,\Omega$$

$$I = 10/266.7 = 37.5 \text{ mA.}$$

This current divides at points C in the inverse ratio of the resistances of the parallel paths. Therefore,

$$I_{SC} = I_N = \frac{200}{200+1000} \times 37.5 = 6.25 \text{ Amp.}$$

(ii) On removing the short circuit, thereby leaving terminals A and B open, and replacing the battery by its zero internal impedance, the resistance R_N of the circuit as viewed back from terminals A and B Fig. (b).

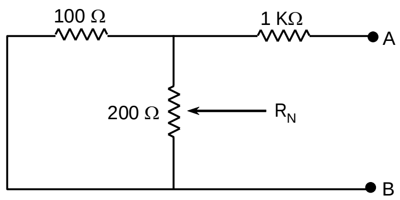


Fig. (b)

$$R_N = 1000 + 200 \,||\, 100$$

$$= 1000 + \frac{200}{3}$$

$$= \frac{3200}{3} = 1066.7\ \Omega$$

$$= 1067\ \Omega$$

Accordingly Norton's equivalent of is drawn in Fig. (c)

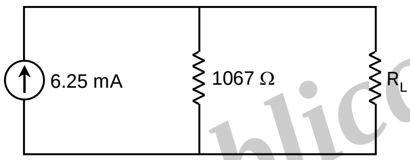
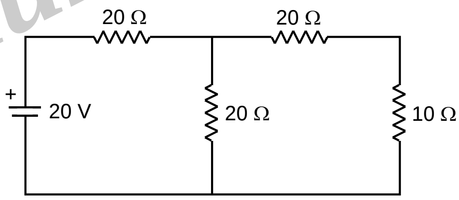


Fig. (c)

6. Find the current in load from the figure given below using Norton's theorem.



Sol:

(i) on removing the load and short circuiting the terminals, where load is removed, is obtained. From it

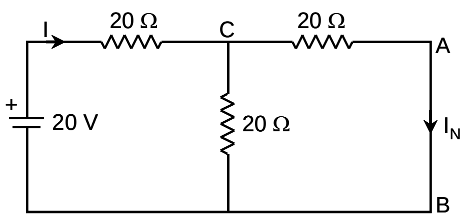


Fig.: (1)

$$I = \frac{20}{20 + (20||20)} = \frac{20}{30} = 0.66 \text{ Amp.}$$

It is divided equally at C and hence

$$I_N = \frac{0.66}{2} = 0.33 \text{ Amp.}$$

(ii) To find R_N load is removed and voltage source is short circuited. (Figure). Hence

$$R_N = (20 \,||\, 20) + 20 = 30\,\Omega$$

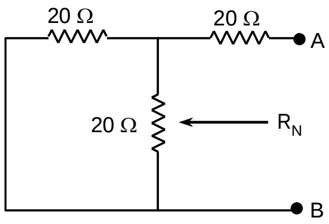


Fig (2)

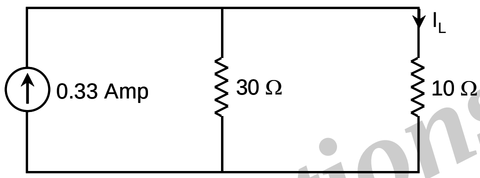
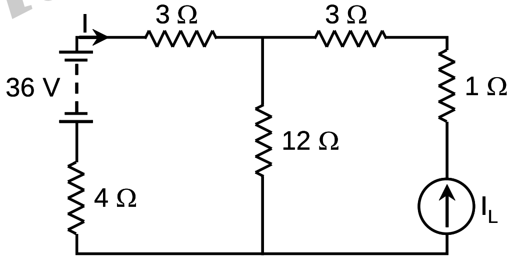


Fig (3)

Accordingly Norton's equivalent circuit is shown in Fig. (3) Current in load resistor 10 Q can be found by proportional current formula.

$$I_L = 0.33 \times \frac{30}{30+10} = 0.2475 \text{ Amp.}$$

7. Prove reciprocity theorem for 1Ω branch from the given figure.



Sol :

From above figure total current in the circuit

$$I = \frac{36}{[(3+1)||12] + 2 + 4} = \frac{36}{3+2+4} = 4 \text{ Amp.}$$

Current can be found by proportional current formula (current is inversely proportional to resistance).

Hence

$$I_L = 4 \times \frac{12}{12+3+1} = 3 \text{ Amp.}$$

According to reciprocity theorem, replacing voltage source in 1Ω branch, Fig. (1) is obtained. From it

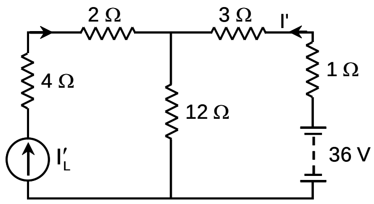


Figure (1)

$$I' = \frac{36}{[(4+2)||12]+3+1} = \frac{36}{4+3+1} = 4.5 \text{ Amp.}$$

$$\therefore \qquad I_L' = 4.5 \times \frac{12}{12+4+2} = \text{ Amp.}$$

Evidently $I_L = I_L' = 3$ Amp.

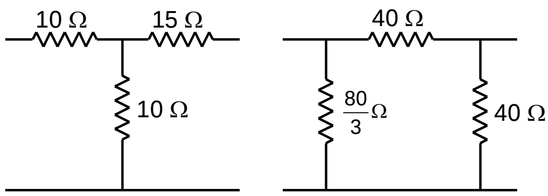
8. The z-parameters of a two port network are z_{12} = 20Ω, z_{12} = z_{21} = 10Ω and z_{22} = 25Ω. Find its equivalent T and π networks.

Sol:

We know that given by

$$z_A = z_{21} - z_{21} = 20 - 10 = 10\Omega$$
$$z_B = z_{22} - z_{12} = 25 - 10 = 15\Omega$$
$$z_C = z_{12} = z_{21} = 10\Omega$$

The equivalent T network will be as shown in Fig. (a)



(a) (b)

By z_A,z_B, z_C can be expressed as

$$Z_{A_\pi} = \frac{(z_1 z_2 + z_1 z_3 + z_2 z_3)}{10} = \frac{(10.5 + 10.10 + 15.10)}{10}$$

$$= (150 + 150 + 100) / 10 = 40\Omega$$

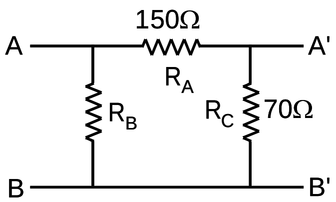
Here $z_1 = z_{AT}$, $z_2 = z_{BT}$, $z_3 = z_{CT}$

$$Z_{B\pi} = \frac{(z_2 z_3 + z_1 z_3 + z_1 z_3)}{z_2} = \frac{400}{15} = \frac{80}{3}\,\Omega$$

$$Z_{C\pi} = \frac{(z_2 z_3 + z_1 z_3 + z_1 z_2)}{z_1} = \frac{400}{10} = 40\Omega$$

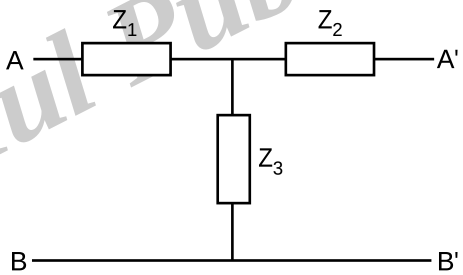
The equivalent π network is shown in Fig (b)

9. Convert the following π network into T-network.



Sol:

The equivalent T-network (see section) is that shown in figure.



The values of z_1, z_2 and z_3 are given by

$$z_1 = \frac{R_A \cdot R_C}{R_A + R_B + R_C} = \frac{150 \times 70}{150 + 70 + 70} = 36.2\Omega$$

$$z_2 = \frac{R_A \cdot R_B}{R_A + R_B + R_C} = \frac{150 \times 70}{150 + 70 + 70} = 36.2\Omega$$

and $$z_3 = \frac{R_C \cdot R_B}{R_A + R_B + R_C} = \frac{70 \times 70}{150 + 70 + 70} = 16.9\,\Omega$$

Short Question and Answers

1. Explain the phenomenon of critical damping.

Ans :

"A circuit with a value of resistor that causes it to be just on the edge of ringing is called critically damped".

Consider a series LCR circuit (one that has a resistor, an inductor and a capacitor) with a constant driving electromotive force (emf) "E". The current equation of the circuit is

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = E$$

This is equivalent : $L\frac{di}{dt} + Ri + \frac{1}{C}q = E$

Differentiating we have

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

This is a second order linear homogeneous equation the corresponding auxiliary equation is,

$$Lm^2 + RM + \frac{1}{C} = 0$$

with roots

$$m_1 = -\frac{R}{2L} + \sqrt{\frac{(R^2 - 4L/C)}{2L}} \; ; \qquad M_2 = -\frac{R}{2L} - \frac{\sqrt{(R^2 - 4L/C}}{2L}$$

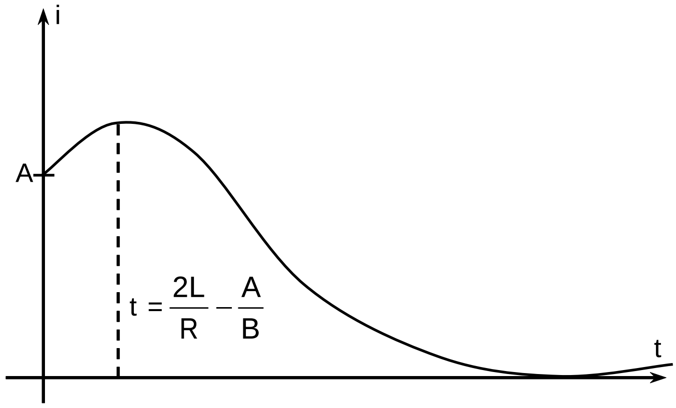
$$= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad\qquad = -\infty - \sqrt{\alpha^2 - \omega_0^2}$$

Now

$\alpha = \frac{R}{2L}$ is called the damping coefficient of the circuit.

Here,

If $R^2 = 4L/C$ then the circuit is said to be critically damped.



Graph of Critically Damped Case

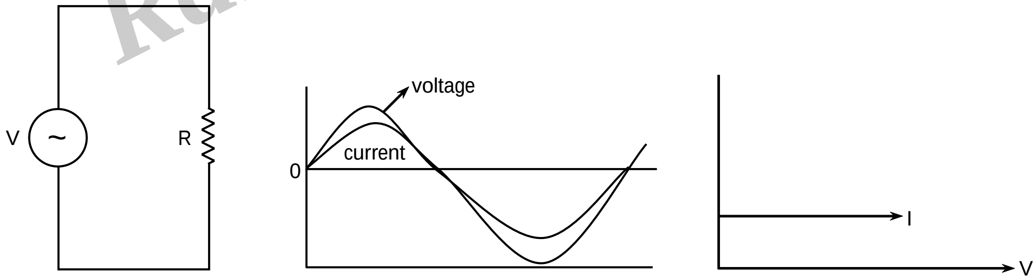
2. Explain about the AC circuit which is having pure resistance?

Ans :

When an alternating voltage is applied across a pure ohmic resistance it produces an alternating current through the resistance.

i) Which is in phase with the voltage.

ii) Whose r.m.s value is given by $I = V/R$.



(a) (b) (c)

If the expression of applied voltage is

$$V = V_0 \sin \omega t \quad \text{... (1)}$$

Then the equation of current is

$$I = I_0 \sin \omega t \quad \text{... (2)}$$

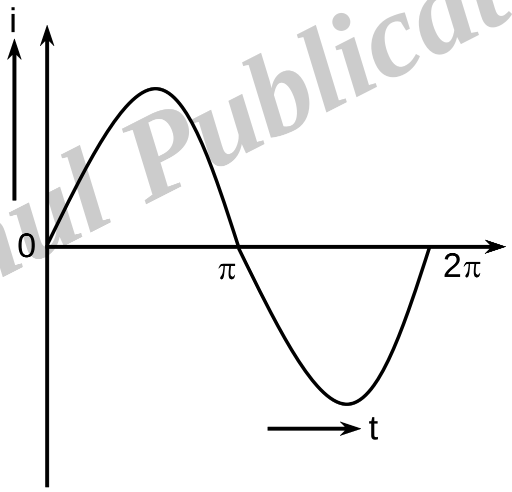
Comparing (1) and (2) equations it is obvious that in a pure resistor the current is always in the same phase as the applied voltage which is graphically represented figure (b). The power dissipated in the circuit in the form of heat is I^2R.

3. Write about Alternating current.

Ans :

The current that changes its magnitude and polarity at regular intervals of time is called an alternating current.

- The major advantage of using the alternating current instead of direct current is that the alternating current is easily transformed from higher voltage to lower level voltage.
- The wave shape of the source voltage and the current flow thought he circuit (i.e., load resistor) is shown in the figure below.



Wave shape of alternating current

- The graph which represents the manner in which an alternating current changes with respect to time is known as wave shape or waveform usually the alternating value is taken along the y-axis and the time taken to the x-axis.
- An alternating current which varies according to the sine of angle ‘θ’ is known as sinusoidal alternating current.

4. Write about comparison between series and parallel resonant circuit?

Ans :

Series resonant circuit		Parallel Resonant Circuit	
1.	An acceptor circuit	1.	A rejector circuit.
2.	Resonant frequency $\gamma_r = \frac{1}{2\pi\sqrt{LC}}$	2.	Resonant frequency $\gamma_r = \frac{1}{2\pi\sqrt{LC}}$
3.	At resonance the impedance is a minimum equal to the resistance in the circuit	3.	At resonance the impedane is maximum nearly equal to infinity.
4.	Selective	4.	Selective
5.	Used in the turning circuit to separate the wanted frequency from the incoming frequencies by offering low impedance at that frequency	5.	Used to present a maximum impedance to the wanted frequency, usually in the plate circuit of value.

5. What is a single phase motor?

Ans :

A single phase motor is an electrically powered rotary machine that can Aurn electric energy into mechanical energy.

- It works by using a single-phase power supply. They contain two types of wiring hot and neutral.
- Their power can reach 3 kw and supply voltages vary in unison.
- They only have a single alternating voltage. The circuit works with two wires and the current that runs across them is always the same.
- In most cases these are small motors with a limited torque. However, there are single phase motors with a power of up to 10 hp that can work with connections of upto 440 V.

- This type of motor is used mainly in homes, offices, stores and small non-industrial companies. Their most common uses include home appliance.
- These do not generate a rotating magnetic field they can only generate an alternate field. Which means that they need a capacitor for startup.

6. What do you know about a DC motor?

Ans :

- A DC motor is any of a class of rotary electrical motors that converts direct current electrical energy into mechanical energy.
- The most common types rely on the forces produced by magnetic fields. Nearly all types of DC motors have some internal mechanism, either electromechanical or electronic, to periodically change the direction of current in part of the motor.
- DC motor were the first form of motor widely used, as they could be powered from existing direct-current lighting power distribution systems.
- A DC motor's speed can be controlled over a wide range, using either a variable supply voltage or by changing the strength of current in its field windings. Small DC motors are used in tools, toys, and appliances.
- Larger DC motors are currently used in propulsion of electric vehicles, elevator and hoists, and in drives for steel rolling mills.
- The advent of power electronics has made replacement of DC motors with AC motors possible in many applications.

7. Explain about 3-phase AC-motors?

Ans :

A three phase alternator is shown in the diagram below. It consists of three similar rectangular coils displaced equally from each ones, i.e., 120º. Each oil is provided with its own brushes and slip rings.

Three emfs are generated when they are rotated at a constant velocity in a uniform magnetic field. They are of the same frequency and of equal values. Each of the three sources of voltage is called a phase. Each phase voltage lags 120º behind that of the one preceding it [Figure (b), (c)]

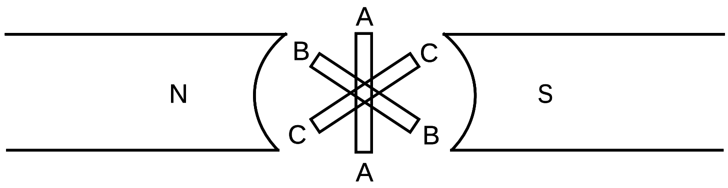


Figure (a)

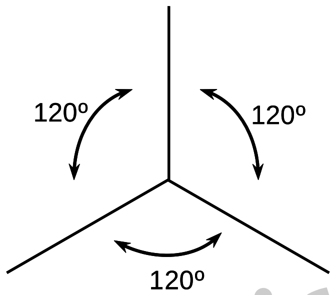


Figure (b)

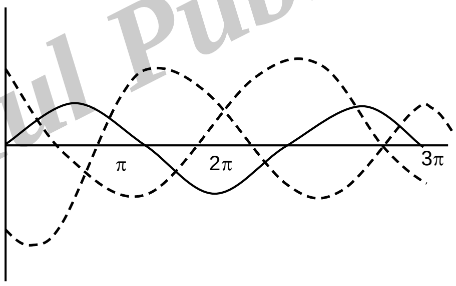


Figure (c)

- The instantaneous values of emf in each coil may be written as:

$$E_1 = E_0 \sin \omega t$$

$$E_2 = E_0 \sin \left(\omega t + \frac{2\pi}{3}\right)$$

$$E_3 = E_0 \sin \left(\omega t + \frac{4\pi}{3}\right)$$

- It can be used to supply a three phase system of thee single phase circuits.

8. Advantages of 3-phase System

Ans :

i) In 3-phase alternators the total power does not fluctuate. While in a single phase generator the current fluctuates.

ii) The output power of a 3-phase alternator is always greater than that of a single phase generator of the same size.

iii) Three phase system is superior for transmission of electrical energy. It involves lot of saving.

9. State thevenin's theorem ?

Ans :

Any combination of batteries and resistances with two terminals can be replaced by voltage source and single series resistor.

The voltage source is the open circuit voltage at the terminals A & B and the value of single series resistor is open circuit voltage divided by current with the terminals short circuited.

10. State & Explain reciprocity theorem ?

Ans :

If an e.m.f. applied in one mesh of network of linear impedance produces a certain current in the second mesh, then the same e.m.f. acting in the second mesh will give an identical current in the first mesh.

Proof : To establish the theorem, let us consider the arrangement, shown in Fig. 1.5(a), where the source of e.m.f. E is in the first mesh. Let the current in the first and second mesh be I_1 and I_2 respectively.

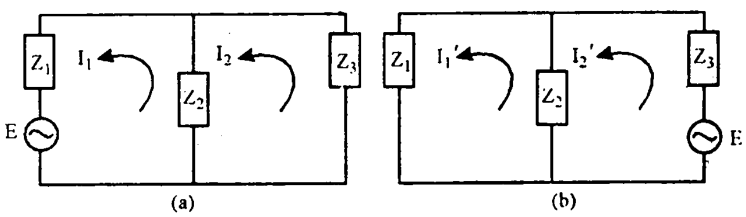


Fig. 1.5

Applying Kirchoff's second law to the two meshes, we get

$I_1(Z_1 + Z_2) - I_2Z_2 = E$ ...(1)

and $I_2(Z_2 + Z_3) - I_1Z_2 = 0$ ...(2)

Substituting the value of I_1 fro eqn. (2) into (1), we get

$$I_2\left[\frac{(Z_1+Z_2)(Z_2+Z_3)}{Z_2}-Z_2\right]=E$$

or $$I_2 = \frac{EZ_2}{(Z_1+Z_2)(Z_2+Z_3)-Z_2^2} \quad ...(3)$$

Again considering the network of Fig. 1.6 (b) in which the source of e.m.f. is in the second mesh. Let the current in the first and second mesh be I_1' and I_2' respectively. Application of Kirchoff's second laws to the two meshes now gives,

$I_1'(Z_1 + Z_2) - I_2'Z_2 = E$ ...(4)

and $I_2'(Z_2 + Z_3) - I_1'Z_2 = 0$ ...(5)

Substituting the value of I_2' from eqn. (4) into (5) we get

$$I_1'\left[\frac{(Z_1+Z_2)(Z_2+Z_3)}{Z_2}-Z_2\right]=E$$

or $$I_1' = \frac{EZ_2}{(Z_1+Z_2)(Z_2+Z_3)-Z_2^2} \quad ...(6)$$

By equations (3) and (6), we have,

$I_2 = I_1'$

which proves the reciprocity theorem.

11. What are input and output parameters of network ?

Ans :

The behaviour of a two port network may be expressed in terms of the four electrical quantities-the input and output currents (I_1, I_2) and the voltages (V_1, V_2) at the two ports (Fig. 1.8). The functional

Fig. : 1.8

relationship between these electrical quantities may be expressed in different ways. Among the four terminal variables, any two may be arbitrarily chosen as independent variables, leading to two equations which may be solved for the other two dependent variables. Out of the twelve possible independent variable pairs, three pairs [eg. (I_1, I_2); (V_1, V_2) and (I_1, I_2)] lead to three sets of circuit parameters and have been frequently applied in the study of active electronic devices.

12. Define ABCD parameters. Determine its values.

Ans :

The transmission parameters express voltage and current at port 1 in terms of voltage and current at port 2 of a two port network, as given by the equations :

$$\upsilon_1 = A\upsilon_2 - Bi_2 \quad ...(1)$$

$$i_1 = C\upsilon_2 - Di_2$$

In the matrix form

$$\begin{bmatrix} \upsilon_1 \\ i_1 \end{bmatrix} = \begin{Bmatrix} A & B \\ C & D \end{Bmatrix} \begin{Bmatrix} \upsilon_2 \\ -i_2 \end{Bmatrix}$$

A, B, C and D are called the transmission or ABCD parameters, because these parameters are widely used in transmission line theory. In transmission line theory, input port 1 - 1' is called the sending end and the output port 2 - 2' is called receiving end.

With port 2 open-circuited, $i_2 = 0$ and eqns. (1) and (2) give,

$$A = \left|\frac{\upsilon_1}{\upsilon_2}\right|_{i_2=0}$$

= open circuit reverse voltage gain

and $C = \left|\frac{i_1}{\upsilon_2}\right|_{i_2=0}$ ...(3)

= open-circuit transfer admittance (= $1/z_{21}$)

With port 2 short-circuited, $v_2 = 0$ and eqns. (1) and (2) give,

$B = \left|\frac{\upsilon_1}{-i_1}\right|$

= short - circuit transfer impedance (= $-1/y_{21}$) ...(4)

and $D = \left|\frac{i_1}{-i_2}\right|_{\upsilon_2=0}$

= short-circuit reverse current gain

13. State superposition theorem.

Ans :

In a network containing linear impedances (impedance of which value does not change with the flow of current) and energy sources (generators), the current flowing at any point is the vector sum of the currents which would exist if each source of e.m.f. were considered separately, all the other sources being replaced on that time by their impedances.

14. Explain the types of power sources.

Ans :

Powers

Power sources can be classified as independent sources and dependent sources.

(i) Independent Sources

An independent source maintains the same voltage (or) current regardless of other elements present in circuit. Its value is either DC or AC. The strength of the voltage (or) current is not changed by any variation in the connected network.

(ii) Dependent Sources

Dependent sources depend upon particular element of the circuit for delivering voltage or current depending upon type of source it is .

15. Differentiate between Active and Passive elements.

Ans :

	Active elements		Passive elements
(i)	Active elements deliver power (or) energy to the circuit.	(i)	Passive elements ulilizes power (or) energy in the circuit.
(ii)	Diodes, tranistors SCR, Integrated circuits are Active elements.	(ii)	Resistors, Capacitors, inductors are passive elements
(iii)	These elements produce energy in the form of voltage (or) current	(iii)	These devices stores energy in the form of voltage (or) current
(iv)	They control the flow of current	(iv)	They can not control flow of current
(v)	They require external source for operation	(v)	They do not require any external source for operation.
(vi)	These elements are energy donor	(vi)	These elements are energy acceptor

Choose the Correct Answer

1. According to superposition theorem [b]

   (a) $I_1' = I_1'' + I_1$ (b) $I_1 = I_1' + I_1''$

   (c) $I_1'' = I_1 + I_1'$ (d) None

2. Expression for the load current in Therening theorem [a]

   (a) $I_2 = \frac{E^1}{Z^1 + Z_2}$ (b) $I_2 = \frac{E}{Z + Z_2}$

   (c) $I_2 = \frac{E^1}{Z + Z_2}$ (d) None

3. In case of z-parameter independent variables are [b]

   (a) V_1, V_2 (b) I_1, I_2

   (c) I_1, V_2 (d) V_1, I_2

4. Expression for output impedance in z- parameter [d]

   (a) $z_{11} = \frac{v_1}{i_1}$ (b) $z_{21} = \frac{v_2}{i_1}$

   (c) $z_{12} = \frac{v_1}{i_2}$ (d) $z_{22} = \frac{v_2}{i_2}$

5. y- parameters in matrix form [c]

   (a) $\begin{bmatrix} i_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} i_2 \\ v_2 \end{bmatrix}$ (b) $\begin{bmatrix} i_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_1 \end{bmatrix}$

   (c) $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ (d) None

6. Reverse voltage ratio with input circuit open is represented as [a]

   (a) h_{12} (b) h_{21}

   (c) h_{22} (d) h_{11}

7. Which expression is correct one [a]

(a) $z_1 = \frac{z_A + z_B}{z_A + z_B + z_C}$ (b) $z_1 = \frac{z_A \div z_B}{z_A + z_B + z_C}$

(c) $z_1 = \frac{z_A . z_B}{z_A + z_B + z_C}$ (d) $z_1 = \frac{z_A}{z_A + z_B + z_C}$

8. Which one is incorrect expression [d]

(a) $z_A = \frac{z_1z_2 + z_1z_3 + z_2z_3}{z_3}$ (b) $z_B = \frac{z_2z_3 + z_1z_3 + z_1z_2}{z_2}$

(c) $z_c = \frac{z_2z_3 + z_1z_3 + z_1z_2}{z_1}$ (d) $z_A = \frac{z_2z_3 + z_1z_3 + z_1z_2}{z_2}$

9. The value of B = [c]

(a) $\left|\frac{-v_2}{i_2}\right|_{v_1=0}$ (b) $-\left|\frac{v_1}{i_1}\right|_{v_1=0}$

(c) $\left|\frac{-v_1}{i_1}\right|_{v_2=0}$ (d) None

10. y_B = ________ [d]

(a) $\frac{1}{z_A}$ (b) $\frac{1}{z_C}$

(c) z_C (d) $\frac{1}{z_B}$

Fill in the blanks

1. When the current in all branches is directly propotional to voltage then such a network is known as ________.
2. A Net work with circuit elements without any energy soruce is known as ________ net work.
3. In thevenin's theorem any combination of batteries and resistances can be replaced by ________ source.
4. According to Norton theorem several energy sources & resistance can be replaced by ________.
5. The power absorbed by the load will be ________ when resistive components of load & generator are equal .
6. The value of load impedance for maximum power is ________.
7. Ratio of input current to input voltage in y-parameter is known as ________.
8. Independent variables in h-parameter ________.
9. Forward current ratio with output short circuited is designated as ________.
10. The value of D- parameter is = ________.

ANSWERS

1. Linear
2. Passive
3. Voltage
4. Single constant current generator
5. Maximum
6. $R_G - j\,X_G$
7. Input admittance
8. (I_1, V_2)
9. h_{21}
10. $\left[{}^{e_1}\!/_{i_2}\right]_{v_2=0}$

One Mark Answers

1. Define maximum transfer theorem ?

Ans :

A two terminal network will absorb maximum power from generator if load impedance is complex conjugate of internal impedance of generator.

2. Define forward transfer impedance in z-parameter.

Ans :

Ratio of output voltage to input current denoted as z_{21}.

3. Express the π network elements interms of y- parameter.

Ans :

$y_A = y_{21}$

$y_B = y_{11} + y_{21}$, $y_c = y_{22} + y_{21}$

4. Define Reciprocity theorem ?

Ans :

If e.m.f applied in one much of network of linear impedance produces a certain current in second much, then the same emf acting in the second mesh will give identical current in first mesh.

5. Define network.

Ans :

An Electrical circuit containing impedances and generators is known as network.

FACULTY OF SCIENCE

B.Sc. III - Semester (CBCS) Examination

Model Paper - I

ELECTROMAGNETIC THEORY

Physics - Paper - III

Time : 3 Hours] [Max. Marks : 80

PART - A (8 × 4 = 32 Marks)

[Short Answer Type]

Note : Answer any EIGHT of the following questions

ANSWERS

1. Define coulomb's law. **(Unit-I, SQA-4)**
2. Explain about Electric flux. **(Unit-I, SQA-7)**
3. The charge on the spherical conductor is 3×10^{-9} C. Radius of conductor is 0.1 m. Find the potential of spherical conductor. **(Unit-I, Prob. 3)**
4. Explain Bio-Savart law. **(Unit-II, SQA-1)**
5. Define charge sensitivity & current sensitivity. **(Unit-II, SQA-9)**
6. The magnetic susceptibility of medium is 948×10^{-11}. Calculate the permeability and relative permeability. **(Unit-II, Prob. 4)**
7. What is displacement current? **(Unit-III, SQA-4)**
8. Explain the types of polarisation. **(Unit-III, SQA-10)**
9. Write boundary conditions for D, E, B, H. **(Unit-III, SQA-13)**
10. Write about comparison between series and parallel resonant circuit? **(Unit-IV, SQA-4)**
11. Explain about the AC circuit which is having pure resistance? **(Unit-IV, SQA-2)**
12. Define ABCD parameters. Determine its values. **(Unit-IV, SQA-12)**

PART - B (4 × 12 = 48 Marks)
[Essay Answer Type]

Note : Answer all the following questions

13. (a) State Gauss law in electrostatics and derive its Integral and differential form ? **(Unit-I, Q.No. 6)**

(OR)

(b) Explain about energy density in electrostatic field? **(Unit-I, Q.No. 17)**

14. (a) Explain the various properties of magnetic field. **(Unit-II, Q.No. 5)**

(OR)

(b) Derive the Expression for energy stored in magnetic field interms of 'L' and 'i'. **(Unit-II, Q.No. 12)**

15. (a) State and Explain Faraday's laws of Electromagnetic Induction. Derive the differential and Integral forms of Faraday's Law. **(Unit-III, Q.No. 1)**

(OR)

(b) Explain energy conservation law in electromagnetism? **(Unit-III, Q.No. 17)**

16. (a) State and prove Thevenin's theorem. **(Unit-IV, Q.No. 25)**

(OR)

(b) Explain about power in AC circuit. **(Unit-IV, Q.No. 9)**

FACULTY OF SCIENCE

B.Sc. III - Semester (CBCS) Examination

Model Paper - II

ELECTROMAGNETIC THEORY

Physics - Paper - III

Time : 3 Hours] [Max. Marks : 80

PART - A (8 × 4 = 32 Marks)

[Short Answer Type]

Note : Answer any EIGHT of the following questions

ANSWERS

1. Write about Electric lines force? **(Unit-I, SQA-6)**
2. Explain the Concept of Electric Potential. **(Unit-I, SQA-9)**
3. What is Electripotential at the surface of nucleus of gold? The radius of nucleus is 6.6×10^{-15} m. The atomic number of gold is 79. **(Unit-I, Prob. 4)**
4. Define energy stored in magneticfield. **(Unit-II, SQA-8)**
5. Define Amperes law. **(Unit-II, SQA-7)**
6. An infinitely long conductor carries a current of 10 mA. Find the magnetic field intensity at a point 20 cm away from it. **(Unit-II, Prob. 6)**
7. Mention the differences between self induction and mutual induction? **(Unit-III, SQA-3)**
8. Define Mutual Induction. **(Unit-III, SQA-6)**
9. Explain the terms polarization reflection & transmission. **(Unit-III, SQA-14)**
10. Explain the phenomenon of critical damping. **(Unit-IV, SQA-1)**
11. What is a single phase motor? **(Unit-IV, SQA-5)**
12. State superposition theorem. **(Unit-IV, SQA-13)**

PART - B (4 × 12 = 48 Marks)

[Essay Answer Type]

Note : Answer all the following questions

13. (a) Give mathematical description on of electric field at points inside, outside and on the surface of uniformly charged cylinder? **(Unit-I, Q.No. 10)**

(OR)

(b) Derive Expression for potential & Electric field due to Uniformly Circular disc. **(Unit-I, Q.No. 19)**

14. (a) Obtain the relationship between magnetic flux density 'B' magnetising force 'H' intensity of magnetisation? **(Unit-II, Q.No. 6)**

(OR)

(b) Derive an expression for the magnetic induction due to long straight conductor carrying current. **(Unit-II, Q.No. 3)**

15. (a) State and Explain Lenz's Law obtain an Expression for induced E.M.F. **(Unit-III, Q.No. 2)**

(OR)

(b) Derive Boundary conditions for D, B, E, and H. **(Unit-III, Q.No. 12)**

16. (a) Write a brief note on Growth of current in LR circuit? **(Unit-IV, Q.No. 1)**

(OR)

(b) Express the elements of π-network interms of Y and ABCD parameter. **(Unit-IV, Q.No. 21)**

FACULTY OF SCIENCE

B.Sc. III - Semester (CBCS) Examination

Model Paper - III

ELECTROMAGNETIC THEORY

Physics - Paper - III

Time : 3 Hours] [Max. Marks : 80

PART - A (8 × 4 = 32 Marks)

[Short Answer Type]

Note : Answer any EIGHT of the following questions

		ANSWERS
1.	Obtain the expression for electric field due to infinite sheet of charge.	**(Unit-I, SQA-3)**
2.	Define Electric field and Intensity of electric field.	**(Unit-I, SQA-5)**
3.	A point charge $q = 2 \times 10^{-7}$ coulomb is placed at centre of spherical cavity of radius 3.0 cm in metal piece . Find Electric intensities at a + b.	**(Unit-I, Prob. 2)**
4.	What are the properties of magnetic field?	**(Unit-II, SQA-2)**
5.	Define magnetic field (B), magnetic flux.	**(Unit-II, SQA-5)**
6.	Write the uses of Ballastic Galvanometer.	**(Unit-II, SQA-10)**
7.	Write Maxwell's equations in differential and integral forms.	**(Unit-III, SQA-8)**
8.	Derive the expression for velocity of light in vacuum and medium?	**(Unit-III, SQA-11)**
9.	State and Explain Lenz's law?	**(Unit-III, SQA-1)**
10.	Differentiate between Active and Passive elements.	**(Unit-IV, SQA-15)**
11.	Advantages of 3-phase System	**(Unit-IV, SQA-8)**
12.	Explain about 3-phase AC-motors?	**(Unit-IV, SQA-7)**

PART - B (4 × 12 = 48 Marks)

[Essay Answer Type]

Note : Answer all the following questions

13. (a) Show that electric field is conservative in nature. **(Unit-I, Q.No. 11)**

(OR)

(b) Derive the Expression for Electric field from potential due to charged Spherical conductor. **(Unit-I, Q.No. 18)**

14. (a) Calculate the intensity of magnetic field at a point on the axis of circular coil carrying current? **(Unit-II, Q.No. 10)**

(OR)

(b) Write about the damping correction of ballastic galvanometer **(Unit-II, Q.No. 20)**

15. (a) Define Mutual Induction. Derive an expression for the coefficient of mutual induction between a pair of coils. **(Unit-III, Q.No. 6)**

(OR)

(b) Derive the Maxwell's Electromagnetic wave equation for E & B in dielectronic medium and vacuum (or) free space. **(Unit-III, Q.No. 11)**

16. (a) Explain the LCR circuit in series and parallel resonant condition? **(Unit-IV, Q.No. 11)**

(OR)

(b) Explain about 3-phase AC-motors? **(Unit-IV, Q.No. 13)**